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THE ROLE OF SYMMETRY IN RECIPROCAL FRAME STRUCTURES

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Abstract: *Reciprocal frames (RF) consist of elements which mutually support each other. The development of surface-related reciprocal frames is very dynamic. There are few architectural issues related closer to geometry, and demanding more work from innovative architects. The present study observes the regularities in different occurrences of RFs from medieval wooden slabs to digitally designed and fabricated amorphous structures. Our purpose is to analyse the main reasons why we should apply RF, and to describe the varicosity of symmetries which it implies. Furthermore, we examine the development process of more and more complex applicable geometries.*

Keywords: architecture, structural reciprocity, reciprocal frames, nexorade, Zollinger roof, lamella roof, mutually supported elements, spatial structures, timber constructions.

1 THE DEFINITION OF RECIPROCAL STRUCTURES

The most characteristic feature of reciprocal structures is that the elements mutually support each other. In opposition to hierarchical structures, where parallel supports are applied, the dislocation of mutually supported elements threaten with disintegrating either other mutually tended parts, or the whole construction. However, the advantage of such constructions is the capability of covering large spans with short-length elements.

In the related literature, we find two terms for the same entity: reciprocal structure, and nexorade. Reciprocity means mutuality in mathematics, and nexorade is originated from the Latin word *nexus*/*nectus*, which means to connect, to bind together. Some resources go so far to call the elements of a reciprocal frame (RF) nexors. Using this idea, one must visualize a construction of interwoven elements which simultaneously hold each other (Douthe, Baverel, and Caron, 2009).

There are three different types of RF structures. The central type results in a conic roof with a lantern, while the linear type forms a vault-like structure. There are surface related RF applications as well, which are mostly discussed today. These constructions are not numerous, except for two examples. The first one is an ancient Chinese bridge building technique, called the rainbow bridge, which is a linear structure used throughout China (Yang, Chen, and Gao, 2007), and the other one is the lamella roof system which has appeared during the first half of the 20th century. However, RF constructions have never been so common, they have not disappeared from architectural praxis, which is the main interest of the present investigation.

2 A REVIEW OF OCCURRENCES

The general literature on the topic varies between the development and the tradition of RF. These descriptions are usually about the built heritage, but there are also illustrations and useful commentary to be found. Even the shortest texts may be extremely helpful in understanding the intentions behind RF's historical applications. 20th and 21st century occurrences are easier to understand for the rich literature, in some cases, written by the designers themselves. However, an abundance of significant information, e.g., software developments inspired by actual projects, manufacturing and construction features are not included in the present discussion. For the identification of geometric regularities, crystallographic symbols are applied (Burkus, 2011).

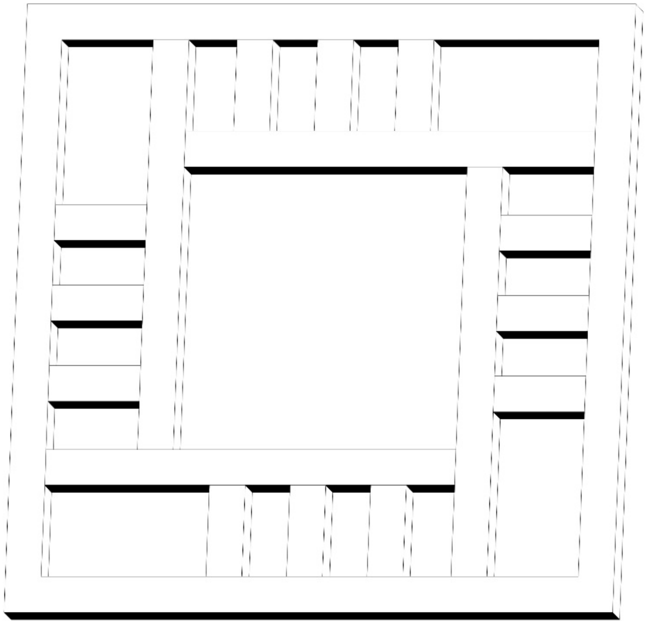

Figure 1: RF according to the sketchbook of Villard de Honnecourt.

2.1 Master builders and geometry

The first known representation of surface related RF has been found in the architectural sketchbook of Villard de Honnecourt (Bibliothèque Nationale de France), which was written between 1225 and 1235. The sketchbook was discovered during the middle 19th century and consists of individual layouts on different sized sheets holding together a wider range of subjects (Gruson, 2019). As the author's foreword says, it provides insight into the field of masonry, carpentry, and the art of geometry. The illustration depicting RF frame is captioned as follows: „*Ensi poe's ovrer a one tor u a one maison de bas, si sunt trop cor*", which means "how to build a tower or house, when the beams are too short". The drawing shows a symmetric group (n = 4) of beams (Fig. 1). One shall note that the illustration has been coupled with carpentered wheels. In addition, it has not been arranged by function, but by the geometry of the timber structure which has a rotational symmetry (Pugnale and Sassone, 2014).

The solution did not sink into oblivion, since it reappears in the book of the Mannerist architect Sebastiano Serlio (1475-1554). His book titled *I sette libri dell'architettura* was published in 1537 in Italian and has been translated into several languages. The *Sette libri*, written in “vulgar” tongue, i.e., Italian, was the first architectural treatise that documented the five orders of architecture in detail, and, in addition, the first to address practicing professionals. According to the commentary, Serlio proposes this solution mostly for building structures with unusually large spans. One of Serlio’s drawings show a tetragonal timber slab consisting of four beams in each direction. The shape has a rotational symmetry ($n = 4$), and the elements in directions x and y separately make frieze groups with 180º rotations (Fig. 2). In fact, it is the Honnecourt structure developed into a surface grid ($p4$)[1]. Nothing is more peculiar than this illustration is situated in Serlio’s first book on geometry. Further building structures appear in the fourth book only. In this case, RF structures are arranged according to their geometric quality instead of their architectural function (Serlio, 1537).

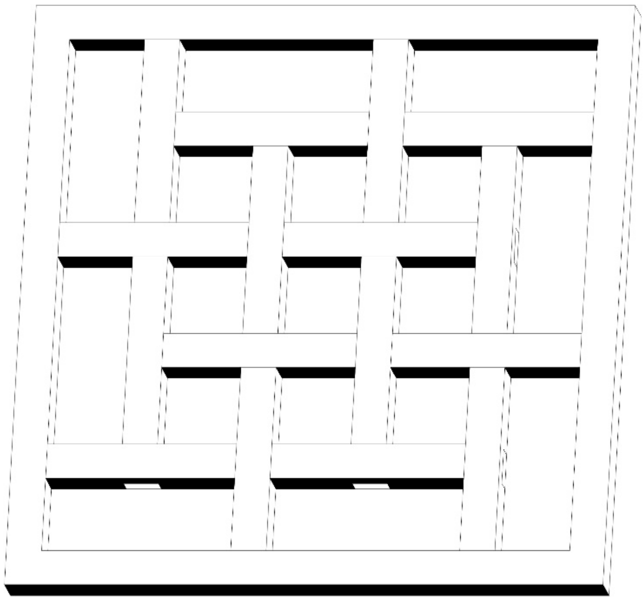

Figure 2: Timber RF according to Sebastiano Serlio’s *Sette libri*.

The number of examples from the same epoch is many. RF timber slabs were applied over the music room of Palazzo Piccolomini, dating from the 15th century Pienza, with a purely decorative purpose. Although larger proportions of the building are covered with straight beams, the oblong-shaped room has four beams in a windmill-like arrangement.

[1] The ‘p’ symbols in the text represent the structure’s wallpaper group, also known as plane symmetry group or plane crystallographic group.

The fields are lapped with panels and divided into coffers surrounding the coat of arms in the tetragonal centre. It is noticeable that the timber frame suggests the users to occupy the moderately lengthwise room in a central pattern.

On the contrary, there are purely technical applications of RF as well, and William Morris' house at Kelmscott Manor, UK, built in 1570, is one of them for certain. Here, the most typically arranged beams are hidden behind the ceiling panels. The harsh treatment of the timber surface and the accidental arrangement of the secondary parts exclude the possibility that the construction was ever designed to be visible. The same situation found in Wollaton Hall in Nottingham, where an irregular RF is hidden behind a suspended ceiling which does not reflect a reciprocal nature of any kind (Pugnale and Sassone, 2014).

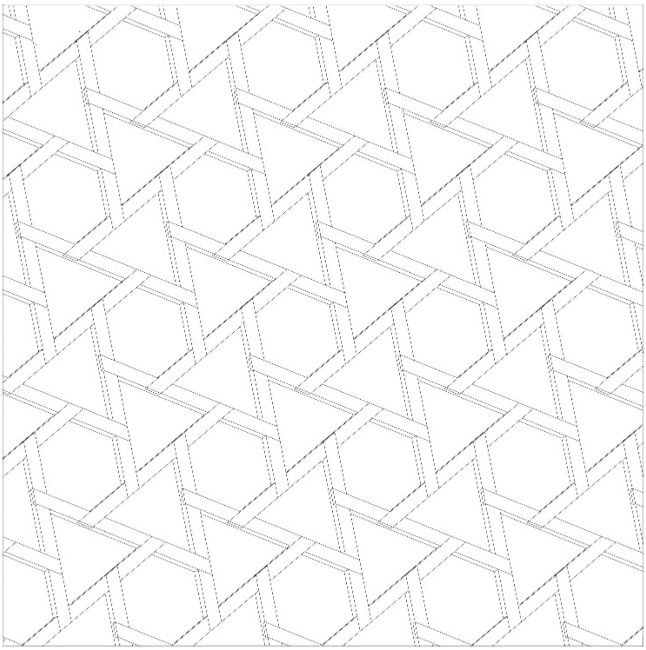

Figure 3: RF according to the *Codex Atlanticus*.

2.2 The mathematics of reciprocal frames

Leonardo da Vinci (1452-1519) also knew about RF. His sketches are collected in the *Codex Atlanticus* (Biblioteca Ambrosiana, Milan, 1480-1518), which includes a page on surface related RF structures. Although RF is presented as a building structure in the commentary, Leonardo was the first one to study RF from a universal geometric point of view. The renaissance polymath has also spotted the structure's connection to semi-regular tessellations (Fig. 3). Among his drawings, there is a tessellation dividing the Euclidean plane to quadrats and other rectangles (*p*4*g*, 4*2) in addition to the square tiling (*p*4, 442), and another one which consists of triangles and hexagons (*p*6, 632) (Williams, 2008; Roelofs, 2008; 2010; Pugnale and Sassone, 2014).

John Wallis (1616-1703) made his own RF timber system in response to the one of Serlio. In his paper *Mechanica: sive, de motu, tractatus geometricus*, first published in 1670, he executed calculations to analyze the stability of the structure. Later it was re-published in *Opera Mathematica* together with his selected studies in 1695. In comparison to that of Serlio, the model of Wallis is more advanced, because it consists of thirty beams by direction, and is capable of spanning greater distances (Fig. 4). The model remarkably reshapes Serlio's joining system which placed tenons alternately on the opposite sides of the beams. Instead of interlocking parts, Wallis' model consists of dovetail tenons on the same side, thus enhancing reflectional symmetry.

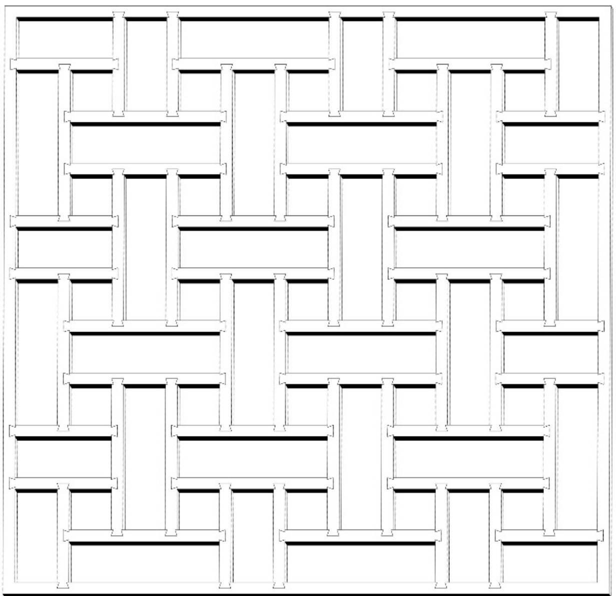

Figure 4: RF according to John Wallis' *Opera Mathematica.*

The fields of the slabs construct a semi-regular tiling out of squares and rectangles (*p*4*g*, 4*2). The positions of beams are given via reflection (*pmm*). The importance of Wallis' work lays in its pioneering analytic study on building construction, more specifically, the static calculus. This was the first time when RF structures became the subject of scientific research. But why did Wallis choose RF for his experiment? Probably because of the symmetries which penetrate the whole structure, resulting in the homogeneity of beams and their details. Such abilities facilitate analytic interpretation.

2.3 Economic structures

The possibility of covering large spans with congruent parts and the repetition of a single joint offers enormous potential benefits. The research of economic structures begun in the first decade of the 20th century (Michell, 1904), and soon spread to the field of timber construction. At the beginning of the 1920s, Fritz Zollinger (1880-1945) registered his patent in the USA. His lamella roof consists of prefabricated standard timber segments forming a rhomboid pattern as the frame of an optional roof surface. The axes of the elements meet in skewed angles to make an RF. Zollinger's lamella roof was originally a pointed arch vault constructed from congruent rhomboid fields that were bound together with one screw per junction. From the beginning on, Zollinger's principles have been going through several advancements. Nowadays, for instance, we can use lamella roofs to cover domes, and in some countries, it is already known as the *oikos* roof (Tamke, Riiber, and Jungjohann, 2010). Timber structures have been changed to metal by Emil Mauritz Hünnebeck, Caspar Heinrich Jucho, and Hugo Junkers. However, the speciality of the lamella roof is still the edges' curved configuration which provides smooth bends on the roof without the need for manufacturing any curved beams. Since RFs are made from a large number of short elements, they are precisely suited for the construction of plastically formed surfaces (Weller, Tasche, and Baatz, 2009).

2.4 Computer aided design

Zollinger lamella roof gained many applications in the field of timber architecture. In fact, lamella roofs similar to the the original ones are still being constructed. In Bad Sulza, Germany, a spa complex is built using the traditional lamella roof system. The architectural design was carried out by Ollertz Architekten, while Trabert und Partner were employed as project engineers. The building's double curvature bending surface was designed using a digitally generated model. Just like in the case of Zollinger's roof, the composition of straight segments appears to be curve, but it has a catenary instead of an arc shape. The architectural application of catenary curvature is reasonable due to its beneficial structural properties which make it highly economic. Combined with catenary models, RF provides close access to ideal geometry with straight elements (Chilton and Tang, 2017).

Probably the most famous timber lamella roof was constructed as the frame of London's Serpentine Pavilion in 2005. That year, the pavilion was erected according to the plans of Alvaro Siza and Eduardo Souto de Moura, together with Cecil Balmond from the Arup Company. The architectural mass of the pavilion was a distorted cuboid, which was built without any intermediate supports, yet all the surfaces were curved. Applying digital modelling and CNC milling, 427 decks were manufactured with diverse length and different bevels. The shape of the pavilion was inspired by mammal skeletons, therefore the RF covered a free formed double curvature bending surface. The intense regularity of the RF enabled access to a highly complex irregular morph (Simmonds, Self, and Bosia, 2007).

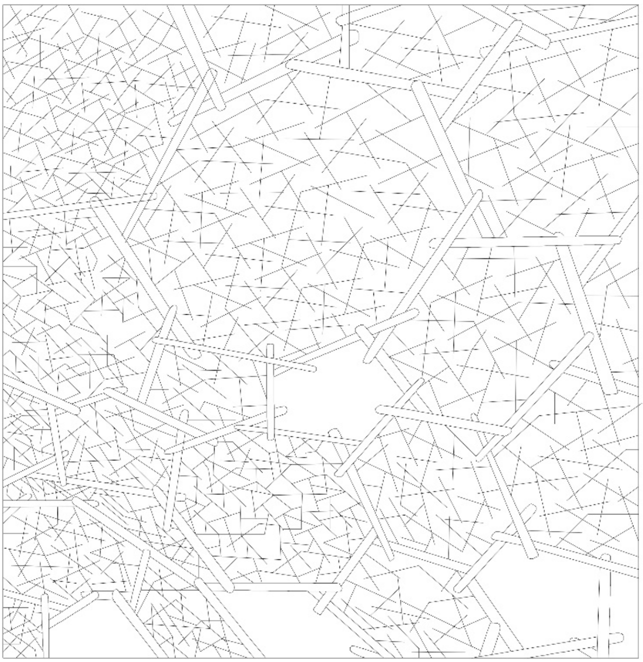

Figure 5: RF, with self-similar Voronoi cells.

2.5 Aperiodicity

Rokko Shidare Observatory in Kobe, Japan, was designed by Hiroshi Sambuichi Architects in 2011. The tower is surrounded by an amorphous bubble shape, similar to an artificial tree, casting shadow on the observatory and its environment. The RF canopy has a major frame of steel tubes supplemented by a densely woven net of wooden sticks (Fig. 5). In the frame, denser, looser and empty fields randomly follow each other. Compared to the previous examples of RFs, the aperiodic tessellation is a special feature in this case. Although the arrangement of staves is not irregular, the canopy is tiled with Voronoi cells in order to give the viewer a more familiar impression. Voronoi cells occur in nature, and its aperiodic tessellation is capable of modifying the distance between the frame elements in accordance with the actual stresses (Popovic Larsen, 2014; Goto, Ryota, and Matsuo, 2011).

3 CONCLUSION

The polyphony of symmetry types defining the position of elements is among the most important properties of RF structures. These symmetries enable the construction of either homogenous or less diverse short elements to smoothly re-arrange stresses, which can save material over a certain span. However, the design of an ideally applicable system of symmetries takes significant work, which may question the role of symmetry as the simplification of structures (Lynn, Reiser, and Umemoto, 1995). In fact, the simplification of structural details helps designers to add more complex geometries to architecture.

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