Analysis of Model Reference Control Based on Modified Laguerre Network with Integrator

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***Abstract*—The model reference control based on modified Laguerre network is an open-loop control structure; therefore it has to be modified to ensure the disturbance rejection. In the paper a new modification using the closed loop with integrator is proposed. This Laguerre-based model reference control algorithm is compared with the control structure augmented by internal model and its stability, robustness and performances are analyzed.**

***Keywords*—model reference control; Laguerre model; internal model control**

I. INTRODUCTION

Laguerre series are a well-known and efficient tool in system identification and modeling. Due to the orthonormal properties of Laguerre networks the system identification can be executed easily. The advantage of Laguerre networks is that a priori knowledge of the system order and dead time are not definitely necessary [1]. The Laguerre models have been used mainly in predictive control design [2], [3], [4], [5], [6] which profits from their orthonormality properties.

The model reference control objective is to design a controller such that the controlled system output resembles the output of the reference model. In this paper the control design is based on the Laguerre models and the controller structure results in the form of modified Laguerre network (called *x*-expansion) [7]. Basically, it is the open loop controller and the calculation of its parameters is very simple. However, such control structure may not compensate for disturbances in the system. In order to ensure the disturbance rejection capabilities the control structure has been augmented by an internal model controller [8], [9]. The main drawback of this control structure is that the exact model of system is required and if the plant model mismatch occurs, then the control performances may exhibit the nonzero steady-state error.

The aim of the paper is to propose a new closed loop model reference control structure, where instead of the internal model an integrator is used in the control scheme, which improves the control performances in the presence of disturbances and modeling errors. Robust stability with respect to the plant model mismatch is also investigated.

The paper is organized as follows. First the continuous Laguerre network and its modification called *x*-expansion are introduced. Next the model reference control design based on the Laguerre expansion is presented. Two closed loop model reference control algorithms are proposed and their stability analysis is performed. Finally, the control performances and robust stability of both controllers are illustrated using an example.

II. LAGUERRE NETWORK

Laguerre networks are suitable for modeling of stable processes with the impulse response decaying to zero [1], [2], [10]. The quality of approximation depends on the choice of the Laguerre network parameters, namely the number of Laguerre coefficients and the value of scaling parameter α. The model identification is simple due to the orthogonality of the Laguerre functions.

A. Continuous Laguerre Network

A function $f(t)$ which satisfies $\lim_{t\to\infty} f(t) = 0$ can be approximated by continuous Laguerre network as follows

$$f(t) = \sum_{i=0}^{\infty} c_i l_i(t), \tag{1}$$

where $c_0, c_1, \ldots, c_\infty$ are the weighting coefficients and $l_i(t)$, $i = 0, 1, \ldots, \infty$, is the output of *i*-th continuous Laguerre function. The expansion (1), in theory, has an infinite number of coefficients. However, for practical purposes only the first M terms of the expansion are taken into account.

The continuous Laguerre functions form an orthonormal set over the interval $(0, \infty >$ with ortonormal properties.

The Laplace transform of the Laguerre function $l_i(t)$ has the form

$$L_i(s) = \frac{\sqrt{2\alpha}}{s+\alpha}\left(\frac{s-\alpha}{s+\alpha}\right)^i. \tag{2}$$

The parameter α is called the time scaling factor and determines the exponential decay rate of the Laguerre functions. For stability reasons it has to be chosen from the interval $(0, \infty >$.

The transfer function can be approximated by Laguerre network as follows

$$H(s) = \sum_{i=0}^{n} c_i L_i(s) \,. \tag{3}$$

The Laguerre weighting coefficients $c_0, c_1, \ldots, c_n$ can be calculated from the system data by minimizing the sum of the squared error function provided the time scaling factor α and the number of Laguerre terms $M = n + 1$ are chosen [2]. It should be noted that the Laguerre model can be used to approximate only the strictly proper transfer functions.

B. *x-expansion*

The non-strictly proper transfer functions can be approximated by a modification of the continuous Laguerre network called *x*-expansion [10]. This is particularly suitable for controllers with transfer functions whose numerator and denominator degrees are the same.

The *x*-expansion is given by following relationship

$$H(s) = \sum_{i=0}^{\infty} d_i x^i = \sum_{i=0}^{\infty} d_i \left(\frac{s-\alpha}{s+\alpha} \right)^i , \tag{4}$$

where function x forms a complete orthonormal system.

III. Model-Reference Control Based on the Laguerre Expansion

Model reference control is a well-known method which designs a controller so that the overall response of a plant plus controller asymptotically approaches that of a given reference model [7]. Therefore the aim of model reference control design is to find such a controller $H_2(s)$ that satisfies (at least approximately) the following equation

$$H_2(s) H_1(s) = H_m(s) \,, \tag{5}$$

where $H_m(s)$ is the transfer function of the reference model and $H_1(s)$ is the system model, both in the form of Laguerre network. The controller is expressed in the form of *x*-expansion

$$H_2(s) = \sum_{j=0}^{n} d_j x^j = \sum_{j=0}^{n} d_j \left(\frac{s-\alpha}{s+\alpha} \right)^j , \tag{6}$$

where

$$\begin{aligned} d_0 &= m_0 / c_0 \\ d_1 &= (m_1 - d_0 c_1) / c_0 \\ &\vdots \\ d_k &= \left(m_k - \sum_{i=0}^{k-1} d_i c_{k-i} \right) \Big/ c_0 \end{aligned} \,, \tag{7}$$

c_i , $i = 0, \ldots, n$ are the system model weighting coefficients and m_i , $i = 0, \ldots, n$ are the reference model weighting coefficients.

A. *Closed-loop Control Design*

The model reference control design results in an open-loop controller and therefore it cannot reject disturbances without feedback. For this reason two closed loop control design algorithms have been proposed.

The first one is based on the internal model control (IMC) [8], [9] as shown in Fig. 1, where $r(t)$ is the reference value, $u(t)$ is the control action, $y(t)$ is the measured output of the system, $y_{di}(t)$ and $y_{do}(t)$ are the input and output disturbances respectively, $y_m(t)$ is the output of the Laguerre model of the system. The controller parameters are calculated according to (7).

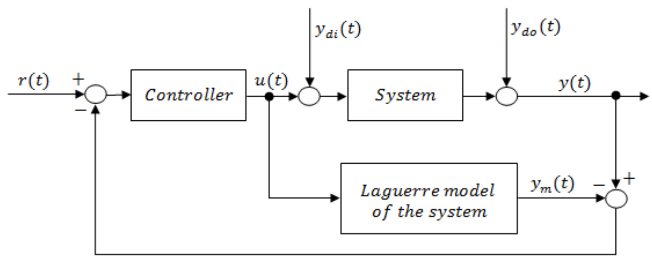

Figure 1. The internal model control

This control structure requires the knowledge of an exact system model which has to be expressed in the form of Laguerre network. If there exist modeling errors, then the control performances may exhibit the nonzero steady-state error.

To avoid these drawbacks, a new control structure has been proposed, where an integrator has been incorporated into the model reference control scheme (Fig. 2, Fig. 3). In this case the controller has two inputs: the set-point value and the error.

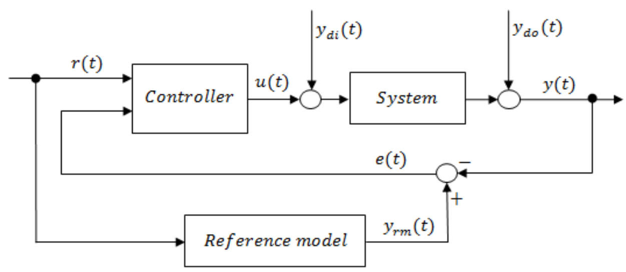

Figure 2. Model reference control with integrator

The controller structure is shown in Fig. 3. The integrator eliminates the steady-state error with tuning parameter I_g. The E_g value influences the disturbance rejection dynamics. This control structure allows separating the tracking behavior and regulation dynamics. The closed loop control design is realized so that (5) is satisfied.

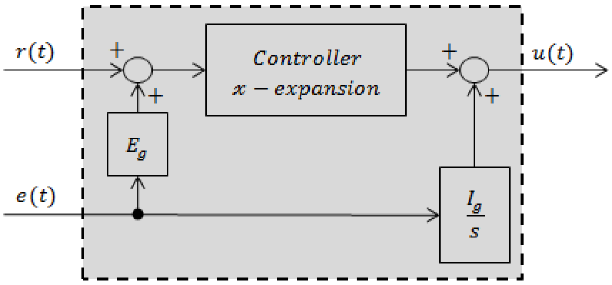

Figure 3. Controller structure

IV. STABILITY ANALYSIS OF THE MODEL REFERENCE CONTROL BASED ON LAGUERRE EXPANSION

Stability of the model reference control with integrator (Fig. 2, Fig. 3) is analyzed supposing inaccuracy resulting partly from the truncation of the Laguerre model and partly from other sources of plant model mismatch.

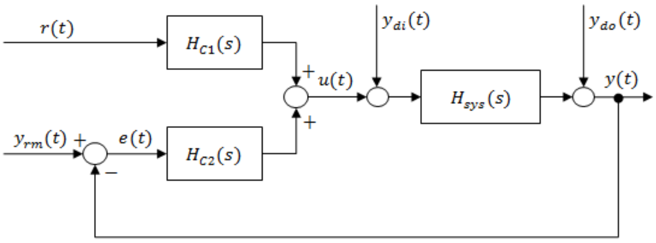

Figure 4. Equivalent scheme to model reference control with integrator

The model reference control structure with integrator shown in Fig. 2 is equivalent to the control structure in Fig. 4, where the controller (Fig. 3) is divided into two parts $H_{c1}(s)$ and $H_{c2}(s)$. The controller $H_{c1}(s)$ is in the form of x-expansion given by (6) and (7). As this controller is in open loop and the reference model is stable, the path $H_{c1}H_{sys}(s)$ is stable. The structure of $H_{c2}(s)$ is shown in Fig. 5. The stability of the closed loop with controller $H_{c2}(s)$ has to be verified.

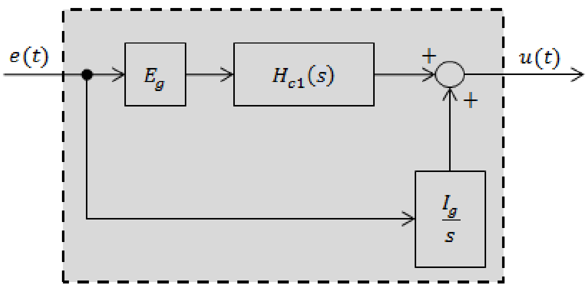

Figure 5. Structure of $H_{c2}(s)$

The closed-loop transfer function is

$$T(s)=\frac{H_{c2}(s)H_{sys}(s)}{1+H_{c2}(s)H_{sys}(s)}=\frac{\left(E_g\sum_{j=0}^{n}d_jx^j(s)+I_g/s\right)H_{sys}(s)}{1+\left(E_g\sum_{j=0}^{n}d_jx^j(s)+I_g/s\right)H_{sys}(s)}\,. \tag{8}$$

Let the system $H_{sys}(s)$ is given by multiplicative uncertainty $\delta(s)$:

$$H_{sys}(s)=H_1(s)+\delta(s)H_1(s)\,, \tag{9}$$

where $H_1(s)$ is the nominal Laguerre model of the system given by (3). Then the nominal closed-loop transfer function is

$$\hat{T}(s)=\frac{H_{c2}(s)H_1(s)}{1+H_{c2}(s)H_1(s)}=\frac{E_g\sum_{j=0}^{n}d_jx^j(s)\sum_{i=0}^{n}c_iL_i(s)+I_g/s\sum_{i=0}^{n}c_iL_i(s)}{1+\left(E_g\sum_{j=0}^{n}d_jx^j(s)\sum_{i=0}^{n}c_iL_i(s)+I_g/s\sum_{i=0}^{n}c_iL_i(s)\right)}=\frac{E_g\sum_{i=0}^{2n}m_iL_i(s)+I_g/s\sum_{i=0}^{n}c_iL_i(s)}{1+\left(E_g\sum_{i=0}^{2n}m_iL_i(s)+I_g/s\sum_{i=0}^{n}c_iL_i(s)\right)}\,, \tag{10}$$

In the case of Laguerre-based IMC control the closed loop transfer function with nominal model is given by the following relationship [9]

$$\hat{T}(s)=\sum_{i=0}^{2n}m_iL_i(s)\,. \tag{11}$$

The characteristic equation of the closed-loop system (10) must be stable with the nominal Laguerre model $H_1(s)$ and simultaneously for the robust—stability the following relationship must be fulfilled

$$|\hat{T}(j\omega)|<\frac{1}{|\delta(j\omega)|};\ \forall\omega\,. \tag{12}$$

The Nyquist stability criterion can be used to check the stability of (10).

A. Stability Analysis Considering Truncation of the Laguerre Model

Relative inaccuracy of the system $H_{sys}(s)$ is

$$\delta(s) = \frac{\Delta H_{sys}(s)}{H_1(s)}, \qquad (13)$$

where

$$\Delta H_{sys}(s) = H_{sys}(s) - H_1(s). \qquad (14)$$

In case of the Laguerre model [9]

$$\Delta H_{sys}(s) = \sum_{i=0}^{\infty} c_i L_i - \sum_{i=0}^{n} c_i L_i = \sum_{i=n+1}^{\infty} c_i L_i. \qquad (15)$$

Then

$$\delta(s) = \frac{\Delta H_{sys}}{H_1} = \frac{\sum_{i=n+1}^{\infty} c_i L_i}{\sum_{i=0}^{n} c_i L_i}. \qquad (16)$$

For both closed-loop control structures the robust stability can be verified using (12) where the closed loop transfer function with nominal model is given by (10) for the control structure with integrator or by (11) for the IMC control structure. In both cases there are two possibilities for calculation of δ:

- (16) can be used. Indeed, the infinite sum of values cannot be used in calculations, but infinite index can be substituted by a higher index after which the values of the coefficients become negligibly small.
- ΔH_{sys} can be given by (14), where ΔH_{sys} is response of the system in frequency domain (if it is available).

B. Design of Robust Model Reference Control Based on Laguerre Expansion

Let be given m different models of system $H_{sys}(s)$ considering different uncertainties. The aim is to design a robust controller based on (5) which is robustly stable in this domain.

The uncertainty δ is given by the set Ψ which considers m stable Laguerre models. So then we have m vectors of Laguerre coefficients and m values of coefficient α and all Laguerre models are of the same order n.

Let be $H_1(s)$ the stable nominal Laguerre model given by mean values of the Laguerre coefficients of all Laguerre models [9]

$$\eta_{nominal} = \sum_{i=1}^{m} \frac{1}{m} \sum_{j=0}^{n} [c_{ji}], \qquad \alpha_{nominal} = \sum_{i=1}^{m} \frac{1}{m} \alpha_i \qquad (17)$$

where $\eta_{nominal} = [c_0 \; c_1 \; \dots \; c_n]$ is a vector of M Laguerre coefficients of $H_1(s)$ and $\alpha_{nominal}$ is the Laguerre parameter of $H_1(s)$. The control design for the nominal model has been described in Section III.

We need to find the maximum values of module on set Ψ

$$\Delta M(j\omega) = \max_{H_L(j\omega)\in\Psi} | H_L(j\omega) - H_1(j\omega) |; \quad \forall\omega. \qquad (18)$$

So

$$\delta(j\omega) = \frac{\Delta M(j\omega)}{| H_1(j\omega) |}. \qquad (19)$$

The robust stability condition (12) with (19) and (10) is to be checked. In the case of IMC the nominal closed loop transfer function is given by (11).

V. EXAMPLE

Consider a simple cylindrical tank. The tank is a nonlinear system whose time constant and gain vary considerably throughout the operating range. The controlled variable is the liquid height h and the control variable is the inlet flow rate Q_1 with the operating range from 0 to 0.04 m^3/s. The outlet flow rate depends on the liquid height according to the Torricelli's law. The control objective is to follow the changes of liquid height reference value as well as to reduce the effect of disturbance represented by the additional inlet flow rate $Q_3 = 0.017 \; m^3/s$ subject to the input/output constraints as well as changing plant dynamics caused by the operating point changes.

Both closed loop model reference control design algorithms described in Section III will be used. Let us denote the IMC controller as IMC and the controller with integrator as INTE.

Two examples will be presented. In the first one, the performances of both controllers will be investigated by simulations. In the second example robust controllers for a given operating range of the system will be designed and the robust stability analysis will be performed.

A. Control Performances

Let us first focus on the control performances of the controllers in the presence of the plant model mismatch.

The Laguerre model of the liquid height dynamics has been identified around an operating point given by $h_0 = 0.5\ m$ with the following parameters

$$H_1 = \begin{Bmatrix} c = \{4.4146;\ -2.0244\} \\ \alpha = 0.05,\ M = 2 \end{Bmatrix} \tag{20}$$

The small number of Laguerre coefficients has been intentionally chosen in order to increase the plant model mismatch.

For the open loop control design the same values of α and M have been used also for the Laguerre reference model and the controller x-expansion coefficients.

The Laguerre reference model is

$$H_m = \begin{Bmatrix} m = \{0.1581;\ 0\} \\ \alpha = 0.05,\ M = 2 \end{Bmatrix} \tag{21}$$

Let $E_g = 10$, $I_g = 0.01$ and the x-expansion coefficients are

$$d_0 = 0.0358;\ d_1 = -0.0164 \tag{22}$$

The simulation results for both control structures are compared in Fig. 6 and in Fig. 7. It can be seen, that the performances of INTE are much better than those of IMC. In the case of INTE controller, the zero steady-state error is obtained and the system output tracks the output of the reference model despite the system model uncertainty. For the IMC controller the knowledge of the exact system model is crucial; the model uncertainty results in the nonzero steady state error.

At time 1100 s the step change of Q_3 occurs. INTE controller is able to eliminate this disturbance; the regulation dynamics can be tuned by the parameters E_g and I_g. In the case of IMC, the offset caused by disturbance is not eliminated.

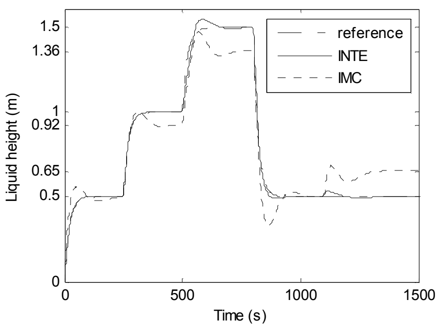

Figure 6. Simulation results – liquid height

The input constraints deteriorate the control performances of both controllers. If the calculated control signal exceeds its limit, it is truncated to the corresponding limiting value. For this reason the INTE controller exhibit small overshoot after the set point step change to 1.5 m.

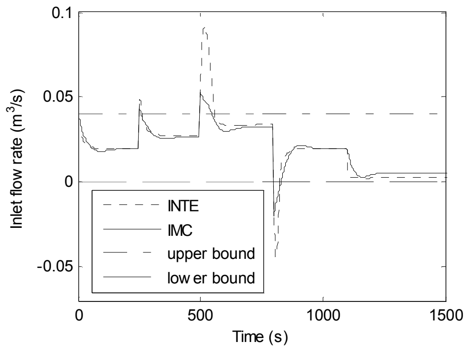

Figure 7. Simulation results – inlet flow rate

B. Robust Stability

Now the robustly stable controller will be designed for the tank operating range $h \in <0.5, 1.5>$ m.

First the nominal Laguerre model must be calculated from the Laguerre models at the endpoints of the operating range. Let $M = 5$, then the following Laguerre models have been obtained:

- for the operating point 0.5 m:

$$\begin{gathered} c_{0.5} = \{4.7644;\ -0.4017;\ 0.0339;\ -0.0029;\ 0.0002\} \\ \alpha_{0.5} = 0.02 \end{gathered} \tag{23}$$

- and for the operating point 1.5 m:

$$\begin{gathered} c_{1.5} = \{6.6601;\ 0.3859;\ 0.0224;\ 0.0013;\ 0.0001\} \\ \alpha_{1.5} = 0.01 \end{gathered} \tag{24}$$

Notice that the number of Laguerre coefficients for both models is the same, but the scaling parameter values are different. Using (17) the nominal Laguerre model is obtained

$$\begin{gathered} c_{nom} = \{5.7122;\ -0.0079;\ 0.0281;\ -0.0008;\ 0.0002\} \\ \alpha_{nom} = 0.015 \end{gathered} \tag{25}$$

which corresponds to the operating point 0.85 m.

Let $E_g = 10$ and $I_g = 0.01$. The following x-expansion coefficients of the controller are calculated based on the nominal Laguerre model

$$d = \{0.0233;\ 0.0126;\ 0.0067;\ 0.0036;\ 0.0019\} \tag{26}$$

for the reference model in the form

$$m = \{0.1322;\ 0.0717;\ 0.0386;\ 0.0208;\ 0.0112\} \quad \alpha = 0.015,\ M = 5 \tag{27}$$

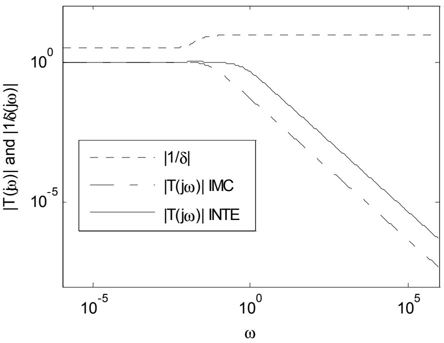

Figure 8. Amplitude characteristics of complex functions $|\hat{T}(j\omega)|$ and $|1/\delta(j\omega)|$

The sufficient condition for the robust stability is formulated by (12) and depicted in Fig. 8. It can be seen that both controllers are robustly stable. $|\delta(j\omega)|$ has been calculated using (19), $|\hat{T}(j\omega)|$ for INTE is given by (10) and for IMC (11) must be used. The simulation results are shown in Fig. 9 and in Fig. 10.

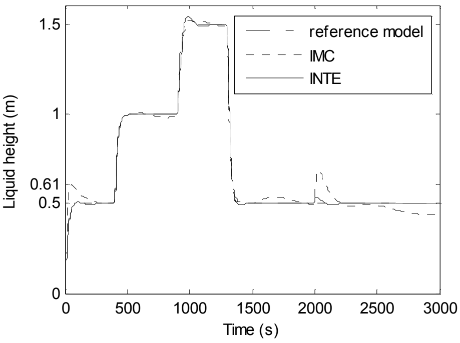

Figure 9. Simulation results – liquid height

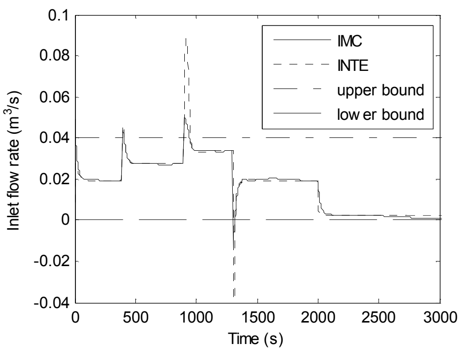

Figure 10. Simulation results – inlet flow rate

VI. Conclusion

In the paper the model reference control based on the modified Laguerre network has been presented. The control design results in an open-loop controller and therefore it cannot reject disturbances without feedback. To overcome this drawback, the IMC control strategy has been proposed. However, this control design necessitates the exact model of the system; otherwise it can result in control with nonzero steady state error. For this reason a new closed loop control algorithm using an integrator has been proposed which eliminates the shortcomings of the IMC controller. Moreover, it allows separating the tracking and regulations dynamics. The robust stability condition in the presence of the uncertainty in the Laguerre model of the system has been derived for both control structures. The performances of both controllers have been compared by simulations using a simple nonlinear system.

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