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Relationship between the critical dissipated energy per unit volume and the mechanical properties of different rocks

B.Vásárhelyi, Á.Deli & M.Gálos
Department of Engineering Geology, Technical University of Budapest, Hungary

P.Ván
Department of Chemical Physics, Technical University of Budapest, Hungary

ABSTRACT: The aim of this paper is to present a laboratory method determining the critical dissipated energy (CDE) per unit volume of homogenous-isotropic rock. In case of uniaxial compression it can be calculated easily with the difference of the work done by external force and the energy connected to the change of internal structure. These energies are measured applying different stress or strain rates, i.e. it tends to infinite and zero. The CDE is material dependent: it is influenced by the rock texture, internal bonds, the cohesion, the strength of the minerals, the porosity, etc. Using seven different type of rocks the CDE was measured and the relationship with the compressive and tensile strength, ultrasonic wave velocity and porosity was calculated.

1 INTRODUCTION

The schematic stress-strain curves for different constant strain rates ($\dot{e}(t)$ = constant) and stress rates ($\dot{s}(t)$ = constant) of the Poynting-Thomson (standard) model of linear viscoelasticity are shown in Figures 1a, b, respectively. The stress-strain curves are linear if loading rates tend to infinite or zero although for finite loading rates these curves are nonlinear (see e.g. Asszonyi & Richter 1979).

Comparing the theory with the practice (Fig. 2) the stress-strain curves for rocks at the experimentally available rates are non-linear and strongly velocity dependent: with increasing loading rate the entire stress-strain curve rises, i.e. time-dependent. An unloading process (dotted line in Figure 2) starting from any stress level reveals an irreversible component of the strain. For the two above-mentioned reasons one can conclude that for loading process which starts from the stress-free state the yield stress for most rocks is zero (Cristescu 1989). The curves shown in Figure 2 also reveal that the loading rate also influences failure; with increasing loading rate the stress at failure increases while strain decreases.

According to the usual explanation higher stress or strain rate leads to greater (crack) localization, stress relaxation (achieved by means of visco-elastic deformation or breaking off of inter-particle bonds – grains and atoms – and stress redistribution) and dissipation is limited (Andreev 1995). As a consequence, more elastic energy is stored within the body and its failure (fracture) is burst-like (Houpert 1979).

The dissipated energy amount depends on the ratio between the time of attaining constant strain t_0 and

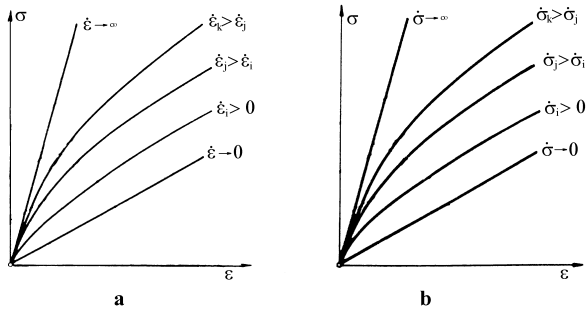

Figure 1. Schematic stress-strain curves for different constant a) strain, ?(t), and b) stress, ?(t), rates using the Poynting-Thomson (standard) model (Asszonyi & Richter 1979).

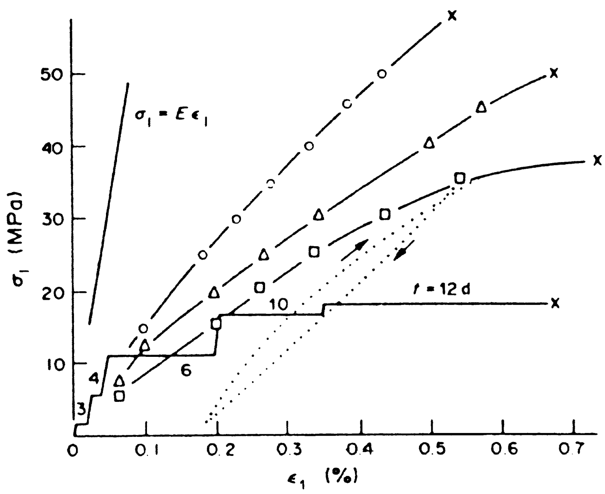

Figure 2. Stress-strain curves for schist according to Cristescu (1993). The broken lines are the curves obtained for three stress rates: 0.49 MPa/sec (circles); 0.065 MPa/sec (triangles); and 0.002 MPa/sec (squares). The solid line is a uniaxial curve, showing at each stage the number of days taken to achieve stabilization.

the time of relaxation t_{rel}. In test conditions of high stress and strain rates t_0/t_{rel} is small and there is no dissipation. In case of t_0 > 10 t_{rel}, almost all energy dissipates (Andreev 1995).

The Poynting-Thomson (standard) model gives a minimal theoretical foundation to these observed properties.

2 CALCULATION THE DISSIPATED ENERGY

The deformation of the rock-sample depends on the loading rate. The difference between the work of external forces per unit volume (U) and the structural energy per unit volume (Φ) is the dissipating energy (L). According to the Poynting-Thomson model and the related physical view this dissipated energy seems to be a good candidate as a measure of the failure. When the dissipated energy per unit volume ($L = U - \Phi$; dimension: kJ/m^3) reaches a critical value the failure of the material occurs. This critical value should be rock-type dependent.

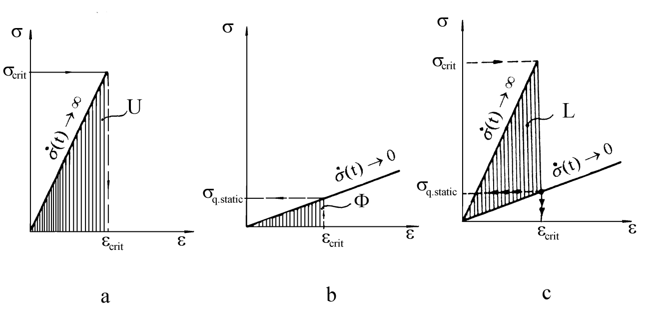

Figure 3. Energies per unite volume in case of uniaxial compressive test: a) definition of work of external forces per unite volume (U); b) structural energy per unit volume (Φ); and c) dissipating energy per unit volume (L).

Knowing the stress-strain curves for different loading rates these energies can be determined. When the strain or stress rate tends to infinite, i.e. the loading rate is "very fast" the work of external forces per unit volume (U) is represented by the area under the stress-strain curve. Figure 3a shows the geometrical meaning of this work for uniaxial stress. The internal energy per unit volume (Φ) is represented by the area under the stress-strain curve when the strain or stress rate tends to zero (see Figure 3b for uniaxial stress). The dissipating energy (L) is the area between the two lines, as it can be seen on Figure 3c. The area is between the two lines is maximal at the critical stress and this is the Critical Dissipating Energy CDE (L_c) per unite volume.

If the material is quasi-homogeneous, isotropic and linear elastic, the work or energy per unit volume (W) supplied during the deformation can be calculated (Asszonyi & Richter 1979):

a) In case of known main strains:

$$W = \int_0^{?} ?d? = \frac{1}{2(1+n)(1-2n)} E_{(e(t))}\left[(1-n)P_1 + 2nI_2\right] \quad (1)$$

where s and e are the main stress and the main strain tensor, respectively, ν is the Poisson ratio, and P_2 and I_2 are the invariants of strain tensor, i.e.:

$$P_1 = e(t)_1^2 + e(t)_2^2 + e(t)_3^2$$

$$I_2 = e(t)_1 e(t)_2 + e(t)_2 e(t)_3 + e(t)_1 e(t)_3$$

b) In case of known main stresses:

$$W = \int_0^{?} ?d? = \frac{1}{2E_{(s(t))}}\left[P_1' - 2nI_2'\right] \quad (2)$$

where s and e are the main stress and the main strain tensor, respectively, ν is the Poisson ratio, and P_1' and I_2' are the invariants of stress tensor, thus:

$$P_1' = s(t)_1^2 + s(t)_2^2 + s(t)_3^2$$

$$I_2' = s(t)_1 s(t)_2 + s(t)_2 s(t)_3 + s(t)_1 s(t)_3$$

When the strain or stress rates tend to infinite, i.e. the loading is "very fast" the slope of the stress-strain curve is the Young's modulus of the material (E). In this case the stress-strain curve is independent of time, i.e. the influence of the rheological constants (relaxation times) is not significant.

When the strain or stress rates tend to zero, i.e. the loading is "quasi-static", the slope of the stress-strain curve (E*) is smaller than the previous Young's modulus.

The ratio of these two material constants is the same as the ratio of the "very fast" and "quasi-static" loading at the same strain (i = 1, 2, 3):

$$\frac{s_{q.static,i}}{s_{v.fast,i}} = \frac{E_i^*}{E_i} \quad (3)$$

Using Equation 3 the CDE (L_c) can be determined applying Equation 1 and Equation 2 in case of main strains and main stresses theory, respectively.

a) In case of known main strains:

$$L = U - \Phi = \frac{E - E^*}{2(1+n)(1-n)}\left[(1-n)P_1 + 2I_2\right] \quad (4)$$

where E is the Young's modulus, E* is the slope of the stress-strain curve in case of infinite slow (i.e. "quasi static") strain rate, ν is the Poisson ratio, P_1 and I_2 are the invariants of strain tensor.

b) In case of known main stresses:

$$L = U - \Phi = \frac{1}{2}\frac{E - E^*}{E^2}\left[P_1' - 2nI_2'\right] \quad (5)$$

where ν is the Poisson ratio, P_1‘ and I_2‘ are the invariants of stress tensor (see above).

According to the previous equations in case of uniaxial stress the CDE (L_C) of the material can be calculated:

a) In case of main strain (Eq. 4):

$$L_C = \frac{E - E^*}{2} e^2 \tag{6}$$

b) In case of main stress (Eq. 5):

$$L_C = \frac{1}{2} \frac{E - E^*}{E^2} s^2 \tag{7}$$

If $e = e_{crit}$ (main strain theory) or $s = s_{crit}$ (main stress theory) (Fig. 3), the CDE can be determined. Consequently there are two methods for determining the CDE (L_C) in the laboratory using uniaxial compression tests. The main stress theory was chosen in this research.

3 EXPERIMENTS

The aim of this research was to examine the influence of the uniaxial compressive and tensile strength, porosity and the longitudinal ultrasonic wave velocity of the rock on the CDE. Three different andesites, granite, dolomite, sandstone and rhyolite-tuff rock-samples were investigated for determining the above mentioned material properties. Table 1 contains the material properties before the investigation (i.e. ultrasonic wave velocity, density and porosity).

The investigations were done under air-dry condition at room temperature (constantly 22 °C) for measuring the slope of the stress-strain curves both for “very fast” and “quasi-static” stress rates. All experiments were performed on two specimens. All the specimens were circular cylindrical having a length to diameter ratio between 2 and 3, and the diameters were approximately 54 mm and they were prepared according to ISRM standard (ISRM 1979). In the case of the Brazilian tests the length/diameter ratio for the cylindrical specimen was approximately 1. The Brazilian tests were done according to ISRM standard (ISRM 1978), too.

Table 1. The average values of the measured longitudinal ultrasonic wave velocity, density and the porosity of the different rock samples in air-dry condition.

Rock type	wave velocity v [km/sec]	Density r_0 [kg/m^3]	Porosity n` [V%]
1 Andesite I	5.85	2730	2.9
2 Andesite II	4.49	2480	5.8
3 Andesite III	3.91	2330	9.4
4 Granite	6.02	2760	0.4
5 Dolomite	4.45	2660	1.5
6 Sandstone	3.67	2480	4.2
7 Rhyolite tuff	2.85	1630	15.0

The Young’s moduli were determined at a stress rate of about 5 MPa/sec ($\dot{s}(t) \to \infty$) (according to the standard tests the normal stress rate is 0.5-1.0 MPa/sec). For larger stress rates it was found that the slope of the stress-strain curve does not change significantly.

The strains in the quasi static stress rate ($\dot{s}(t) \to 0$) conditions were obtained performing creep tests. Using the solution of the differential equation of the Poynting-Thomson (standard) model the strain as a function of the time can be calculated (Asszonyi & Richter 1979):

$$e = \frac{s_0}{E^*} - \left(\frac{s_0}{E^*} - e_0\right) e^{-\frac{E^*}{l}t} = e_\infty - (e_\infty - e_0) e^{-\frac{E^*}{l}t} \tag{8}$$

where: s_0 is the constant uniaxial stress; E^* is the “quasi-static” modulus; l is the linear viscosity factor; t is the time and e is the strain at different time.

Using the creep test equipment which was designed and built at the Department of Engineering Geology at the Technical University of Budapest (Gálos 1982) the asymptote of the strain curve (using Equation 8) was calculated with the measured strain-time curve after 30 days for different the stress levels.

Figure 4 shows schematically the method for determining the stress-strain curve at a loading rate tending to 0 (two steps are shown). The first creeping test was carried out at 5 MPa uniaxial compressive stress and after 30 days it was increased with 5 MPa (it was assumed that after 30 days the deformation of the samples were finished). E^* value was determined using least square method but only the points under the area of the stress-strain curve in case of infinite stress rate (see Fig. 3) were used.

4 RESULTS AND DISCUSSION

The measured Young’s modulus (E) with “very fast” stress rate and “quasi-static” modulus (E^*) (using the above written method) are shown in Table 2. According to the applied Poynting-Thomson rheological model the ratio of these material parameters is the ratio of the two characteristic relaxa-

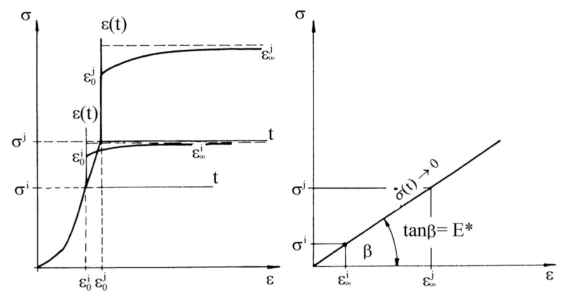

Figure 4. The determination of slope of the stress-strain curve (E^*) in case of infinite slow ($\dot{s}(t) \to 0$) stress rate with creep tests. In this figure two steps are shown schematically.

tion times of the model and can be large. For example for granite and sandstone specimen the Young's modulus (E) is nearly ten times higher than the "quasi-static" modulus (E^*) and this number is less than two for a high porosity andesite: the internal structure of the rock seems to have a large influence on these parameters.

The calculated CDE (using Eq. 7) comparing with the uniaxial compressive and the tensile strengths are presented in Table 3.

Figures 5-8 show the relations between the CDE and the uniaxial compressive and tensile strength, the longitudinal ultrasonic wave velocity and porosity, respectively. The following conclusions can be drawn:

Table 2. Measured Young's modulus and quasi-static modulus of the investigated rocks.

Rock type	Young's modulus E [GPa]	Quasi-static E^*[GPa]	E/E^*
1 Andesite I	55.6	9.4	5.9
2 Andesite II	18.6	6.4	2.9
3 Andesite III	22.0	11.3	2.0
4 Granite	52.6	5.4	9.7
5 Dolomite	58.9	7.7	7.7
6 Sandstone	11.7	1.2	9.8
7 Rhyolite tuff	3.7	1.5	2.5

Table 3. Calculated critical dissipated energy (CDE) comparing with the uniaxial compressive and tensile strengths.

Rock type	Compressive strength s_c [MPa]	Tensile strength s_t [MPa]	CDE L_C [kJ/m^3]
1 Andesite I	151	17	172
2 Andesite II	79	6	110
3 Andesite III	86	7	82
4 Granite	157	15	210
5 Dolomite	118	10	104
6 Sandstone	46	6	78
7 Rhyolite tuff	17	3	25

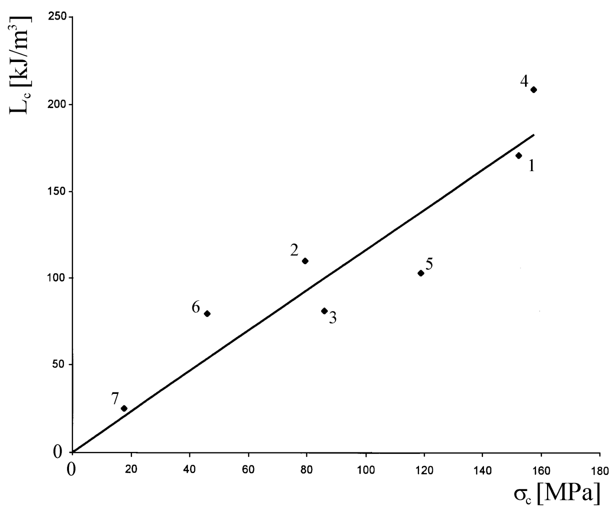

Figure 5. The critical dissipated energy per unit volume (L_C) as a function of the uniaxial compressive strength (s_c) – Numbers show the type of the rock according to the Tables.

a) The relation between the CDE (L_C) and the unaixial compressive strength (s_c) is linear (see Fig. 5):

$$L_C = 1.17\ s_c \qquad (R^2 = 0.85) \qquad (9)$$

b) Figure 6 shows the CDE (L_C) as a function of the measured tensile strength (s_t). The calculated equation is:

$$L_C = 11.87\ s_t \qquad (R^2 = 0.86) \qquad (10)$$

c) There is a parabolic relation (Fig. 7) between the CDE (L_C) and longitudinal ultrasonic wave velocity (v) of the rock and it can be expressed as:

$$L_C = 6.11\ v^2 - 3.75v \qquad (R^2 = 0.97) \qquad (11)$$

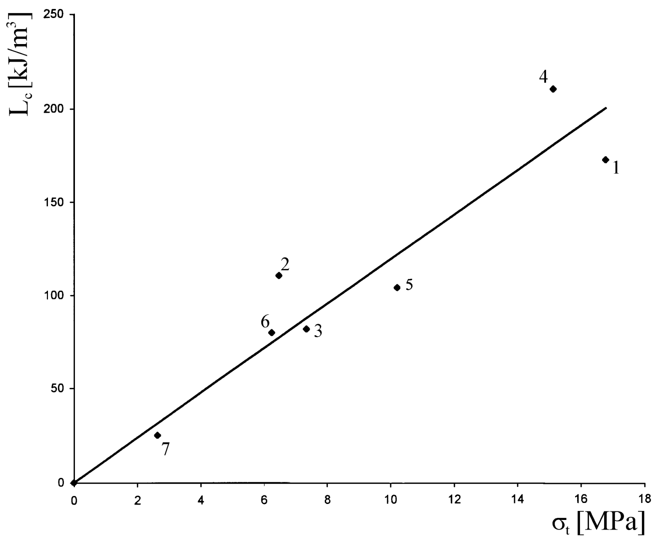

Figure 6. The critical dissipated energy per unit volume (L_C) as a function of the tensile strength (s_t) – Numbers show the type of the rock according to the Tables.

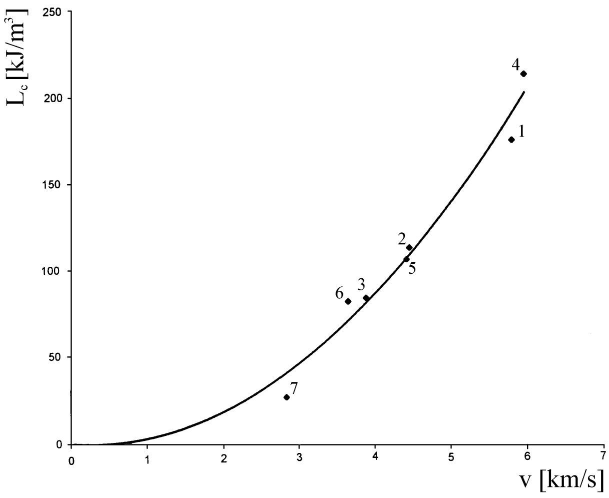

Figure 7. The critical dissipated energy per unit volume (L_C) as a function of the longitudinal ultrasonic wave velocity (v) – Numbers show the type of the rock according to the Tables.

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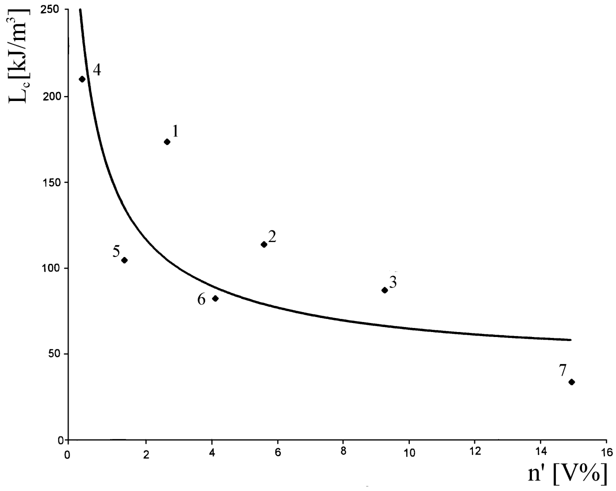

Figure 8. The critical dissipated energy per unit volume (L_C) as a function of the porosity (n') – Numbers show the type of the rock according to the Tables.

d) With increasing rock porosity (n') the CDE (L_C) is decreasing (Fig. 8) according to the following equation:

$$L_C = 162.77\,(n')^{-0.442} \quad (R^2 = 0.62) \qquad (12)$$

5 CONCLUSION

The CDE was calculated for seven different types of rocks in air-dry condition. Using the Poynting-Thomson (standard) model the CDE can be calculated as the difference between the work of external forces per unit volume (U) and the structural energy per unit volume (Φ) at the strength of the rock. These energies were determined with "very fast" ($\dot{s}(t) \to \infty$) and "quasi-static" ($\dot{s}(t) \to 0$) stress rate, respectively.

Let us remark here that the investigated question (can be the CDE a material property or not?) is independent on the applied theoretical model. Since here (and in the rheological theories in general) the failure/yield of the materials is independent on the structural changes modeled by rheology, it appears as an independent 'yield criteria'. Moreover, the continuum damage mechanical theories where the thermodynamic internal variables are qualified to take into account structural changes leading to failure, suffer from the same illness: the changes in elastic properties are mostly independent on the failure described by the 'damage surface'. Therefore, lacking a suitable theory, the posed question cannot be answered on a theoretical level. A proper theoretical explanation should connect the ductile-brittle changes of the elastic properties to the failure, a stability loss, of the materials.

An other remarkable point is whether the supposed material properties are really characterize the material, the rock? Experimental evidences show that the compressive strength of brittle rocks measured by the standardized methods is strongly rate dependent (Bieniawski 1967, Martin & Chandler 1994). Here further experimental (and theoretical) investigations are necessary. For determining the stress-strain lines in case of "infinite" stress rate 5 MPa/sec was chosen and "quasi-static" condition creeping tests were carried out: the final strain was measured after 30 days and the load was increased in 5 MPa steps (i.e. the average stress rate was 0.000002 MPa/sec).

The CDE (L_C) depends linearly on the uniaxial compressive strength (s_c) and the tensile strength (s_t), parabolicaly with the longitudinal ultrasonic wave velocity (v) and hyperbolically with the porosity (n').

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