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Numerical results on noisy blown-up matrices*

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Abstract

We study the eigenvalues of large perturbed matrices. We consider an Hermitian pattern matrix P of rank k. We blow up P to get a large block-matrix B_n. Then we generate a random noise W_n and add it to the blown up matrix to obtain the perturbed matrix $A_n = B_n + W_n$. Our aim is to find the eigenvalues of B_n. We obtain that under certain conditions A_n has k 'large' eigenvalues which are called structural eigenvalues. These structural eigenvalues of A_n approximate the non-zero eigenvalues of B_n. We study a graphical method to distinguish the structural and the non-structural eigenvalues. We obtain similar results for the singular values of non-symmetric matrices.

Keywords: Eigenvalue, symmetric matrix, blown-up matrix, random matrix, perturbation of a matrix, singular value

MSC: 15A18, 15A52

1. Introduction

Spectral theory of random matrices has a long history (see e.g. [1, 5–8] and the references therein). This theory is applied when the spectrum of noisy matrices is

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considered. In [2] and [3] the eigenvalues and the singular values of large perturbed block matrices were studied. In [4] we extended the results of [2] and [3] and we suggested a graphical method to study the asymptotic behaviour of the eigenvalues. In this paper we consider the following important question for the numerical behaviour of the eigenvalues. How large the blown-up matrix should be in order to show the asymptotic behaviour given in the mathematical theorems? We also study the influence of the signal to noise ratio for our results.

Thus we consider a fixed deterministic pattern matrix P, we blow up P to obtain a 'large' block-matrix B_n, then we add a random noise matrix W_n. We present limit theorems for the eigenvalues of $A_n = B_n + W_n$ as $n \to \infty$ and show corresponding numerical results. We also consider a graphical method to distinguish the structural and the non-structural eigenvalues. This test is important, because in real-life only the perturbed matrix A_n is observed, but we are interested in eigenvalues or the singular values of B_n which are approximated by the above mentioned structural eigenvalues/singular values of A_n.

In Section 2 we list some theoretical results of [4]. In Section 3 the numerical results are presented. We obtained that the asymptotic behaviour of the eigenvalues can be seen for relatively low values of n, that is if the block sizes are at least 50, then we can use our asymptotic results. We also study the influence of the signal/noise ratio on the gap between the structural and non-structural singular values. Our numerical results support our graphical test visualised by Figure 1.

2. Eigenvalues of perturbed symmetric matrices

In this section we study the perturbations of Hermitian (resp. symmetric) blown-up matrices. We would like to examine the eigenvalues of perturbed matrices.

We use the following notation:

- P is a fixed complex Hermitian (in the real valued case symmetric) $k \times k$ pattern matrix of rank r
- p_{ij} is the (i, j)'th entry of P
- $n_1, \dots, n_k$ are positive integers, $n = \sum_{i=1}^{k} n_i$
- $\tilde{B}_n$ is an $n \times n$ matrix consisting of k^2 blocks, its block (i, j) is of size $n_i \times n_j$ and all elements in that block are equal to p_{ij}
- B_n is called a blown-up matrix if it can be obtained from $\tilde{B}_n$ by rearranging its rows and columns using the same permutation

Following [2], we shall use the growth rate condition

$$n \to \infty \quad \text{so that} \quad n_i/n \geq c \quad \text{for all} \quad i, \tag{2.1}$$

where $c > 0$ is a fixed constant. Here we list those theorems of [4] which will be tested by numerical methods.

Proposition 2.1. *Let P be a symmetric pattern matrix, that is a fixed complex Hermitian (in the real valued case symmetric) $k \times k$ matrix of rank r. Let B_n be the blown-up matrix of P. Then B_n has r non-zero eigenvalues. If condition* (2.1) *is satisfied, then the non-zero eigenvalues of B_n are of order n in absolute value.*

Now, we assume that $rank(P) = k$. We shall consider the eigenvalues in descending order, so we have $|\lambda_1(B_n)| \geq \cdots \geq |\lambda_n(B_n)|$. Since k eigenvalues of B_n are non-zero and the remaining ones are equal to zero, we shall call the first k ones structural eigenvalues of B_n. Similarly, we shall call structural eigenvalue that eigenvalue of A_n, which corresponds to a structural eigenvalue of B_n. This correspondence will be described by Theorem 2.2 and Corollary 2.3. We shall see that the magnitude of any structural eigenvalue is large and it is small for the other eigenvalues.

First we consider perturbations by Wigner matrices. Next theorem is a generalization of Theorem 2.3 of [2] where the real valued case and uniformly bounded perturbations were considered.

Theorem 2.2. *Let B_n, $n = 1, 2, \ldots$, be a sequence of complex Hermitian matrices. Let the Wigner matrices W_n, $n = 1, 2, \ldots$, be complex Hermitian $n \times n$ random matrices satisfying the following assumptions. Let the diagonal elements w_{ii} of W_n be i.i.d. (independent and identically distributed) real, let the above diagonal elements be i.i.d. complex random variables and let all of these be independent. Let W_n be Hermitian, that is $w_{ij} = \bar{w}_{ji}$ for all i, j. Assume that $\mathbb{E}w_{11}^2 < \infty$, $\mathbb{E}w_{12} = 0$, $\mathbb{E}|w_{12} - \mathbb{E}w_{12}|^2 = \sigma^2$ is finite and positive, $\mathbb{E}|w_{12}|^4 < \infty$. Then*

$$\limsup_{n\to\infty} \frac{|\lambda_i(B_n + W_n) - \lambda_i(B_n)|}{\sqrt{n}} \leq 2\sigma$$

for all i almost surely.

Corollary 2.3. *Let B_n, $n = 1, 2, \ldots$, be blown-up matrices of a complex Hermitian matrix P having rank k. Assume that condition* (2.1) *is satisfied. Let the Wigner matrices W_n, $n = 1, 2, \ldots$, satisfy the conditions of Theorem 2.2. Then Theorem 2.2 and Proposition 2.1 imply that $B_n + W_n$ has k eigenvalues of order n and the remaining eigenvalues are of order $\sqrt{n}$ almost surely.*

So the structural eigenvalues have magnitude n while the non-structural eigenvalues have magnitude $\sqrt{n}$.

3. Singular values of perturbed matrices

In this section we study the perturbations of arbitrary blown-up matrices. We are interested in the singular values of matrices perturbed by certain random matrices. We use the following notation:

- P is a fixed complex $a \times b$ pattern matrix of rank r

- p_{ij} is the (i,j)'th entry of P
- $m_1, \ldots, m_a$ are positive integers, $m = \sum_{i=1}^{a} m_i$
- $n_1, \ldots, n_b$ are positive integers, $n = \sum_{i=1}^{b} n_i$
- $\tilde{B}_n$ is an $m \times n$ matrix consisting of $a \times b$ blocks, its block (i,j) is of size $m_i \times n_j$ and all elements in that block are equal to p_{ij}
- B_n is called blown-up matrix if it can be obtained from $\tilde{B}$ by rearranging its rows and columns

Following [3], we shall use the growth rate condition

$$m, n \to \infty \quad \text{so that} \quad m_i/m \geq c \quad \text{and} \quad n_i/n \geq d \quad \text{for all} \quad i, \tag{3.1}$$

where $c, d > 0$ are fixed constants. The following proposition is an extension of Proposition 6 of [3] to the complex valued case.

Proposition 3.1. *Let P be a fixed complex $a \times b$ matrix of rank r. Let B be the $m \times n$ blown-up matrix of P. If condition* (3.1) *is satisfied, then the non-zero singular values of B are of order $\sqrt{mn}$.*

Now we consider perturbation with matrices having independent and identically distributed (i.i.d.) complex entries. Let x_{jk}, $j, k = 1, 2, \ldots$, be an infinite array of i.i.d. complex valued random variables with mean 0 and variance σ^2. Let $X = (x_{jk})_{j=1,\ k=1}^{m,\ \ n}$ be the left upper block of size $m \times n$.

Theorem 3.2. *For each m and n let $B = B_{mn}$ be a complex matrix of size $m \times n$ and let $X = X_{mn}$ be the above complex valued random matrix of size $m \times n$ with i.i.d. entries. Moreover, assume that the entries of X have finite fourth moments. Assume that $m, n \to \infty$ so that $K_1 \leq \frac{m}{n} \leq K_2$, where $0 < K_1 \leq K_2 < \infty$ are fixed constants. Denote by s_i and z_i the singular values of B and $B + X$, respectively, $s_1 \geq \cdots \geq s_{\min\{m,n\}}$, $z_1 \geq \cdots \geq z_{\min\{m,n\}}$. Then for all i*

$$|s_i - z_i| = \mathrm{O}(\sqrt{n})$$

as $m, n \to \infty$ almost surely.

Corollary 3.3. *Proposition 3.1 and Theorem 3.2 imply the following. Let P be a fixed complex $a \times b$ matrix of rank r. Let B be the $m \times n$ blown-up matrix of P. Assume that condition* (3.1) *is satisfied. Let X be complex valued perturbation matrices satisfying the assumptions of Theorem 3.2. Denote by z_i the singular values of $B + X$, $z_1 \geq \cdots \geq z_{\min\{m,n\}}$. Then z_i are of order n for $i = 1, \ldots, r$ and $z_i = \mathrm{O}(\sqrt{n})$ for $i = r+1, \ldots, \min\{m,n\}$ almost surely as $m, n \to \infty$ so that $K_1 \leq \frac{m}{n} \leq K_2$, where $0 < K_1 \leq K_2 < \infty$ are fixed constants.*

So the structural singular values (i.e. the largest r values) are 'large', and the remaining ones are 'small'.

4. Numerical results

4.1. Eigenvalues of a symmetric matrix perturbed with Wigner noise

In [4] we concluded the following facts. In the case when the perturbation matrix has zero mean random entries, then the structural eigenvalues are 'large' and the non-structural ones are 'small'. More precisely let $|\lambda_1| \geq |\lambda_2| \geq \dots$ be the absolute values of the eigenvalues of the perturbed blown-up matrix in descending order. Then the structural eigenvalues $|\lambda_1| \geq |\lambda_2| \geq \dots \geq |\lambda_l|$ are 'large' and they rapidly decrease. The other eigenvalues $|\lambda_{l+1}| \geq |\lambda_{l+2}| \geq \dots$ are relatively small and they decrease very slowly. To obtain the structural eigenvalues we can use the following numerical (graphical) procedure. Calculate some eigenvalues of A_n starting with the largest ones in absolute value. Stop when the last 5-10 eigenvalues are close to zero and they are almost the same in absolute value. Then we obtain the increasing sequence $|\lambda_t| \leq |\lambda_{t-1}| \leq \dots \leq |\lambda_1|$. Plot their values in the above order, then find the first abrupt change. If, say,

$$0 \approx |\lambda_t| \approx |\lambda_{t-1}| \approx \dots \approx |\lambda_{l+1}| \ll |\lambda_l| < \dots < |\lambda_1|,$$

that is the first abrupt change is at l, then $\lambda_l, \lambda_{l-1}, \dots, \lambda_1$ can be considered as the structural eigenvalues. The typical abrupt change after the non-structural eigenvalues can be seen in Figure 1.

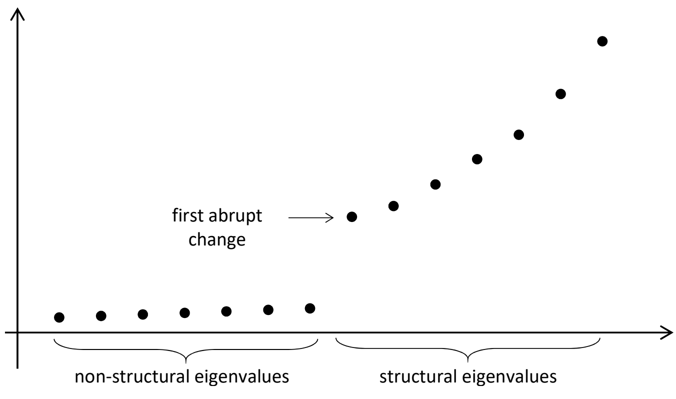

Figure 1: The abrupt change after the non-structural eigenvalues

Our first example supports Theorem 2.2 and Corollary 2.3 in the real valued case. The following results were obtained using the Julia programming language version 1.1.1. The simulations were divided into four steps. Let P be the real symmetric pattern matrix

$$\begin{bmatrix} 8 & 7 & 2 & 5 & 3 & 2 & 4 & 0 & 3 & 1 \\ 7 & 9 & 6 & 3 & 4 & 0 & 2 & 5 & 2 & 0 \\ 2 & 6 & 7 & 6 & 5 & 4 & 2 & 0 & 3 & 4 \\ 5 & 3 & 6 & 8 & 7 & 6 & 0 & 5 & 4 & 2 \\ 3 & 4 & 5 & 7 & 9 & 8 & 8 & 6 & 5 & 1 \\ 2 & 0 & 4 & 6 & 8 & 7 & 6 & 8 & 0 & 4 \\ 4 & 2 & 2 & 0 & 8 & 6 & 9 & 7 & 6 & 6 \\ 0 & 5 & 0 & 5 & 6 & 8 & 7 & 8 & 8 & 4 \\ 3 & 2 & 3 & 4 & 5 & 0 & 6 & 8 & 9 & 6 \\ 1 & 0 & 4 & 2 & 1 & 4 & 6 & 4 & 6 & 5 \end{bmatrix}.$$

In the initial step we have matrix P. It has the following eigenvalues: 45.302, 15.497, 10.914, 7.677, −7.245, 6.188, 4.696, −3.789, −3.381 and 3.141. This matrix is blown up with the help of a vector containing the size informations of the blocks. If the vector is $\mathbf{n} = [n_1, n_2, \dots, n_{10}]$, then the first row of blocks is built with the following block sizes: $n_1 \times n_1$, $n_1 \times n_2$, $\dots n_1 \times n_{10}$, the second row of blocks: $n_2 \times n_1$, $n_2 \times n_2$, $\dots n_2 \times n_{10}$ and we continue this pattern till the last row. In different simulations we used different vectors $\mathbf{n}$ to blow up the matrix, it will be detailed separately for each simulation.

The next step is to generate the noise matrix, which is a symmetric real Wigner matrix, as defined in Section 2. The entries are generated in the following way: let the diagonal elements w_{ii} of W_n be i.i.d. real with standard normal distribution, let the above diagonal elements be i.i.d. real random variables and let all of these be independent. The size of the noise matrix is equal to the size of the blown up matrix. After that, in order to obtain the noisy matrix, in each iteration we generate a new Wigner matrix and add to the blown up matrix.

As the last step we calculate the eigenvalues of the perturbed matrix. To do so we used the *eigvals* function from the *LinearAlgebra* package. Julia provides native implementations of many common and useful linear algebra operations which can be loaded with `using LinearAlgebra`, to install the package we used `using Pkg` and then with the `Pkg.add("LinearAlgebra")` command we installed the package.

We studied 4 different schemes to blow up matrix P, in each of these 4 schemes we applied 6 different block size vectors. These block series are given in Table 1.

We realised 1000 simulations for all block series in Table 1 and checked if the abrupt change like in Figure 1 was seen. For very low values of n_i we did not find the abrupt change. In each case the first well distinguishable abrupt change appeared when n_1 was 50. This result is shown in Figures 2, 3, 4, and 5. At each figure the left side: $|\lambda_{20}(A_n)| < \cdots < |\lambda_1(A_n)|$; right side: $|\lambda_{20}(A_n)| < \cdots < |\lambda_6(A_n)|$ in a typical realization in our example. The right side figures show only a part of the sequence around the change, so the jumps are clearly seen.

Each of the figures shows that there is an abrupt change between the 11^{th} and 10^{th} eigenvalues. So we can decide that the 10 largest eigenvalues are the structural ones (which is the true value, since we used matrix P having rank 10).

Name of the scheme	Series of the blocks
Equal	$n_1 = n_2 = \cdots = n_{10} = 10$
	$n_1 = n_2 = \cdots = n_{10} = 20$
	$n_1 = n_2 = \cdots = n_{10} = 50$
	$n_1 = n_2 = \cdots = n_{10} = 100$
	$n_1 = n_2 = \cdots = n_{10} = 500$
	$n_1 = n_2 = \cdots = n_{10} = 1000$
Two types	$n_1 = n_2 = \cdots = n_5 = 10, n_6 = \cdots = n_{10} = 20$
	$n_1 = n_2 = \cdots = n_5 = 20, n_6 = \cdots = n_{10} = 40$
	$n_1 = n_2 = \cdots = n_5 = 50, n_6 = \cdots = n_{10} = 100$
	$n_1 = n_2 = \cdots = n_5 = 100, n_6 = \cdots = n_{10} = 200$
	$n_1 = n_2 = \cdots = n_5 = 500, n_6 = \cdots = n_{10} = 1000$
	$n_1 = n_2 = \cdots = n_5 = 1000, n_6 = \cdots = n_{10} = 2000$
Arithmetic progression	$n_1 = 10, n_2 = 20, \ldots, n_{10} = 100$
	$n_1 = 20, n_2 = 30, \ldots, n_{10} = 110$
	$n_1 = 50, n_2 = 60, \ldots, n_{10} = 140$
	$n_1 = 100, n_2 = 110, \ldots, n_{10} = 190$
	$n_1 = 500, n_2 = 510, \ldots, n_{10} = 590$
	$n_1 = 1000, n_2 = 1010, \ldots, n_{10} = 1090$
Four types	$n_1 = n_2 = 10, n_3 = n_4 = n_5 = 20,$ $n_6 = n_7 = n_8 = 30, n_9 = n_{10} = 40$
	$n_1 = n_2 = 20, n_3 = n_4 = n_5 = 40,$ $n_6 = n_7 = n_8 = 60, n_9 = n_{10} = 80$
	$n_1 = n_2 = 50, n_3 = n_4 = n_5 = 100,$ $n_6 = n_7 = n_8 = 150, n_9 = n_{10} = 200$
	$n_1 = n_2 = 100, n_3 = n_4 = n_5 = 200,$ $n_6 = n_7 = n_8 = 300, n_9 = n_{10} = 400$
	$n_1 = n_2 = 500, n_3 = n_4 = n_5 = 1000,$ $n_6 = n_7 = n_8 = 1500, n_9 = n_{10} = 2000$
	$n_1 = n_2 = 1000, n_3 = n_4 = n_5 = 2000,$ $n_6 = n_7 = n_8 = 3000, n_9 = n_{10} = 4000$

Table 1: Schemes used to blow up matrix P

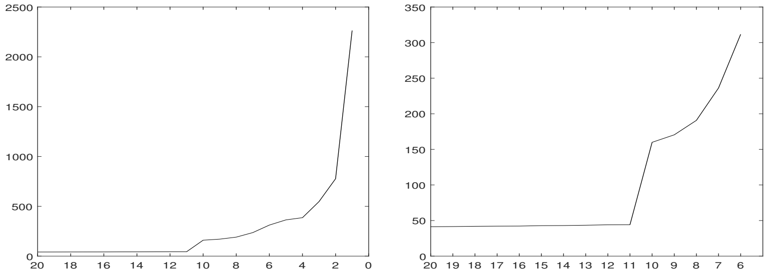

Figure 2: Equal: $n_1 = n_2 = \cdots = n_{10} = 50$

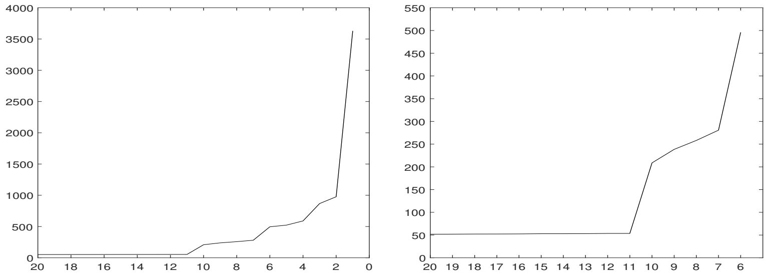

Figure 3: Two types: $n_1 = \cdots = n_5 = 50$, $n_6 = \cdots = n_{10} = 100$

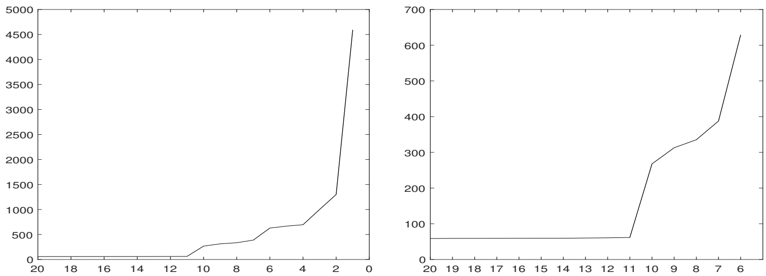

Figure 4: Arithmetic progression: $n_1 = 50, n_2 = 60, \ldots, n_{10} = 140$

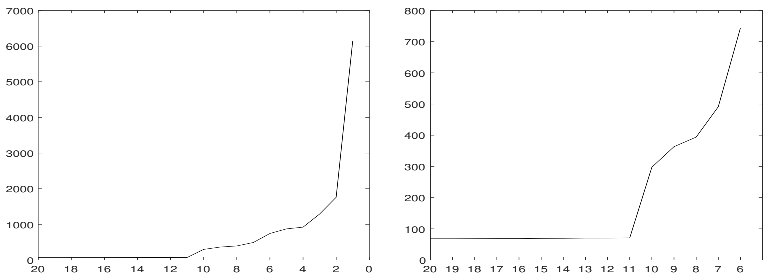

Figure 5: Four types: $n_1 = n_2 = 50, n_3 = n_4 = n_5 = 100,$
$n_6 = n_7 = n_8 = 150, n_9 = n_{10} = 200$

As the block's sizes are increasing, the border between the structural and non-structural eigenvalues is getting more and more significant, as it is shown in Figure 6. We see, that when $n_i = 1000$ (for all i), then the gap is much larger than in the case of $n_i = 50$.

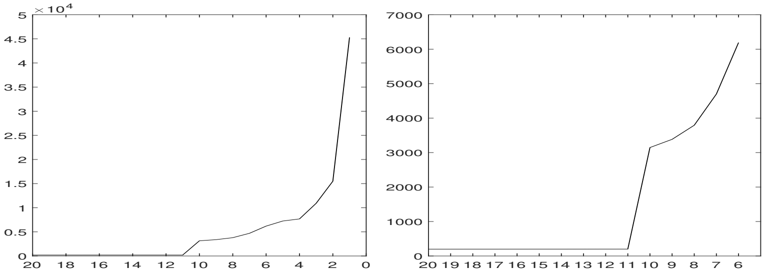

Figure 6: Equal: $n_1 = n_2 = \cdots = n_{10} = 1000$

In Table 2, we list the averages of the absolute values of the eigenvalues making 1000 repetitions in each case. More precisely, n_i means that all blocks were $n_i \times n_i$ and in the column below n_i there are the 20 largest of the averages of the absolute values of the corresponding eigenvalues of different A_n matrices. We see, that there is a gap between the 10^{th} and 11^{th} values even in the relatively small value of $n_i = 10$. The larger the value of n_i, the larger the gap. This table shows that our test is very reliable for moderate (e.g. $n_i = 50$) sizes of the blocks.

j	$n_i = 10$	$n_i = 20$	$n_i = 50$	$n_i = 100$	$n_i = 200$	$n_i = 500$	$n_i = 1000$
1	453.26	906.28	2265.40	4530.50	9060.70	22652.00	45303.00
2	155.64	310.65	775.43	1550.31	3100.00	7748.90	15497.00
3	110.06	219.28	546.65	1092.40	2183.90	5458.00	10915.00
4	77.99	154.82	385.08	768.93	1536.70	3839.60	7677.90
5	73.76	146.21	363.63	725.90	1450.40	3624.10	7246.90
6	63.36	125.32	310.93	620.41	1239.20	3095.40	6189.40
7	48.86	95.95	236.92	471.66	941.21	2349.90	4697.80
8	40.51	78.39	192.04	381.54	760.54	1897.10	3791.70
9	36.30	70.32	171.90	340.95	678.98	1693.20	3383.40
10	34.06	65.84	160.18	317.23	631.47	1573.90	3144.70
11	18.56	27.21	44.03	62.73	89.04	141.14	199.78
12	17.96	26.70	43.58	62.33	88.67	140.84	199.50
13	17.45	26.27	43.20	61.99	88.37	140.58	199.27
14	17.03	25.92	42.90	61.72	88.14	140.38	199.09
15	16.65	25.59	42.62	61.47	87.92	140.18	198.92
16	16.33	25.28	42.36	61.25	87.72	140.01	198.77
17	15.99	25.00	42.11	61.03	87.53	139.85	198.63
18	15.69	24.74	41.89	60.83	87.34	139.69	198.49
19	15.40	24.47	41.67	60.64	87.17	139.54	198.36
20	15.12	24.23	41.47	60.45	87.01	139.40	198.23

Table 2: Eigenvalues in the case of equal size blocks

4.2. Singular values of a non-symmetric perturbed matrix

This example supports Theorem 3.2 and Corollary 3.3. Let P be the 7×8 real non-symmetric pattern matrix

$$\begin{bmatrix} 6 & 5 & 6 & 5 & 3 & 2 & 1 & 2 \\ 3 & 9 & 6 & 7 & 4 & 5 & 6 & 1 \\ 4 & 8 & 9 & 8 & 3 & 4 & 2 & 1 \\ 5 & 7 & 6 & 8 & 7 & 5 & 3 & 2 \\ 2 & 5 & 7 & 8 & 9 & 6 & 5 & 3 \\ 1 & 3 & 4 & 5 & 6 & 7 & 6 & 4 \\ 2 & 1 & 4 & 6 & 7 & 8 & 9 & 9 \end{bmatrix}.$$

In the previous example we showed the effect of the blocks' sizes, now we show the effect of the signal to noise ratio (snr) on the singular values of P. To blow up P, we chose block heights as $[m_1, \dots, m_a] = [500, 750, 500, 600, 750, 550, 500]$ and block widths as $[n_1, \dots, n_b] = [500, 500, 600, 1000, 550, 500, 550, 500]$. Then we blew up P, and we added a noise to get the perturbed matrix. Like in Section 3, we denote by B the blown-up matrix which is the signal, by X the noise matrix, and by $A = B + X$ the perturbed matrix. We shall use the following definition of the signal to noise ratio

$$\texttt{snr} = \frac{\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} b_{i,j}^2}{\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{i,j}^2}.$$

Here $m = \sum_{i=1}^{a} m_i$ is the number of rows, $n = \sum_{j=1}^{b} n_j$ is the number of columns in B and X, $b_{i,j}$ is the element of the signal matrix B, while $x_{i,j}$ is the element of the noise matrix X in the i^{th} row and j^{th} column.

In this example, the entries of the noise matrix X are independent and, at the initial step, they are from standard normal distribution. In each of the following steps we increased the noise, by multiplying each of the elements of the noise matrix by certain multipliers. The first multiplier was 1.002, the second one was 1.004, then $1.006, \dots, 3.000$.

So, during the experiment, we amplify the noise step-by-step and check at each iteration the snr and the ratio of the last (smallest) structural singular value and the first (largest) non-structural one. If the structural/non-structural ratio reaches a certain point, the signal gets so noisy, that it is not possible any more to distinguish the structural singular values from the non-structural ones. On Figure 7 the dashed line is the signal to noise ratio and the solid line is the ratio of the the last (smallest) structural singular value and the first (largest) non-structural singular value. On the horizontal axis the scale is given by the variance of the noise. One can see, that when the noise grows, then the snr decreases quickly, but the structural/non-structural ratio decreases quite slowly. If the structural/non-structural ratio decreases close to 1, then we reach a point, where the structural and non-structural singular values are not well distinguishable any more. However, Figure 7 shows that our method is reliable, that is if the noise in not higher than

25% of the signal, then the structural singular values are well distinguishable from the non-structural ones.

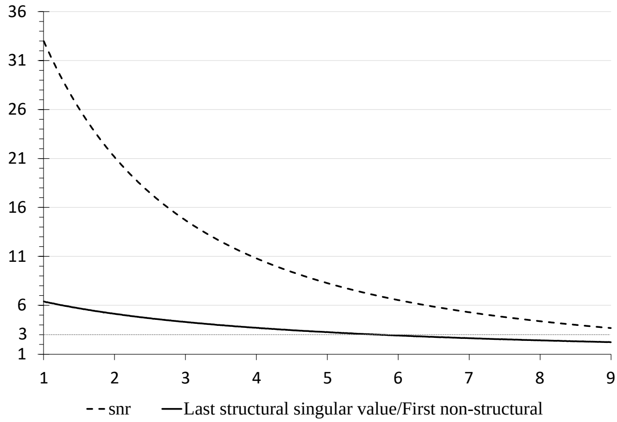

Figure 7: Signal to noise ratio compared to structural/non-structural singular value ratio

5. Conclusion

The shown graphical method works appropriately to distinguish the structural and non-structural eigenvalues and singular values. The size of the blocks has an influence on the 'jump' between the non-structural and the structural eigenvalues. As the block's sizes are increasing, the border between the structural and non-structural eigenvalues is getting more and more significant. If the block sizes are at least 50, then the structural and the non-structural eigenvalues are well distinguishable. Our test is reliable, if the noise is less than 25%, but above this noise level it can be unreliable.

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