

## IMPACT OF DIFFERENTIAL EVOLUTION PARAMETERS ON OPTIMIZATION OF HYDRO- THERMAL COORDINATION

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**Abstract:** Nowadays heuristic methods are one of the most used tools for the optimization of problems. The proof of that is the fact that they are widely used in chemistry, economics and energy. Among the most popular of heuristic methods belongs differential evolution, belonging to the so-called ‘evolutionary algorithms’. They can handle difficult, large-scale problems with many parameters, like the optimization of the hydro-thermal coordination of hydro and thermal power plants. As with any other method, differential evolution also has certain parameters. These parameters, among others, are the size of the population, the maximum number of generations, crossover parameter and mutation factor. The effect of these parameters on the results of an optimization using differential evolution is the focus of this paper. The hydro-thermal coordination of one hydro and one thermal power plant was used as an example to explain this issue.

**Keywords:** Sensitivity analysis, Differential evolution, Optimization, Hydro power plant

### 1. Introduction

Most Hydro Power Plants (HPP) transmit the electricity they produce into a uniform system called a Hydro-Thermal System (HTS). This system also includes Thermal Power Plants (TPP), which transmit the electricity they produce [1]. When planning the production of electricity, it is important for every element to have the same objective, which is defined by common criteria for the optimization of the whole system. This is called Hydro-Thermal Coordination (HTC). HTC is a complicated optimization

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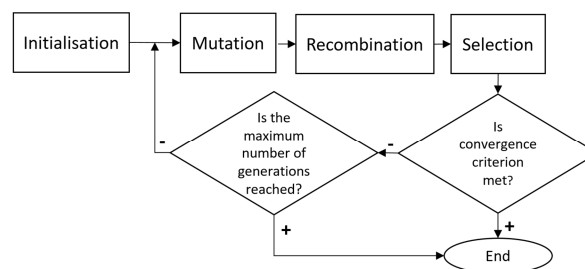
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problem. To solve this problem, optimal control methods are often used. Nowadays in addition to the classic numerical methods, like linear/non-linear programming, modern heuristic methods are more frequently used [2], [3]. The function that describes a HTC problem has a complicated shape of the surface and many local extremes. When solving simpler problems, numerical methods show a high degree of efficiency, but they often tend to have difficulty solving complex problems and can encounter a so-called ‘curse of the size of the problem’ [4]. For complex problems, similar to HTC, which is marked as a large-scale problem, it is better and more suitable to use heuristic methods.

Among the most popular of heuristic methods belongs Differential Evolution (DE), belonging to the so-called ‘Evolutionary Algorithms’ (EA). The optimization method known as DE was originally introduced by Storn and Price [5]. DE is a simple yet powerful heuristic method for solving nonlinear, non-differentiable and multi-modal optimization problems. The DE algorithm has gradually become more popular and has been used in many practical cases, mainly because it has demonstrated good convergence properties and is principally easy to understand. The solution of HTC by DE is demonstrated in [6], [7].

This technique combines simple arithmetic operators with the classical events of crossover, mutation and selection to evolve from a randomly generated initial population to the final individual solution. The key idea behind DE is a scheme for generating trial parameter vectors, which are iteratively combined and updated using simple formulas to form new vectors. Mutation and crossover are used to generate new vectors (trial vectors), and selection then determines, which of the vectors will survive into the next generation [8].

The main difference from evolutionary strategy is that a new individual is created using a differential vector, which is counted as the difference of two randomly selected individuals. Two other individuals enter to the creation of the subject. In the end, the new individual is made up of four individuals. DE works with a population of individuals similar to other EAs. The difference is, however, in the crossing and mutation that run in the reverse order of other EAs (see *Fig. 1*).



*Fig. 1.* General evolutionary algorithm procedure

DE also has disadvantages. High risk of getting stuck in a local extreme and a complicated definition of the penalization functions are some of them. Unlike numerical methods like the simplex method (which is characterized by its straightforward solution

of a problem), solutions found by DE is often dependent on the correct setting of the parameters of the DE [9]-[12].

There has been a trend in recent years to try and make the DE parameters automatically adapt to new problems during optimization, hence alleviating the need for the practitioner to select the parameters by hand, see for example [10]-[16]. But these DE variants with so-called adaptive parameters just introduce new parameters that must then be set by the practitioner and has therefore merely deferred this difficult issue without actually eliminating it. Furthermore, we have previously demonstrated that basic DE variants with properly tuned parameters have comparable performance [17], [18].

The basic parameters of DE are:

$F$  is called a mutation factor. The mutation operator is used to generate a new trial solution by adding a weighted difference vector between two other different individuals. Storn and Price in [5] recommended a value of  $F$  of between 0.4 and 1, with 0.5 as a good initial choice;

$CR$  is the crossover parameter in the range of  $\langle 0, 1 \rangle$ . The trial vector is developed from the elements of the target vector and the elements of the donor vector. Elements of the donor vector enter the trial vector with probability  $CR$ . Storn and Price in [5] list  $CR$  values of 0.1 for a thorough (but slower) optimization, to 1.0 for speedier (but risky) convergence, with 0.5 being recommended. Previous evolutionary algorithm studies have shown that most forms of recombination work well, across quite a wide range of rates, so 0.5 would appear an adequate first choice;

$NP$  is population size. Price and Storn in [5] recommended a population size of 5-20 times the dimensionality of the problem.

The DE's parameters  $CR$  and  $F$  that need to be adjusted by the user are generally the key factors affecting the DE's convergence. Choosing suitable parameter values are difficult for DE, which is usually a problem-dependent task.

In the following section this paper will focus on the impact of parameter settings on the solution of an HTC problem solved by DE. As an example, models of one hydro power plant (HPP Žilina) and one thermal (coal) power plant (TPP Nováky) were used.

## 2. Basics and methods

### 2.1. Description of the HTC problem

A criterion for the optimal solution of an HTC is achieving minimal production and distribution costs, while following all the restrictions that the system has. This criterion is called a regime economy and ensures an optimal solution for distributing an electricity load between electricity producers. The regime economy can be described by criteria function:

$$C = \int C(P_{TPP}) dt = \int C(P_{TPP+HPP} - P_{HPP}) dt \rightarrow \min, \quad (1)$$

where  $C$  is then total thermal production cost;  $C(\dots)$  is the fuel cost function (function of  $P_{TPP}$ );  $P_{TPP}$  is the power output of thermal units;  $P_{HPP}$  is the power output of hydro units and  $P_{(TPP+HPP)}$  is the overall production requested of the HTS.

The optimization of 1 HPP and 1 TPP is planned for the next 24 hours (the so-called D+1 plan) and can be described in terms of the Peak Shaving (PS) method by function:

$$F = \sum_{i=1}^{T=24} \left( P_{(TPP+HPP)_i} - 9.81 Q_{HPP_i} H_{HPP_i} \eta \right)^2 \rightarrow \min, \quad (2)$$

where  $i$  is the index of the time step of the solution (hour);  $P_{(TPP+HPP)_i}$  is the overall production requested of the HTS in  $i$ -hour in [MW]; the variable the DE is searching for is  $Q_{HPP_i}$ , which is the discharge flowing through HPP Žilina during an  $i$ -hour in [ $\text{m}^3 \cdot \text{s}^{-1}$ ];  $H_{HPP_i}$  is net head in [m] and  $\eta$  is efficiency of the turbines.

Peak shaving methods is based on the assumption that electricity production at an HPP should cover peak sections of an electricity load and that the rest should be covered by a TPP. A solution using the peak shaving method assumes that the configuration of blocks in a TPP are constant during the planning period along with the characteristics of the running costs.

## 2.2. Optimization model of the hydro and thermal power plant

The goal of the HTS is to plan the distribution of the load between HPP Žilina and TPP Nováky to achieve the lowest fuel costs possible. The set of inputs for the HTS model includes prediction of peak loads of daily electricity load (Table I), prediction of inflow into reservoir (Table II) and data based on the manipulation orders of the HPP and on the actual operations.

Table I

Prediction of peak loads of daily electricity load -  $P_{(TPP+HPP)_i}$

| Hour | Load [MW] | Hour | Load [MW] | Hour | Load [MW] | Hour | Load [MW] | Hour | Load [MW] |
|------|-----------|------|-----------|------|-----------|------|-----------|------|-----------|
| 1    | 114       | 6    | 113       | 11   | 136       | 16   | 132       | 21   | 130       |
| 2    | 114       | 7    | 113       | 12   | 138       | 17   | 138       | 22   | 124       |
| 3    | 114       | 8    | 119       | 13   | 136       | 18   | 140       | 23   | 120       |
| 4    | 113       | 9    | 125       | 14   | 134       | 19   | 138       | 24   | 114       |
| 5    | 114       | 10   | 130       | 15   | 131       | 20   | 133       |      |           |

The parameters of TPP Nováky (which represents a thermal system) that are important for this model are the regulatory scope of the TPP ranging from  $P_{TPP_{\min}}=50$  MW to  $P_{TPP_{\max}}=440$  MW and the characteristics of the running costs represented by equation  $N=0.0132P_{TPP}^2+1.024P_{TPP}+1456$  €/hour. In the case of a TPP covering the whole predicted load itself, the running costs would be 43,053.00 €/day.

The parameters of HPP Žilina (which represents a hydro system) that are important for this optimization are two Kaplan type turbines with a regulatory scope ranging from  $P_{HPP_{\min}}=12$  MW to  $P_{HPP_{\max}}=72$  MW; the discharge capacity of each turbine is in a range

from  $Q_{HPPmin}=50 \text{ m}^3 \cdot \text{s}^{-1}$  to  $Q_{HPPmax}=150 \text{ m}^3 \cdot \text{s}^{-1}$ ; the reservoir above HPP Žilina has a maximum reservoir storage  $V_{max}=3.918 \text{ mil. m}^3$  and a minimum reservoir storage  $V_{min}=0 \text{ m}^3$ . At the beginning of the planning period the reservoir storage was  $V_{in}=2,766 \text{ mil. m}^3$ , which is the same storage required to have in the reservoir at the end of the planning period (day)  $V_{in}=V_{fin}$ . The fish ladder at HPP Žilina needs a constant feed of  $2.5 \text{ m}^3 \cdot \text{s}^{-1}$ . The reservoir water level  $HN$  (level above HPP Žilina) can be calculated from:

$$HN = 0.43V + 350.3[\text{m a.s.l.}], \tag{3}$$

where  $V$  is the reservoir storage. The downstream level can be approximated by the polynomial function in the form:

$$DH = 324,856 + 0.0011Q_{HPP} - 0.000006Q_{HPP}^2 [\text{m a.s.l.}] . \tag{4}$$

Table II

Prediction of the inflow into the reservoir from the Váh and the Varinka rivers

| Hour | Inflow<br>[m <sup>3</sup> s <sup>-1</sup> ] | Hour | Inflow<br>[m <sup>3</sup> s <sup>-1</sup> ] | Hour | Inflow<br>[m <sup>3</sup> s <sup>-1</sup> ] | Hour | Inflow<br>[m <sup>3</sup> s <sup>-1</sup> ] | Hour | Inflow<br>[m <sup>3</sup> s <sup>-1</sup> ] | Hour | Inflow<br>[m <sup>3</sup> s <sup>-1</sup> ] |
|------|---|------|---|------|---|------|---|------|---|------|---|
| 1    | 65  | 5    | 75  | 9    | 70  | 13   | 70  | 17   | 90  | 21   | 65  |
| 2    | 65  | 6    | 75  | 10   | 70  | 14   | 70  | 18   | 90  | 22   | 65  |
| 3    | 65  | 7    | 70  | 11   | 70  | 15   | 65  | 19   | 80  | 23   | 60  |
| 4    | 70  | 8    | 70  | 12   | 70  | 16   | 75  | 20   | 70  | 24   | 60  |

The solution to the HTC problem is represented by vector  $\mathbf{s} = [Q_{HPP1}, \dots, Q_{HPP24}]$ . The values of the elements of the vector  $\mathbf{s}$  (i.e. the operating plan of HPP Žilina in a one-hour range) are the result of the minimization of the function (2), which must be modified by constraining conditions (5)-(8), which are based on the constraints defined in the handling regulations of HPP Žilina:

$$0 \leq Q_{HPP_i} \leq 300 [\text{m}^3/\text{s}], \tag{5}$$

$$0 \leq P_{HPP_i} \leq 72 [\text{MW}], \tag{6}$$

$$0 \leq V_i \leq 3.918 [\text{mil.m}^3], \tag{7}$$

$$V_0 = V_{in} = 2.766 [\text{mil.m}^3], \quad V_{24} = V_{fin} = 2.766 [\text{mil.m}^3]. \tag{8}$$

### 2.3. Using a DE

DE in general solves unconstrained optimization problems by following block scheme shown in Fig. 2. It is necessary to modify function (2) in order to use it for solving the HTC problem into a constrained form. This was achieved using penalization functions. Function (2), which represents the fitness values, is modified into a pseudo-fitness function (9) with the following constraints (10)-(13):

$$\varphi = - \underbrace{\sum_{i=1}^{T=24} (P_{(TPP+HPP)_i} - 9.81Q_{HPP_i}H_{HPP_i}\eta)}_{Fitness}^2 - \underbrace{W_{pen} \left( \sum_{j=1}^4 S_j \frac{pen_j(\cdot)}{\max(pen_j(\cdot))} \right)}_{Penalization} \rightarrow \max, \quad (9)$$

$$pen_1(V_i, V_{\min}) = \begin{cases} \sum_{i=1}^{N_1} (V_{\min} - V_i)^2, & V_i < V_{\min}, \\ 0, & V_i \geq V_{\min}, \end{cases} \quad (10)$$

$$pen_2(V_{24}, V_{fin}) = \begin{cases} \sum_{i=1}^{N_2} (V_{24} - V_i)^2, & V_i < V_{fin}, \\ 0, & V_i \geq V_{fin}, \end{cases} \quad (11)$$

$$pen_3(P_{HPP_i}, P_{HPP_{\max}}) = \begin{cases} \sum_{i=1}^{N_3} (H_{HPP_i} - P_{HPP_{\max}})^2 & H_{HPP_i} > P_{HPP_{\max}}, \\ 0, & H_{HPP_i} \leq P_{HPP_{\max}}, \end{cases} \quad (12)$$

$$pen_4(P_{HPP_i}, P_{HPP_{\min}}) = \begin{cases} \sum_{i=1}^{N_4} (H_{HPP_i} - P_{HPP_{\min}})^2, & H_{HPP_i} < P_{HPP_{\min}}, \\ 0, & H_{HPP_i} \geq P_{HPP_{\min}}, \end{cases} \quad (13)$$

where  $pen_j$  are the penalization functions representing constraints (6), (7) and (8);  $W_{pen}$  is the penalty weighting factor and  $S_j$  describes how strictly the restrictions will be followed. By means of the mutual proportion of the individual factors, it is possible to

‘tighten’ or ‘loosen’ the individual limits of the task. Constraint (5), i.e. the limits of variable  $Q_{HPPi}$  is set as the upper and lower boundaries of the interval from which the DE can select the elements of solution vector  $s$  is the individual genes. If there is the assumption, that the strictness of observing the boundaries is the same for all the constraints  $S_1=S_2=S_3=S_4=1$ , the value of penalty factor  $W_{pen}$  has a significant impact on the process of the selection of the best individual. The setting of  $W_{pen}$  is a complex problem, and its value should be set so that the pseudo-fitness value will never exceed/falls short the global maxima/minima. Therefore, every solution that violates the constraints should be penalized in a way that it will be worse than a solution that follows all the constrictions. On the other hand, too big a  $W_{pen}$  can cause a premature convergence to a solution. The closest to the best solution were the DE using  $W_{pen}=10^{10}$ .

The model used for the optimization was based on the results achieved and the above-mentioned equations. The model of the hydro-thermal system consisting of 1 HPP (Žilina) and 1 TPP (Nováky) was modeled using Visual Basic 6 programming language with an integrated .dll library, which is a part of the XLOptimizer made by TechnoLogismiki, Inc. The result of the maximization of function (2) and the solution to the HTC problem is the best individual from the final population of individuals represented by vector  $^{FIN}s=(Q_{HPPi})_{24}$ .

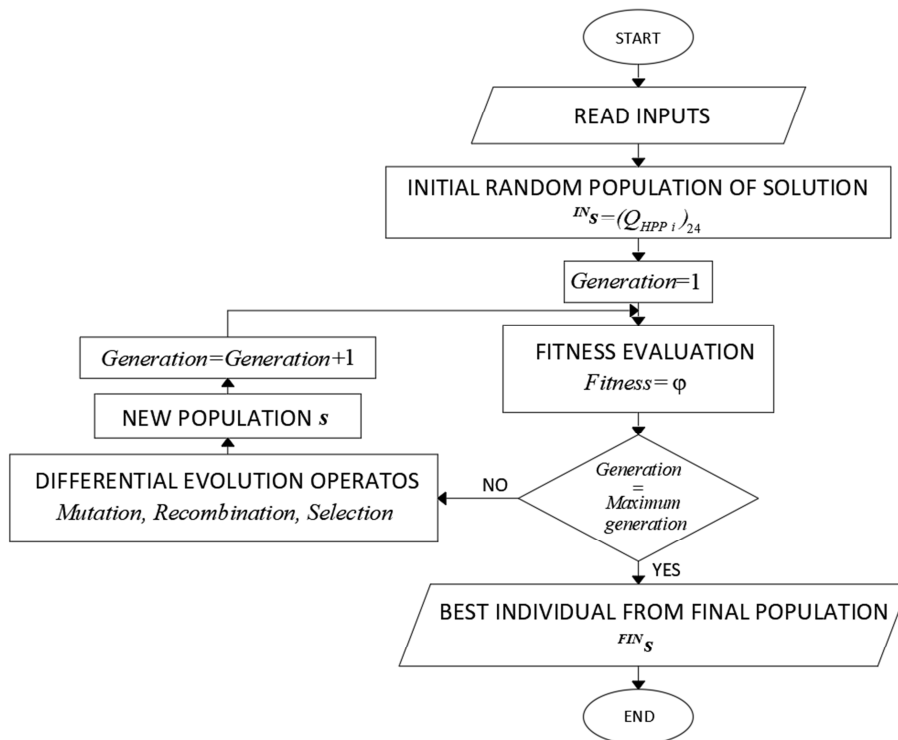
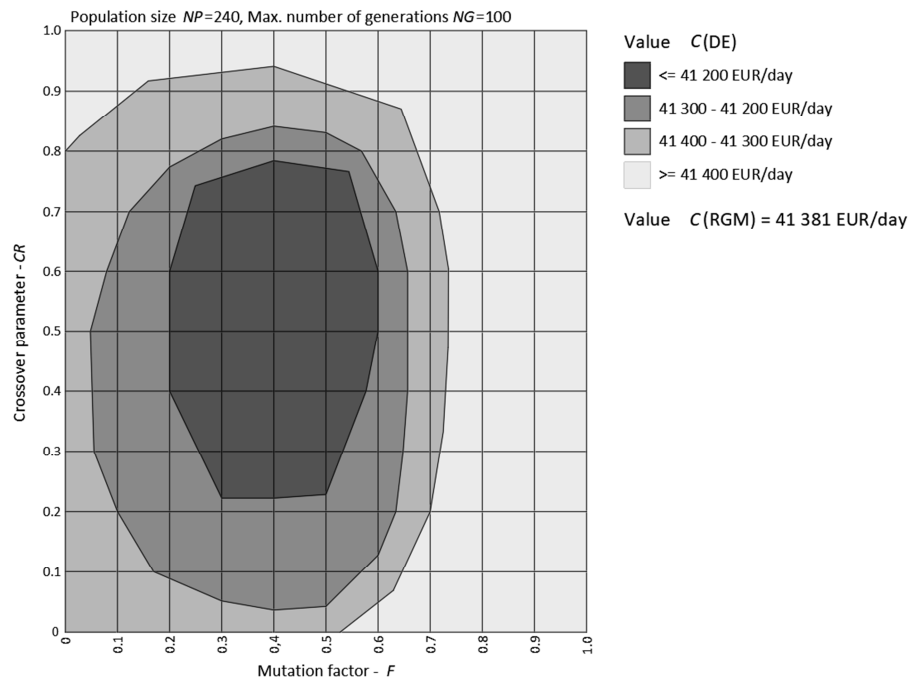


Fig. 2. Block scheme of the optimization model

### 3. Results of the sensitivity analysis

A sensitivity analysis of the impact of crossover parameter  $CR$  and mutation factor  $F$  was made using the above-mentioned optimization model. In the sense [5], the population size  $NP$  was set to a value of 10 times the dimensionality of the problem, i.e.  $NP = 240$ . The parameters given in this paper have been tuned for the  $DE/rand/1/bin$  algorithm. Using the terminology of [5], this version of algorithm is the standard and robust.

In *Fig. 3* the results of the sensitivity analysis of the effect of the crossover parameter  $CR$  and mutation factor  $F$  on the minimization of the criteria function (1), i.e. the minimization of the total thermal production cost. The values shown represent 2000 simulations of the DE for all combinations of  $CR$  and  $F$ . There was realized 5 ‘runs’ for each combination of  $CR$  and  $F$ . The achieved function value (1) was the average of these 5 ‘runs’.  $CR$  and  $F$  were considered in the range of  $\langle 0, 1 \rangle$ . Simulations that violated any of the constraints were not included. The maximum number of generations, where the population evolved, was determined by value  $NG=100$ .



*Fig. 3.* The effect of  $F$  and  $CR$  on the minimization of the total thermal production cost  $C$  in HTS HPP (Žilina and TPP Nováky)

The reference method for DE was the Reduced Gradient Method (RGM). The RGM is one of the traditional numerical methods of nonlinear programming suitable for



solving nonlinear optimization problems with nonlinear constraints. The best solution (reference minimum) calculated with RGM is  $C(\text{RGM})=41,381.0$  €/day.

The evidence of the high efficiency of DE is the fact that in the ‘overwhelming majority’ the solution has achieved a better result than RGM. Even better results are achieved within a wide range of the parameter values  $F <0\sim 0.72>$  and  $CR <0\sim 0.93>$ . The best solution calculated with DE is  $C(\text{DE})=41.092.0$  €/day. This represents an improvement over the reference minimum of about 0.7%.

It is clear from the results that  $F$  has a more significant effect on the results than  $CR$ . The best values (the values less than 41,200.0 €/day) are reached within a relatively wide range of the parameter values  $F <0.2\sim 0.6>$  and  $CR <0.2\sim 0.8>$ .

#### 4. Conclusions

The focus of this paper is on the impact of the parameters of DE on the solution of a hydro-thermal system of 1 hydro power plant and 1 thermal (coal) power plant. Function (1), which describes the problem, was modified to (2) and solved in a nonlinear form by DE, so it could be compared with the RGM results.

For the above mentioned set of inputs, the impact of the individual DE parameters on 2000 simulations was evaluated as follows:

- *crossover parameter*,  $F$  has a more significant effect on the results than *mutation factor*  $CR$ ;
- ‘good’ results are achieved within a wide range of the parameter values  $F <0\sim 0.72>$  and  $CR <0\sim 0.93>$ ;
- the best values are reached within a relatively wide range of the parameter values  $F <0.2\sim 0.6>$  and  $CR <0.2\sim 0.8>$ ;
- the sensitivity analysis proved that the best settings for these parameters are converging to the ones recommended by the literature (i.e.  $CR=F=0.5$ ).

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