PÓSA METHOD: TALENT NURTURING IN WEEKEND MATH CAMPS

Péter Juhász and Dániel Katona
Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences
& ELTE Eötvös Loránd University, Hungary

Abstract*: Lajos Pósa has been organizing weekend mathematics camps for highly gifted students to foster their development with a special method since 1988. During these 30 years, he and his disciples led more than 350 camps for more than 1500 students. Students' work dominantly takes the form of a special team work, and is based on a five-year-long coherent curriculum organized around problem threads, which form a complex web. These threads run parallel, in 'harmony', supplementing and assisting each other's development. Effectiveness of the camps is reflected in the fact that in the past 25 years, almost all members of the Hungarian IMO teams were participated in these camps before.*

Key words: mathematically gifted learners, learner autonomy, discovery learning, inquiry based mathematics education, connected task-design, problem thread, teamwork

BACKGROUND AND THE BEGINNINGS OF THE PÓSA CAMPS

In the beginning of the 1970s, Lajos Pósa joined the "complex mathematics education" movement in Hungary led by Tamás Varga (Halmos & Varga, 1978), having also been inspired by the ideas of many influential Hungarian mathematician and mathematics educators of the middle of the 20th century, like the work of György Pólya on problem solving (Polya, 1957). In the 1980s, Pósa became a member of the group at the Rényi Institute of Mathematics, led by János Surányi, studying the applicability of the Varga method in high schools. Between 1982 and 1991, he taught 2 high school classes for 4 years each, during which he developed teaching materials with the aim of making the learning of mathematics enjoyable, and focusing not on mechanically applying formulas and algorithms, but on relaxed and cheerful, autonomous and logical thinking.

He then turned to talent care, which has had a well-established tradition in Hungary. There were a national network of special mathematics circles for students and several 'high-quality' competitions were (and are) organized, as well as a mathematics journal for students, the KöMaL has been published since 1893, currently also available in it's website (http://www.komal.hu/info/bemutatkozas.e.shtml). However, Pósa felt that the school environment, the short learning sessions (maximum 90 minutes study circle sessions) do not enable students to be involved in the intense thinking that highly talented students need. Besides he did not consider preparing for competitions to be at the core of nurturing talent, but rather that students shall think on exciting, interesting problems and discover the beauties of mathematics autonomously.

Pósa launched a talent nurturing weekend math camp for one single group in 1988. The first author was a member of this group of roughly 15 students. Camps' schedule had been evolved continuously, in these early times the main activity was students' individual autonomous work, together in a large room. While in these first camps Lajos Pósa was the only teacher and organizer, there are presently 2-7 'assistant teachers offering help to the 'camp leader'. For more details, see (Győri & Juhász, 2018)

PARTICIPATING STUDENTS

Hungary has an elaborate system of mathematics competitions, with individual and team ones, multiple choice tests and ones requiring detailed written solutions or proofs. Based on the results of the most prestigious competitions, and recommendations of teachers and parents we invite approximately 60 students (grade 7) to the beginner groups of our math camp. As from 2010 we launch 2 beginner groups at grade 7 each year, which allows us to invite approximately 60 students altogether, there are some other, much less exact invitation methods, based on teachers' and parents' recommendations. In these cases, students have to solve a set of 'entrance' problems, as we think that participating in the camps among peers being substantially 'better' is not good for the students.

All camps are free for all students, so that no gifted student left out for financial reasons. There is a continuously growing demand for participation.

There are 10-12 groups running parallel at the same time (1 or 2 at each grades 7-12), that is roughly 300-350 students are involved. We organize yearly 2.5 camps for one group, in average, and the groups usually have 10-13 camps from grade 7 until grade 11 or 12.

A TYPICAL CAMP – WHAT DOES IT LOOK LIKE?

A typical camp starts at 5 pm on Friday, and ends at 3 pm on Sunday, with a total of 13-15 full hours spent purely on math, at least half of which is devoted to students' autonomous thinking. The mathematical work can typically be categorized into 4+1 types of situations: 1) 'Autonomous Thinking' in small groups, with special rules (see section on Special Team Work); 2) 'Plenary Discussion' on problems, solutions and conclusions; 3) 'Individual Thinking' (typically) on 'lightning round' questions (with short time limit), physically the whole group together; 4) 'Team Contest': autonomous thinking in small teams with different rules and partially different aims than that of 'Autonomous Thinking'; +1) 'Homework', with problems to be thinking on between two camps.

Type 1) will be discussed in details in the section on Special Team Work.

During 'plenary' discussions (type 2) we talk about homework and those problems that have been solved by enough students during autonomous group thinking. Students present their solutions, often several different ones (if there are). It is prominently crucial that students also pose new questions in view of the discussed solutions. As a significant part of our pedagogy, we consider students' learning to pose good questions very important. Posing questions are rewarded by chocolate and students regularly vote for student-posed new questions to be built into the camp's curriculum, as they wish to think them on. These questions are then named after the students who posed them. Over the years, camps' curriculum is being broadened by many student-posed problems. Discussing problems is, at times, supplemented by 'tales' about connected moments in the history of mathematics or current problems and results in mathematical research.

Once in every camp, in average, students think on problems individually, but at the same room (type 4). These are usually blocks for 'lightning round' questions', with the aim of evoking previously discussed topics or consider it from a different point of view. These may also be loaded questions (trick questions), trying to lure solvers into traps, in order for calling attention to the limits of certain previously used methods or techniques, that they can only be used under certain circumstances. High numbers of good answers to 'lightning round' questions are also rewarded by chocolate.

Saturday afternoons are always devoted to the team contests (type 3), where the special rules of special team work (type 1) do not apply, instead the teams work with the widespread methods of 'group work'. Teams need to do their best in solving 5 problems in usually 2.5-3 hours. These problems are special in that teams also compete with each other problem by problem (not only on the whole). They need to find the sole different weight with as few number of weighting as they can (balance puzzles); select as many elements from a set, with a particular property to be hold, as possible; construct a triangle with a maximized area or perimeter, under certain boundary conditions; solve a certain problem with the lowest cost possible; etc. Initially, there were no team contests in the camps, as competition is (almost) the last thing we connect to the real nature of doing mathematics. Instead, we regard it as mainly characterized by relaxed and deep thinking, in which time does not play an important role. Moreover, we prioritize the importance of each individual's thinking on problems appropriate to their own 'level', and that they shall, first of all, compete with themselves, and not with others. However, it turned out that children are energized by these team contests in an extremely intense and positive way, so it has become an integral part of the camps, but still not significant in time, also rewarded by chocolate.

SPECIAL TEAM WORK

In the Pósa method, the main focus is on students' autonomous thinking, that as many students shall discover as many ideas autonomously as possible. This may support the need for a dominance of students' individual work. However, our experience has shown something different. In the first camps, students were mainly thinking individually, together at the same place; keeping track of each other's progress, as their competitive spirit made them being interested in which problem was solved by whom. Faster performing students tended to work less hard, after realizing their relatively outstanding achievements; while - which we consider a more serious problem - slower performing students were discouraged, they had lost their interest in working on, after realizing their relatively slow progress. We, in turn, think time (speed of progress) does not play such an important role. That is the reason why Pósa has changed the main form of students' activity into thinking in groups of 2-4, in separate rooms, having no information on the other groups' progresses.

Being the whole camp together, usually 5-8 problems are posed at the same time, and then the teams go to their separate places 'alone'. The recommended structure of team work is the following: 1) each student think individually on each problem (with a freedom of choosing the order); and if they think they have solved one, they shall not tell it to their peers who don't; 2) if a problem is considered to be solved by all team members, they can discuss their ideas; 3) if a number of them has been struggling with solving the same problem, without much results, than they can brainstorm together. However, during stage 3), if a student discovers a crucial idea leading to the solution, they need to quit the discussion, allowing their peers to discover it too. In grade 7, this method does not work smoothly, as students usually do not understand why they are not free to talk when working in groups. It depends on the individual students, and the particular groups when they start to realize that this 'rule' is for their own interest; however, until the second term of grade 8, this system is usually well developed. Around grade 10, many students go to the other extreme, and though teams exist, they insist on solely autonomous work with a decreased wish for cooperation, they want to solve each problem themselves. However there are some, who continue doing the 'restricted cooperation' (or 'limited group work').

FEATURES OF THE PROBLEM SET – THE WEB OF PROBLEM THREADS

Research on reconstructing the method in a theoretical level

The Pósa method as a complex teaching practice with special task-design and classroom management elements is based in "teaching as craft knowledge" (Watson & Ohtani, 2015, p. 5) and has never been completely developed on a theoretical level with a conceptual basis of Pósa's didactical conceptions. From 2016, research is being carried out by the second author mainly to reconstruct the theoretical model behind the method as an "intermediate-level framework (Watson & Ohtani, 2015, pp. 19-81) or as a variant of already established frameworks, such as the Anthropological Theory of the Didactics (ATD) (Bosch & Gascón, 2014), so that the concepts of our introduced WPT model (see next paragraph) is studied and explained by ATD tools, such as praxeological analysis (Katona, 2018, January).

Our research is also to contribute to the international discourse on conceptualizing Inquiry Based Mathematics Education (Artigue & Blomhøj, 2013; Bosch & Winsløw, 2016). As a first step of theorizing, a mathematical content analysis was conducted, focusing on the connection between the problems, as well on the attribute of the learning goals of the Pósa method that they are essentially realized in long-term. This resulted in introducing the term and 'web of problem threads' for a particular task-design and curriculum development tool (Katona & Szűcs, 2017).

PROBLEM THREADS – A CONNECTED TASK-DESIGN TOOL FOR (A TYPE OF) INQUIRY-BASED LEARNING MATHEMATICS

Problems posed at the Pósa camps are connected to each other in multiple ways. A set of connected problems, with a not completely fixed order, is called a 'problem thread'. These connections manifest themselves in the form of common features of the problems, that they foster the development of specific ways of mathematical thinking, or methods, called the 'kernels of the threads'. Although these kernels are usually not of traditional, content-based categorizations, that is, threads are typically not created by problems just belonging to the same content areas, such as geometry or number theory, there are some content-based threads, like 'remainders', as we think that modular arithmetic and it's effective application in problem solving shall be clear to every student. Threads typically cover many mathematical content areas. Some important kernels are: 'recursive thinking', '(proof of) impossibility' and 'movement'. These threads run for years; there are threads which are part of the curriculum from the first camp until the last one. Students' attention is also drawn frequently to the connections between the problems, we regularly ask in which task they have met a similar line of thought before. Students are asked to tag the problems by the names of the kernels for categorizing them.

Threads are not separated. They regularly cross each other at problems belonging to these crossing threads, forming the 'web of problem threads' (WPT) (Katona & Szűcs, 2017). Astonishing moments of the camps are created by solving the problems that connects two or more threads. Students then realize that in mathematics, apparently remote areas may also be closely connected, and finding solutions may be facilitated by ideas that we may not think of right in the beginning. Threads run simultaneously, crossing each other at the appropriate moments, then go on. Sometimes we do not meet a thread for a longer time, until it comes back.

Using Pósa's allegory it's like a well-composed piece of polyphonic music. Phrases can be heard simultaneously, perfectly supplementing each other. At time, some phrases are quieter, later may be louder, contributing to an ideal musical experience. However, the composition has and will never be finished. The WPT is being developed from the beginnings, and has an inherently dynamic nature. Problems may disappear, for some time (for some groups of students); new ones appear regularly, often posed by students.

EXAMPLE FOR A PROBLEM THREAD

There are a lot of different problem threads in Pósa method. Here we introduce a tiny one to understand what a thread means.

Problem 1. Does there exist an arithmetic progression with six terms so that each pair of terms is relatively prime? (The terms of the AP are distinct positive integers.)

After the students solved this problem we ask them to pose good questions. The following questions are the most typical ones:

Problem 2. What is the maximum length of an AP if each pair of terms is relatively prime?

Problem 3. Does there exist an infinite AP so that each pair of terms is relatively prime?

Some students recommended the following questions:

Follow-up Q1. Does there exist an infinite sequence so that the differences of the consecutive terms form an AP and each pair of terms is relatively prime?

Follow-up Q2. If the terms of the AP are primes then it implies that each pair of terms is relatively prime. Therefore a naturally arising question is the following: Does there exist an arbitrarily long AP whose terms are primes? The answer for this question is very hard. It is yes, and it was proved by Ben Green and Terrence Tao in 2004 (Green & Tao, 2008).

GOALS, EFFICIENCY AND RESULTS

No controlled measurements on the efficiency of the Pósa method have been conducted so far, as we aim at assisting the development of such abilities that are extremely difficult to be measured; efficient and trustworthy method for measurement are not known (by us).

One targeted 'ability' to be developed is that students shall love thinking hard and persistently on difficult and extremely difficult problems. Another one is, that in case of initial failures, they shall not give up, and shall approach the problem from several different 'directions'. The third one is that although the curriculum intends to teach 'thinking methods' (spinned into the WPT), these cannot always mechanically and directly be applied to new problems. The set of these methods is a tool bar to be applied in an intelligent and creative way, sometimes with improved variants of the methods. The fourth and perhaps most important one is that students shall be happy while doing 'high-quality' mathematics.

Based on students' oral feedback, our camps are very successful. Many of them report that the way of thinking they have learnt in the camps help them a lot in their work, which is not infrequently far from doing pure mathematics.

Fostering the development of students' competition problem solving skills is definitely not targeted in our camps. Still, students' feedback indicates that, during the math competitions, they can boneneficially apply their knowledge learnt in the camps. In the past 25 years, with only some exceptions, almost always all the 6 members of the Hungarian IMO teams came from the Pósa camps. There is a similar situation for the 6 member MEMO teams (Middle European Mathematical Olympiad), from its starts in 2007. The following table contains data about the participating Pósa students in the last 10 years.

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
IMO	6	5	5	6	6	6	6	6	6	5
MEMO	5	5	5	6	5	4	6	4	6	6

Table 1: Number of 'Pósa students' in the Hungarian IMO and EMO groups, 2009 - 2018

It is important to note that the Pósa camps are not part of the official IMO and MEMO participatory program in Hungary. Our students have outstandingly good results in the national competitions too. Many of them continue their studies at the world's leading universities.

FURTHER RESEARCH

We believe the principles of the Pósa Method can be used in various settings. One of our further, ongoing research foci is on public education, we started a four-year experiment with three groups of grade-9-students, we are interested in what way the Pósa method can be used in a normal high school setting. Our other research interest is about how to reach out to talents. Our "Flying School" program offers mostly underprivileged high schools a 3-4 hours inspiring activity for grade 9 students. For the talented students we offer a further opportunity of a two-year long talent-nurturing program, organizing them a "mathematical" day every months.

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