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The application of network theory in official statistics*

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The challenges faced by official statistics in the 21st century are manifold. We are surrounded by systems that are becoming substantially more complex. The emergence of new phenomena, such as globalisation, digitalisation, has increased the complexity of the reality that needs to be meaningfully and timeously captured by official statistics; it has resulted in the development of new patterns, routes, and types of data, giving us the opportunity to further improve the relevance of statistics. In response to these trends, we need to find new tools and methods for the measurement of these dynamic phenomena. Network theory is an innovative approach that can help us handle the complexity of the 21st century. However, so far it has not featured in mainstream official statistics.

KEYWORDS: network theory, official statistics, globalization

In the globalized world, various activities (internet, business, migration, etc.) are arranged into networks, and the different phenomena, which could be observed, take place through these skeletons. We should move forward from traditional thinking and traditional distributions. The meaning of 'average' has lost its importance gradually, there aren't average companies, migration countries (just tiny or arbitrarily large ones). We should focus on hubs and networks behind the numbers, if we wish to understand the globalized issues. The complex systems and their collective behaviour cannot be recognized soundly just from the knowledge of the system's components. The global perspective is crucial to gain understanding of the full picture.

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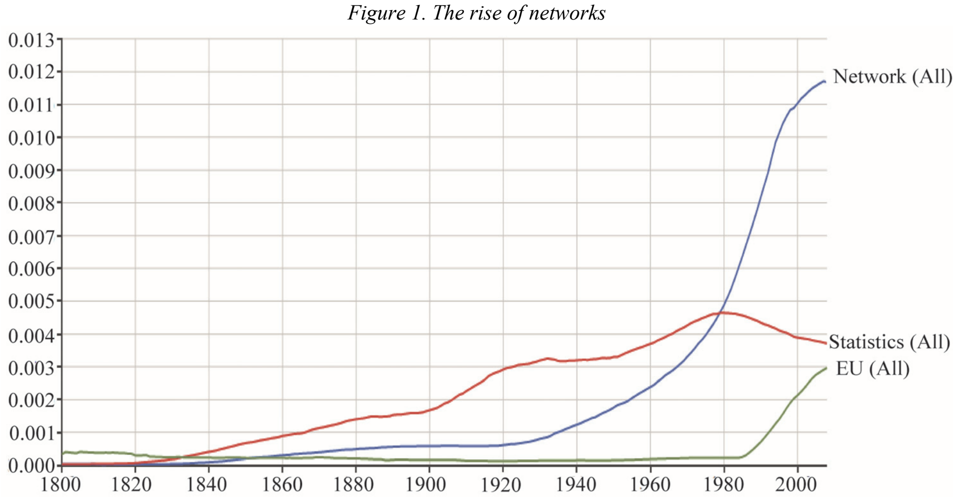
1. Introduction

Figure 1 indicates the frequency of use of the words '*EU*', '*statistics*', and '*network*' in books published since 1800; it reveals a rise in public awareness of networks over the decades.

Figure 1. The rise of networks



Source: The figure was generated through Google's Ngram platform (https://en.wikipedia.org/wiki/Google_Ngram_Viewer).

Interest in network analysis has spiked for a very simple reason: the 21st century requires new, timely, and useable tools and methods that are capable of capturing the essence of new phenomena and complex realities in a simple fashion. Network theory can fulfil this role (*Vante* [1995]).

In order to gain understanding of a complex system (*Lawyer* [2015]), we first need to know the ways in which its components interact with one another. A network is a catalogue of a system's components, often called *nodes*, and the direct interactions, or *links*, between them. This network representation offers a common language for the study of systems that may differ greatly in nature, appearance, or scope (*Barabási* [2009]). The way in which we define the links between two individuals will determine the nature of the questions we can explore (*Newman* [2010]). For instance, social networks reveal the spread of knowledge, news, or behaviours. Communication networks, which describe the interactions between communication devices, are at the centre of the modern communication system. In the networks owned by business enterprises, the aim is to support the informational and operational requirements of the business, such as issues related to sales or manufacturing (*Lewis* [2001]). The variety of relationships within MNEs (multinational enterprises), and that between parent companies and affiliates underpins the importance of dynamic capabilities in the global market. These systems are collectively called complex systems, and they capture the fact that it is difficult to derive collective behaviour just by knowing about the system's components (*Barabási* [2016]).

Table 1

Simplified network maps

Network	Node	Link	Type of link	Average degree of network $\langle k \rangle$
Internet	Routers	Internet connections	Undirected	6.34
WWW	Webpages	Links	Directed	4.60
Power grid	Power plants, transformers	Cables	Undirected	2.67
Mobile-phone calls	Subscribers	Calls	Directed	2.51
Email	Email addresses	Emails	Directed	1.81
Science collaboration	Scientists	Co-authorships	Undirected	8.08
Actor network	Actors	Co-acting	Undirected	83.71
Protein interactions	Proteins	Binding interactions	Undirected	2.90

Note. Some systems, such as those based on phone calls, have directed links, wherein one person calls the other. Other systems, such as transmission lines in the power grid, have undirected links, wherein the electric current can flow in both directions.

Source: http://networksciencebook.com/

2. The nature of networks

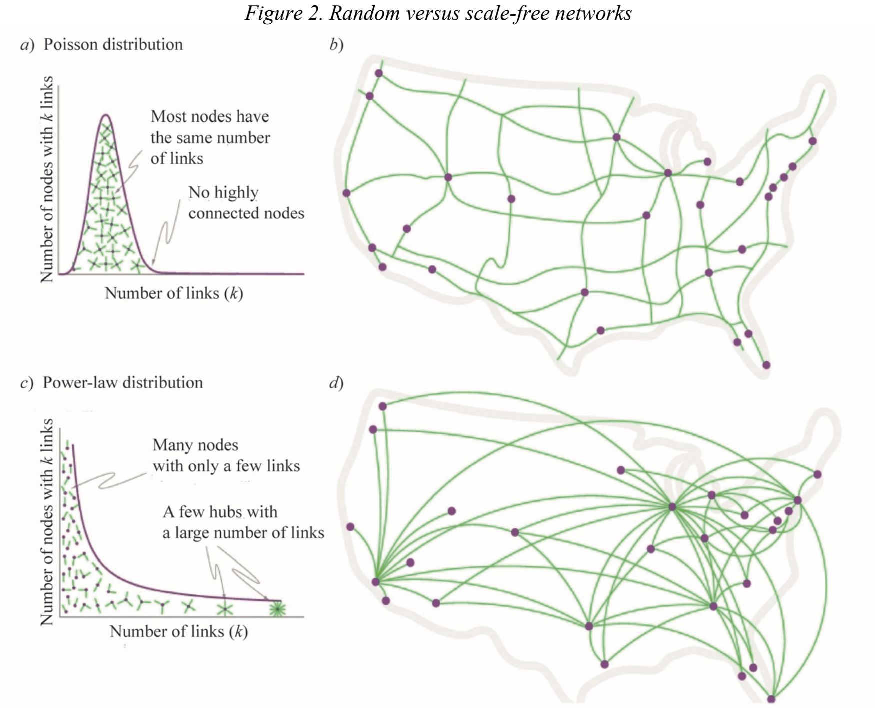
The degree of a node represents the number of links it has to other nodes. The degree distribution (p_k) has a central role in network theory. The reason is that the precise functional form of p_k determines many network phenomena, ranging from network robustness to its ability to evolve. The average degree of a network can be expressed as:

$$\langle k \rangle = \sum_{k=1}^{\infty} k * p_k ,$$

where $\sum_{p=1}^{\infty} p_k = 1$ and $p_k = \frac{N_k}{N}$ (N_k is the number of degree-k nodes[1]).[2]

Based on the degree distribution, we can theoretically differentiate between two types of networks: random and scale-free networks (*Barabási* [2010]).

Figure 2. Random versus scale-free networks



Note. Poisson distribution: $p_k = e^{\langle -k \rangle} * \frac{\langle k \rangle^k}{k!}$; power-law distribution: $p_k = \frac{k^{-\gamma}}{\zeta(\gamma)}$, where $\zeta(\gamma)$ is the Riemann-zeta function: $\zeta(\gamma) = \sum_{k=1}^{\infty} k^{-\gamma}$ (*Bombieri* [1992]). (More about this function, see http://mathworld.wolfram.com/RiemannZetaFunction.html)

Source: http://networksciencebook.com/

The degrees of a random network follow the Poisson distribution, which is similar to a bell curve. Therefore, most nodes have similar number of degrees and there are no

[1] $N_k = N * p_k$

[2] Real networks are supercritical. Once the average degree exceeds $\langle k \rangle = 1$, a giant component should emerge that contains a finite fraction of all nodes. Hence, only when $\langle k \rangle = 1$ can the nodes organise themselves into a recognisable network. For $\langle k \rangle > \ln N$, all components are absorbed by the giant component, resulting in a single connected network.

nodes with a large number of links. The model suggests that the network should be described as purely random; it looks somewhat like the national highway network, in which the nodes are cities and the links are major highways. There are no cities connected to hundreds of highways, and there is no city that is disconnected from the highway system (*Barabási* [2016]).

In a network with power-law degree distribution, most nodes have only a few links. These numerous small nodes are held together by a few highly connected hubs (*Shah–Zaman* [2011]). A scale-free network looks like the air-traffic network, where the nodes are airports and the links are the direct flights between them. Most airports are tiny, with only a few flights. In this network, however, we can reach most destinations via single hubs, such as Chicago. Airlines deliberately build hubs to decrease the number of transfers between two airports. Hubs have the 'small-world nature'. The distances in a scale-free network seem smaller than the distances observed in a similar, but randomly arranged network. These networks are characterised by the small-world phenomenon; thus, the distance between two randomly chosen nodes in a network is short and we are always close to hubs. The presence of hubs fundamentally changes the system's behaviour (*Barabási* [2016], *Battiston–Nicosia–Latora* [2017]).

The key difference between random and scale-free networks is rooted in the different shapes of the Poisson and power-law function; random networks have a scale. In other words, nodes in a random network have comparable degrees, and its average degree $\langle k \rangle$ serves as its 'scale'. Scale-free networks lack a scale; because their average degree does not convey much information, we do not know what to expect when we randomly choose a node. The selected node's degree could be tiny or arbitrarily large. Hence, instead of having a meaningful internal scale, such networks are 'scale-free' (*Barabási* [2016]). The presence of hubs and the small-world phenomenon are *universal* characteristics of the scale-free network.

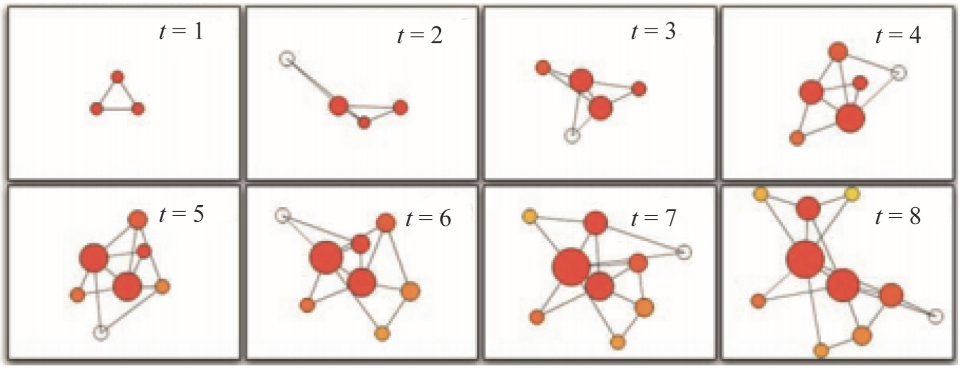
However, *not all networks are scale-free*; in fact, several important networks do not share these features. The networks that appear in materials science describe the bonds between the atoms in crystalline or amorphous materials. In these networks, each node has exactly the same degree, as determined by the laws of chemistry. A carbon atom can share only four electrons with other atoms; regardless of how we arrange these atoms relative to each other, a node in the resulting network can never have more than four links. So, the diamond and graphite networks do not constitute a scale-free phenomenon (*Barabási* [2016]).

Thus, the main question is: what causes the development of scale-free networks? Network growth and preferential[3] attachment are jointly responsible for the scale-free feature (*Barabási* [2009], *Cardillo et al.* [2013], *Kryven* [2016]).

[3] The likelihood of connecting to a node depends on k that the node's degree.

The following is the simplest process that can produce a scale-free topology. Starting with the three connected nodes (top left), a new node (shown as an empty circle) is added to the network in each subsequent image. When deciding to link, new nodes prefer attaching to the more connected nodes – a process known as preferential attachment. Thanks to network growth and preferential attachment, a rich-gets-richer process is observed, meaning that highly connected nodes acquire more links than those that are less connected; this naturally leads to the emergence of a few highly connected hubs. The node size, which is proportional to the node's degree, illustrates the natural emergence of hubs as the largest nodes. The degree distribution of the resulting network follows the power law (*Barabási* [2016]).

Figure 3. The birth of a scale-free network



Source: http://science.sciencemag.org/content/325/5939/412/F1

3. Networks in statistics and their usability

Official statistics offer a new field to which the results of network theory can be applied. Network analysis offers us the opportunity to better understand the processes of globalisation by going beyond the figures, besides improving the relevance and quality of official statistics. Through examples, we provide some of the most important tangible outcomes of network analysis in official statistics (including usability, degree distribution, and consequence). The following networks disseminations and calculations were made with UCINET's NetDraw software (*Borgatti–Everett–Freeman* [2002]).

Table 2

Examined networks overview

Network	Node	Link	Data
Companies' sales	Top 1,000 companies in Hungary (according to domestic sales)	Sales	Data referring to yearly total sales in 2016 (more than HUF 1 million/transaction)
International migration	Countries	Migration	UN migration database, total migrant stock at mid-year by origin and country of destination, 2017

Table 3

Characteristics of the examined networks

Description	Network	
	Companies' sales	International migration[4]
Average geodesic distances within the network	3.8	4.1
Standard deviation	1.10	0.23
Density (matrix average)	0.0093	0.0450
Standard deviation	0.0962	0.2072

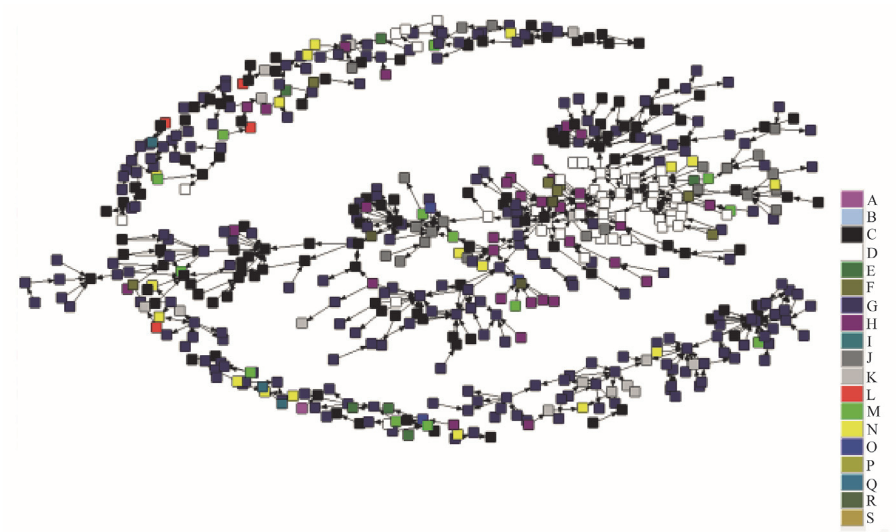
The length of a path in a network is the number of links it contains. The geodesic distance between two nodes is the length of the shortest path. The *distances in these networks are relatively small.*

The density of a network is the total number of existing ties divided by the total number of possible ties. The density of the examined networks is increasing over

[4] The unweighted network takes movements into account above a certain threshold, as small number of international migrants between two large countries does not necessarily represents a real migration relationship. Thus, $q(A,B) = \dfrac{M[A \to B] - M[B \to A]}{N(A) + N(B)}$ is above a fixed threshold μ, where $M[X \to Y]$ is the population density born in country X and living in country Y, while $N(X)$ is the population of country X and $\mu \in \{-1; +1\}$, $\mu \in \boldsymbol{R}$. If $q(A, B) > \mu$, then a migration connection from country A to country B is created, if not, there is no such relationship between the two countries. Thus, depending on the μ parameter different networks can be developed. Here, $\mu = 0.006$ was used.

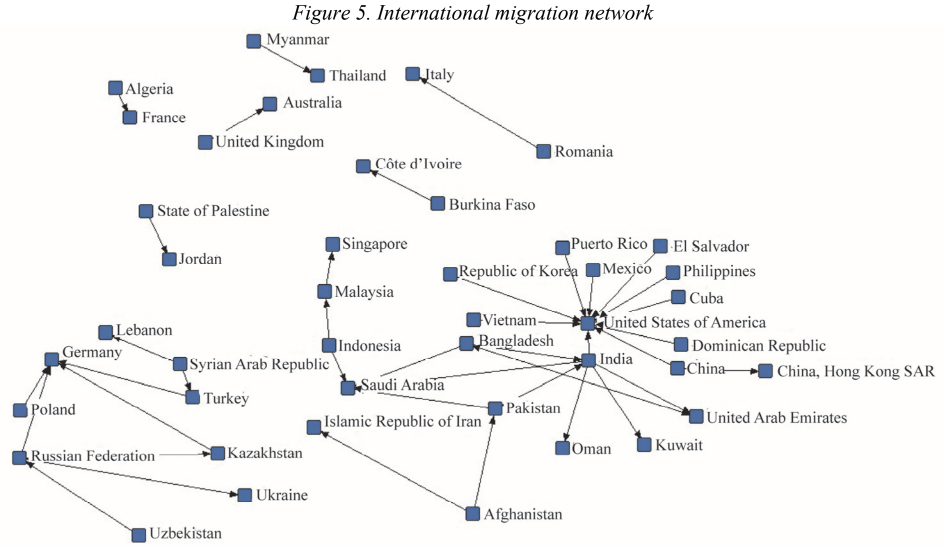
time; this shows that the connectivity is improving. In these networks, the nodes have only a few links; these numerous small nodes are held together by a few highly connected hubs.

Figure 4. Companies' sales network in Hungary by NACE level 1



Note. NACE (Nomenclature des industries établies dans les Communautés européennes): Statistical Classification of Economic Activities in the European Community. To ensure greater visibility, only trades above EUR 10 million are shown. A - Agriculture, forestry, and fishing; B - Mining and quarrying; C - Manufacturing; D - Electricity, gas, steam and air conditioning supply; E - Water supply; sewerage, waste management, and remediation activities; F - Construction; G - Wholesale and retail trade; repair of motor vehicles and motorcycles; H - Transportation and storage; I - Accommodation and food service activities; J - Information and communication; K - Financial and insurance activities; L - Real estate activities; M - Professional, scientific and technical activities; N - Administrative and support service activities; O - Public administration and defence; compulsory social security; P - Education; Q - Human health and social work activities; R - Arts, entertainment, and recreation; S - Other service activities.

Figure 5. International migration network



Note. For ease of depiction, only migration having more than 1,000,000 migrants are shown.

Figure 6. Degree distribution of company network

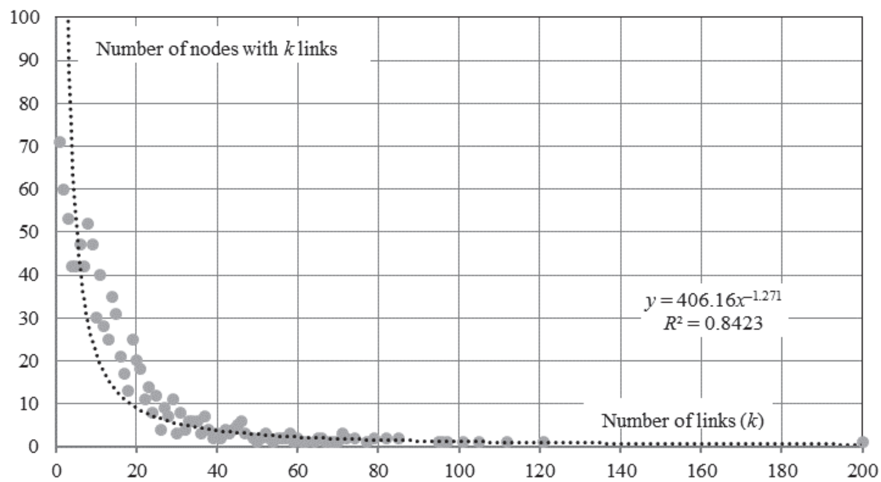
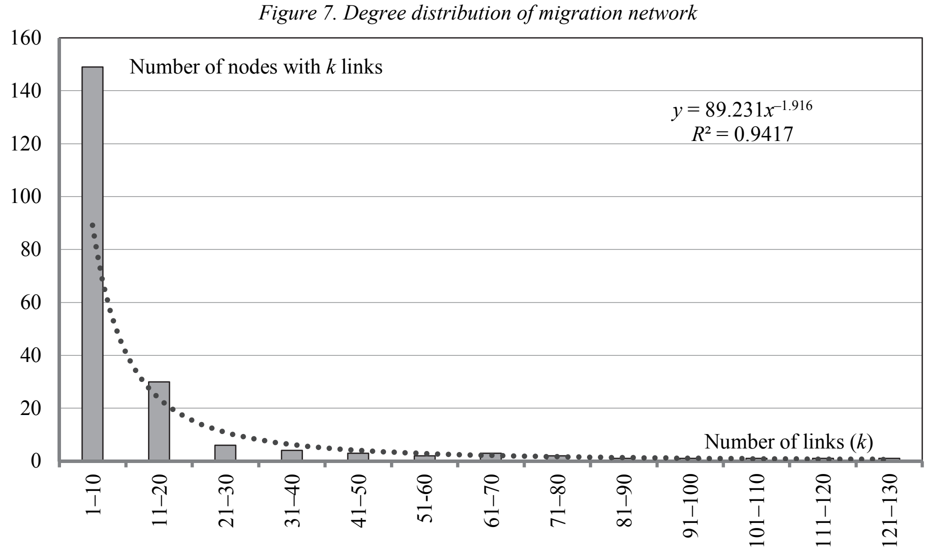


Figure 7. Degree distribution of migration network



The examined networks mostly follow the power-law distribution. The above facts confirm that the networks of official statistics have a scale-free nature; in these cases, we also may assume that the reason for this is network growth and preferential attachment.

The analysed statistics have hubs and exhibit the small-world phenomenon. This means that we can use the consequences and advantages of the *universal characteristics of scale-free models*.

In the globalised world, various activities (business, migration, etc.) arrange into networks with scale-free topology, and, through these skeletons, we can observe with official statistics the different phenomena that occur.

The complex systems and their collective behaviour cannot be recognised soundly just from the knowledge of the system's components. The networks neither stop at the borders of the countries, nor can they be effectively examined at country level; they require collaboration at the EU and global levels. The global perspective is crucial to understand the full picture.

Networks with power-law distribution do not have a meaningful internal scale. The observed units (the data providers) are not equally relevant. There are a few vital nodes and many trivial ones. The hubs and the small-world phenomenon allow us to better understand the processes of globalisation beyond the figures, besides improving the relevance and quality of official statistics. Hence:

– The presence of the LCU (large cases unit) and the European networks of LCU are essential. These units should focus on global networks, hubs, key enterprises, MNEs, core phenomena, and the global supply chain.

– The exchange of microdata is important in general, but the largest enterprises, as well as their activity and connectivity should be the centre of interest.

– Official statistics need to reorganise their own data collection work (few vital and many trivial units):

  – it is necessary to rethink sample selection methods;

  – it is important to foster differentiated checking aspects: more connected nodes are more important than others and we need to prioritise them in our control system;

  – these actions allow us to reduce the data providers' burden, even as more effort is directed to hubs.

4. Conclusions

Network theory is an innovative tool that reflects a new type of thinking in our changing world, and can help us handle the challenges of the 21st century.

The scale-free nature of networks has played an important role in the development of networks as a whole; this is evident in many scientific and practical interest networks. This scale-free property is an unavoidable issue in many disciplines. Once the hubs are present, they fundamentally change a system's behaviour. The statistics of the 21st century have had scale-free features. This means that in the globalised world, various activities (business, migration, etc.) can be depicted by networks with scale-free topology, and, through these skeletal representations, we can observe the different phenomena that take place by using relevant official statistics.

We should move beyond traditional thinking and distributions. The meaning of average has gradually lost its importance because there are few average-sized companies (just tiny or arbitrarily large ones). If we want to increase the quality and relevance of statistics, we should focus on the hubs and networks behind the numbers.

Networks neither stop at the borders of the countries, nor can they be effectively examined at the country level; they require collaboration at the EU level. Given the important roles that complex systems play in our daily lives and in our economy, understanding and eventually controlling them is one of the major intellectual and scientific challenges we face in the 21st century. It is a challenge that European statistics cannot afford to ignore. We see much potential to develop official statistics in line with network analysis and further research at the EU level may be needed.

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