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AN APPLICATION OF SYMMETRICAL OPTIMUM METHOD TO SERVO SYSTEMS WITH VARIABLE INERTIA

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Abstract
The paper presents an application of the Symmetrical Optimum method under the form of the Extended Symmetrical Optimum method to the design of controllers for servo systems with variable inertia. A brushless direct current servo system with variable inertia is considered as the plant. A proportional-integral controller is tuned for the speed control of this plant using the Extended Symmetrical Optimum method. The results are shown for four values of the moment of inertia and two variable reference input shapes.

1 Introduction

Successful applications of brushless direct current servo systems are based on several controllers. Such controllers are: predictive controllers [1], direct torque control [2] and augmented with indirect flux control [3], optimal controllers [4], fault diagnosis in control [5], proportional-integral (PI), and proportional-integral-derivative (PID) and fuzzy control [6]–[10].

The sliding mode / variable structure controllers [11]–[13] are relevant for servo control where simple and fast solutions are needed. In this view the simplicity and robustness of these controllers gives successful servo system control applications reported in [14]–[16].

The Symmetrical Optimum method was initially formulated in [17], [18], to tune PI and PID controllers for benchmark-type plant models. This method became widely applied in the field of electrical drives, servo systems and robotics. Several generalizations and applications are presented as follows.

The Extended Symmetrical Optimum method [19], [20], is characterized by only one design parameter, which offers flexibility in imposing the phase margin and the other performance indices. Gain and phase margins are considered as performance indices in [21] and the damping factor in [22]. Combinations with state feedback control and model predictive control are presented in [23]–[25]. The generalization to plants with more than one integrator is proposed and extensively investigated in [26]–[30]. Applications to fuzzy controllers tuning are given in [31]–[34]. Stability issues including robust stability, controller robustness and pole placement techniques are discussed in [35], [36]. Recent combinations with phase locked loop-based algorithms and robotics are presented in [37]–[42].

This paper applies the Extended Symmetrical Optimum method to the design of a PI controller for a brushless direct current servo system with variable inertia. The controlled plant represented by the brushless direct current motor is modeled using the detailed and simplified equations presented in [6], [7].

The paper is organized as follows: the simplified model of the plant is presented in Section 2. The design of the PI controller for speed control is presented in Section 3. The case study, the simulation results and the conclusions are given in Sections 4 and 5.

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2 Simplified Model of the Plant

The following state-space equations of the detailed model of the plant are obtained from [6] by neglecting the friction in speed control applications if the three state variables are $x_1 = i_a$, $x_2 = \omega_m$ and $x_3 = f_t$:

$$\begin{aligned}
\dot{x}_1(t) &= -\frac{R_a}{L_a}x_1(t) - \frac{k_e}{L_a}x_2(t) - \frac{k_E}{L_a}u(t),\\
\dot{x}_2(t) &= \frac{k_m}{J_e(t)}x_1(t) - \frac{1}{J_e(t)}m_f(t)[x_2(t) - \frac{r_t(t)}{J_e(t)}x_3(t)] - \frac{1}{J_e(t)}\dot{J}_e(t)r_t(t),\\
\dot{x}_3(t) &= -c_b r_t(t)x_2(t) - c_b m_{Load}(t),
\end{aligned} \tag{1}$$

where $J_e(t)$ is the variable moment of inertia, $m_{Load}(t)$ is the load torque, which is also the external disturbance, $i_a(t)$ is the current, $\omega_m(t)$ is the variable angular speed, $v_t(t)$ is the linear speed, $u(t)$ is the control signal, $f_t(t)$ is the force that acts on the strip in the framework of a winding process, and $r_t(t)$ is the measured radius of the strip that is rolled on rotating drum. The other parameters in (1), which are specific to the electrical part of the motor and to the mechanical part of the motor, are constant. As shown in [7], the linearization of the model (1) of the plant at representative operating points leads to the following model as benchmark-type transfer function for speed control:

$$P(s) = \frac{k_P}{s(1+s\,T_\Sigma)(1+s\,T_1)}, \tag{2}$$

where k_P is the gain of the plant, T_1 is the mechanical time constant, T_Σ is the small time constant, $T_1 >> T_\Sigma$.

The parameters k_P and T_1 in (2) are time variant because they depend on $J_e(t)$. The controllers can be designed and tuned relatively easily if the inertia $J_e(t)$ varies within a reasonable range. The variation of $J_e(t)$ is achieved by the continuous variation of the radius $r_t(t)$ in winding processes. However, the computation of $J_e(t)$ is not simple. Moreover, the model presented in (2) is an approximate model that can be better used in position control applications, but the approximation is justified due to the ranges of the time constants, and the tuning method ensures robustness.

3 Design of speed controller

The Extended Symmetrical Optimum method recommends PI controllers for the plant with the transfer function presented in (2). The transfer function of a PI controller is:

$$C(s) = \frac{k_c}{s}(1+s\,T_i), \tag{3}$$

where k_c is the gain of the controller gain and T_i is the integral time constant of the controller. The tuning conditions for the PI controller (3) are [19], [20]:

$$k_c = \frac{1}{\beta\sqrt{\beta}k_P T_\Sigma^2}, \tag{4}$$

$$T_i = \beta\,T_\Sigma, \tag{5}$$

where β is the design parameter. The right choice of this design parameter according to the diagrams given in [19] and [20] offers a compromise to several performance indices imposed to the control system: rise time, settling time, overshoot and phase margin. Kessler's value [17], [18] is

$$\beta = 4, \tag{6}$$

and the recommended extended domain for the parameter β is [19], [20]:

$$4 \le \beta \le 20. \tag{7}$$

The application of the tuning conditions (4) and (5) leads to the following open-loop transfer function $G_{open}(s)$ and to closed-loop transfer function $G_{closed}(s)$ with respect to the reference input:

$$G_{open}(s) = \frac{1+\beta T_\Sigma s}{\beta\sqrt{\beta} T_\Sigma^2 s^2 (1+T_\Sigma s)}, \tag{8}$$

$$G_{closed}(s) = \frac{1+\beta T_\Sigma s}{\beta\sqrt{\beta} T_\Sigma^3 s^3 + \beta\sqrt{\beta} T_\Sigma^2 s^2 + \beta T_\Sigma s + 1}. \tag{9}$$

Equation (9) shows the presence of a zero and of three poles. The compensation of a zero and of one or more poles can lead to performance improvement. Therefore, the two-degrees-of-freedom (2-DOF) control system structure can be used as shown in Figure 1, where r is the reference input, y is the regulated output, $F(s)$ is the transfer function of the reference input filter, r_1 is the filtered reference input, $e = r_1 - y$ is the control error, CU is the comparing unit, u is the control signal and d is the disturbance input.

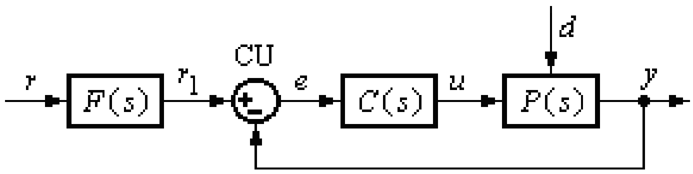

Figure 1. The structure of the 2-DOF control system

The simplest version of reference input filter from [19] and [20] is applied in this paper. The transfer function of the filter is:

$$F(s) = \frac{1}{1+\beta T_\Sigma s}. \tag{10}$$

The filter presented in (10) ensures improved control system performance because the transfer function of the closed-loop system is modified to $G_{2-DOF}(s)$:

$$G_{2-DOF}(s) = F(s) G_{closed}(s) = \frac{1}{\beta\sqrt{\beta} T_\Sigma^3 s^3 + \beta\sqrt{\beta} T_\Sigma^2 s^2 + \beta T_\Sigma s + 1}. \tag{11}$$

4 Results of digital simulation

The design aspects presented in Section 3 are applied to the speed control of a brushless direct current servo system considered as controlled plant and described in [43]. This laboratory servo system is advantageous because of the modification of the inertia. The values of a set of parameters of this laboratory servo system are [6], [7], [43]: $p = 2$, $R_s = 1\,\Omega$, $L_s = 0.02\,\mathrm{H}$, $V_{DC} = 220\,\mathrm{V}$, and the nominal inertia $J_{e0} = 0.005\,\mathrm{kg\,m^2}$. The values of the other parameters are presented in [43]. The values of the parameters in (1) are: $R_a = 1\,\Omega$, $L_a = 0.02\,\mathrm{H}$, $k_e = 0.088$, $k_E = 0.0206$, $k_m = 0.02$, $c_b = 0.054$, $\dot{J}_e(t) = 0$ because $J_e(t) = \mathrm{const}$, and $m_{Load}(t) = 0$.

The application of a simple least-squares identification leads to the well approximated transfer function (2) of the controlled plant, with the following parameters obtained for the nominal value of the inertia $J_e(t)$, namely $J_{e0} = 0.005\,\mathrm{kg\,m^2}$:

$$\begin{aligned} k_P &= 0.3286, \\ T_\Sigma &= 0.0015\,\mathrm{s}, \\ T_1 &= 0.015\,\mathrm{s}. \end{aligned} \tag{12}$$

The value of the design parameter of the PI controller is set as:

$$\beta = 9. \tag{13}$$

The tuning conditions (4) and (5) are applied and the values of the parameters of the PI controller used as a speed controller are:

$$\begin{aligned} k_c &= 5.0094 \cdot 10^4, \\ T_i &= 0.0135\,\mathrm{s}. \end{aligned} \tag{14}$$

The simulation results expressed as the variation of the angular speed as regulated output $y = \omega_m$ for one form of variation of the reference input is illustrated in Figure 2. Figure 2 shows that the reference input is variable because this brushless direct current motor is used as a servo system. The results presented in Figure 2 are obtained for four values of the inertia: the nominal value $J_{e0} = 0.005\,\mathrm{kg\,m^2}$, a five times smaller value $J_e = 0.001\,\mathrm{kg\,m^2}$, a three times larger value $J_e = 0.015\,\mathrm{kg\,m^2}$ and a five times larger value $J_e = 0.025\,\mathrm{kg\,m^2}$.

The simulation results for another form of variation of the reference input are presented in Figure 3. These results are obtained for the same four values of the inertia as those considered in Figure 2.

The simulation results presented in Figure 2 and Figure 3 show the operation of the designed controller on the exact nonlinear, time-varying system described by (1). The disturbance input is not applied.

The simulation results presented in Figure 2 a and Figure 3 a prove the good control system performance of the speed control system. But the zoomed plots presented in Figure 2 b and Figure 3 b show that the performance depends on the inertia. This can be critical for abrupt changes of the reference input as, for example, for step-type reference inputs. The performance can be improved if a more complicated reference input filter is designed.

5 Conclusions

This paper has presented an application of one generalized version of the Symmetrical Optimum method to the design of speed controllers for brushless direct current servo systems. Simple PI controllers are obtained and tested by simulation results for four inertias considering two forms of variations of the reference inputs.

The results of the digital simulations show the good performance of the speed control systems. The effects of the variable inertia are also shown. These effects require the design of adaptive controllers and/or nonlinear controllers.

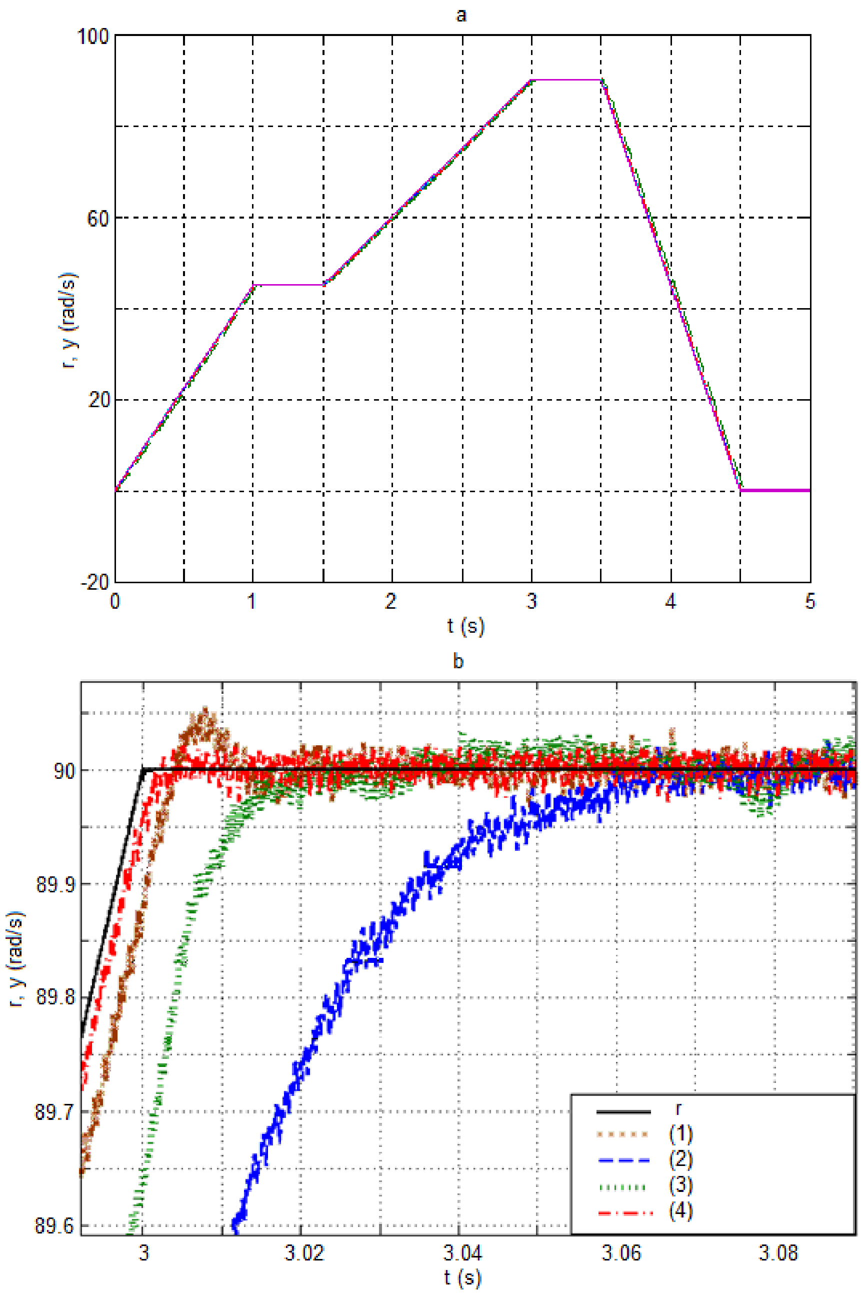

Figure 2. a: the regulated output and the first form of the reference input for four values of J_e*:* $J_{e0} = 0.005\,\text{kg m}^2$ *(1),* $J_e = 0.001\,\text{kg m}^2$ *(2),* $J_e = 0.015\,\text{kg m}^2$ *(3),* $J_e = 0.025\,\text{kg m}^2$ *(4), b: zoomed regulated output and reference input*

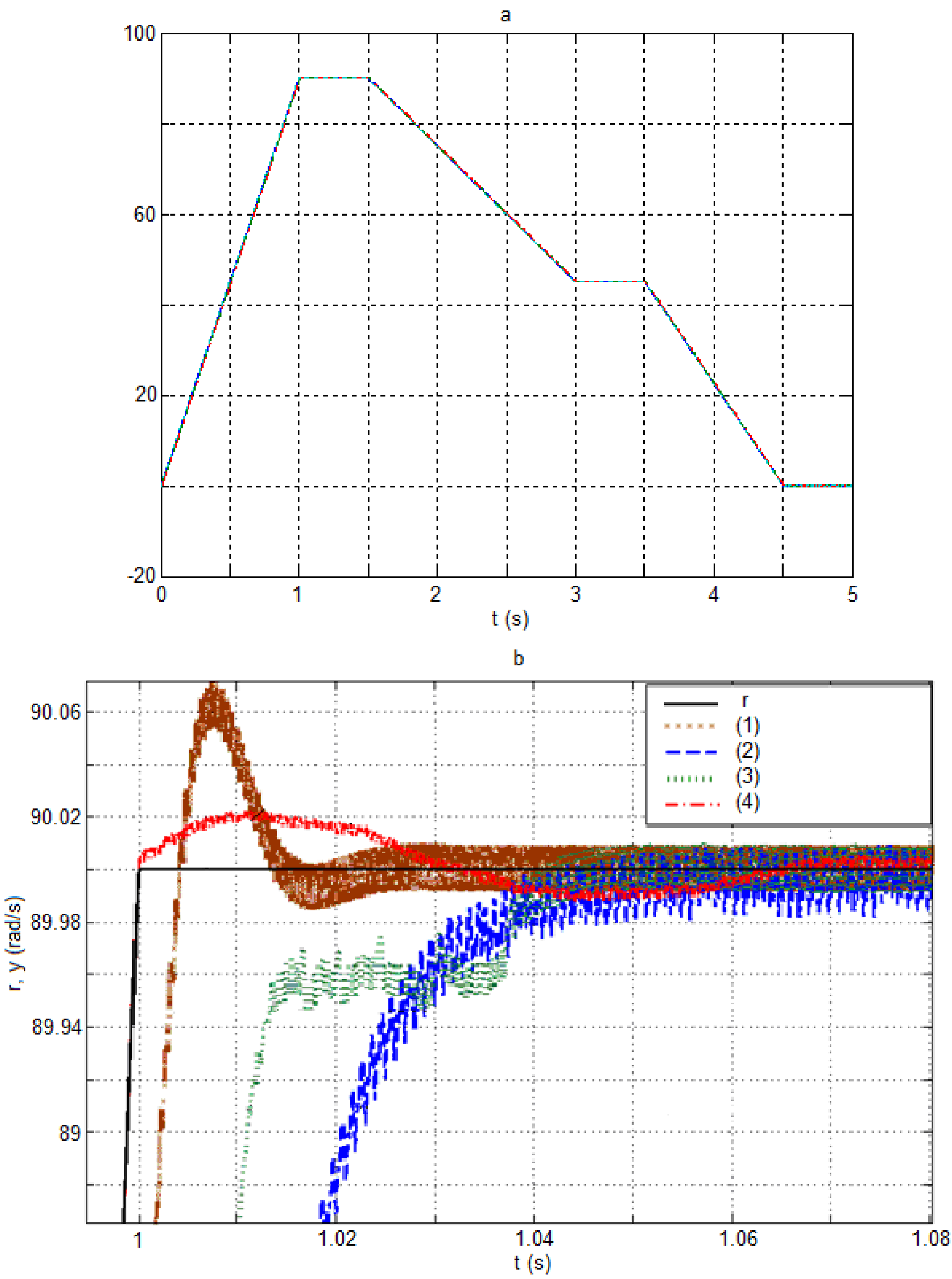

Figure 3. a: the regulated output and the second form of the reference input for four values of J_e*:* $J_{e0} = 0.005\,\mathrm{kg\,m^2}$ *(1),* $J_e = 0.001\,\mathrm{kg\,m^2}$ *(2),* $J_e = 0.015\,\mathrm{kg\,m^2}$ *(3),* $J_e = 0.025\,\mathrm{kg\,m^2}$ *(4), b: zoomed regulated output and reference input*

Future research will be focused on the application of the Magnitude Optimum method in the design of the controllers. Several extensions of this method can be considered [44]–[48]. The validation of the PI controllers as digital controllers in real experiments will be of interest and associated with several specific nonlinearities including the application of digital anti-windup blocks.

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