A Two-Sided Price-Decoupled Pay-As-Bid Auction Approach for the Clearing of Day-Ahead Electricity Markets

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Abstract. In this paper we propose a possible alternative for conventional pay-as-clear type multiunit auctions commonly used for the clearing of day-ahead power exchanges, and analyse some of its characteristic features in comparison with conventional clearing. In the proposed framework, instead of the concept of the uniform market clearing price, we introduce limit prices separately for supply and demand bids, and in addition to the power balance constraint, we formulate constraints for the income balance of the market. The total traded quantity is used as the objective function of the formulation. The concept is demonstrated on a simple example and is compared to the conventional approach in small-scale market simulations.

1 Introduction

Since the process of the liberalization of electricity markets began, auction designs for day-ahead electricity markets have been in the focus of research [1,2]. As electricity is a special good in the sense that it cannot be economically stored, electricity market designs must always reflect the need for supply-demand balance. An additional characteristic feature of European-type portfolio bidding electricity markets is that the bid types available on the supply side usually reflect the technical limitations of plants and the non-convex nature of generation costs, originating e.g. from significant start-up costs of the units. The most common and most recently used bid type which allows the bidder to distribute its start-up costs among several trading periods is the so called block order [3], which, in contrast to standard hourly bids, must fully be accepted or rejected ('fill-or-kill' condition). Further constructions, which allow for the consideration of technological and economic constraints are the so called *minimum income condition* (MIC) orders [4] and *scheduled-stop* condition orders. Although the inclusion of such orders in the market clearing algorithm may imply computational challenges (generally because of the consideration of bid incomes, which include quadratic terms), efficient formulations for these bids have been already proposed [5,6].

All of the above mentioned market models are based on the so-called and dominantly used 'pay-as-clear' or 'marginal pricing' principle. This principle is based on the concept of the *market clearing price* (MCP). The acceptance of standard hourly bids is uniquely determined by the MCP, and the accepted bids are paid off according to the MCP (not according to the bid price). The MCP is usually calculated in order to maximize the

total social welfare (TSW) – the total utility of consumption minus the total cost of production. Due to non-convexities however, a uniform MCP not always exists (see [7]), and this results in the phenomena of so-called *paradoxically rejected block orders* [8].

The most common alternative for the pay-as-clear principle is the pay-as-bid auction, where accepted bids are not paid off according to the MCP, but to the original bid price [9]. Pay-as-bid auctions however typically not used for two-sided auctions as day-ahead electricity auctions in the electricity industry, but rather for one-sided auctions as cross-border capacity auctions.

Regarding conventional pay-as-clear approaches, the decoupling of prices for nodes and zones in the case of connected markets is a commonly researched topic [10], while on the other hand, the decoupling the supply and demand prices has only been recently proposed as a possible approach [11].

Our aim in this paper is to start from the simple formulation of a one-period pay-as-clear market, and propose a pay-as-bid type market, in which limit prices different for the supply and demand side determine the set of accepted/rejected bids in order to maximize the total traded quantity in the market. Bids in this proposed framework are paid off in a pay-as-bid fashion. We formulate the clearing algorithm of the proposed framework to be compatible with the commonly used block orders and other complex orders. The dispatch implied by the proposed clearing method is demonstrated in the case of a simple example with low number of bids, and some of its basic characteristics are analysed and compared to the classical pay-as-clear clearing via simulations of scenarios under market uncertainty (random sets of submitted bids).

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2 Computational formulation

In this section we first consider a simple market clearing model of a classical single period pay-as-clear market, then we propose the price-decoupled pay-as-bid clearing model. In both cases we suppose that the input of the model is the set of bids. For the aim of simplicity, we do not consider block bids or any other complex bids in this first formulation, thus each bid i (supply or demand as indicated by the upper index) is parametrized by a bid quantity (q_i) [MWh] and a bid price (p_i) [EUR/MWh], and may be partially accepted as well. We use sign convention to identify supply and demand bids: for demand bids we suppose $q_i < 0$.

2.1 Pay-as-clear model

The formulation used for the pay-as-clear model is a simplified version of the formalism used in [4-6] (we consider here only one period). The variable vector **x** considered in this case is summarized in eq. (1).

$$\mathbf{x} = [MCP\, \mathbf{y}^S\, \mathbf{y}^D\, \mathbf{z}]^{\mathrm{T}} \qquad (1)$$

MCP stands for the market clearing price, $\mathbf{y}^S$ and $\mathbf{y}^D$ denote the vectors containing the acceptance indicators of supply and demand bids respectively, and $\mathbf{z}$ is the vector of auxiliary binary variables used in the computational formulation of the bid-acceptance constraints. The bid acceptance constraints are described by equations (2-5).

$$y_i^D > 0 \;\rightarrow\; MCP \le p_i^D \qquad (2)$$

$$y_i^D < 1 \;\rightarrow\; MCP \ge p_i^D \qquad (3)$$

$$y_i^S > 0 \;\rightarrow\; MCP \ge p_i^S \qquad (4)$$

$$y_i^S < 1 \;\rightarrow\; MCP \le p_i^S \qquad (5)$$

The implications may be easily implemented with the big-M method, as described e.g. in [12], using the binary variables in $\mathbf{z}$. Eq. (6) formulates the power balance of supply and demand (remember the negative sign of demand quantities).

$$\sum_i y_i^D\, q_i^D + \sum_i y_i^S\, q_i^S = 0 \qquad (6)$$

The objecive function may be written as

$$max\; f = \; - \sum_i y_i^D\, q_i^D\, p_i^D - \sum_i y_i^S\, q_i^S\, p_i^S \quad (7)$$

where the first term corresponds to the utility of consumption (the negative sign is due to the negative quantity of demand bids), while the second term describes the cost of production.

2.2 Price-decoupled pay-as-bid model

The formulation for the price-decoupled pay-as-bid model proposed in this article is as follows. The variable vector may be written as

$$\mathbf{x} = [LP^S\, LP^D\, \mathbf{y}^S\, \mathbf{y}^D\, \mathbf{z}]^{\mathrm{T}} \qquad (8)$$

where LP^S and LP^D denote the limit prices for supply and demand respectively (the remaining notations are the same as before in subsection 2.1). The bid acceptance constraints in this case are as described in eqs. (9-12).

$$y_i^D > 0 \;\rightarrow\; LP^D \le p_i^D \qquad (9)$$

$$y_i^D < 1 \;\rightarrow\; LP^D \ge p_i^D \qquad (10)$$

$$y_i^S > 0 \;\rightarrow\; LP^S \ge p_i^S \qquad (11)$$

$$y_i^S < 1 \;\rightarrow\; LP^S \le p_i^S \qquad (12)$$

In addition to the power balance equation, which is of the same form as before (6), in this case we also have an inequality constraint, describing that the costs of the production should not exceed the income from the accepted demand bids. This constraint is described by as

$$- \sum_i y_i^D\, q_i^D\, p_i^D \;\ge\; \sum_i y_i^S\, q_i^S\, p_i^S \qquad (13)$$

In this case, the objective function is the total traded quantity, as described in eq. (14)

$$f = \; - \sum_i\, y_i^D\, q_i^D + \sum_i y_i^S\, q_i^S \qquad (14)$$
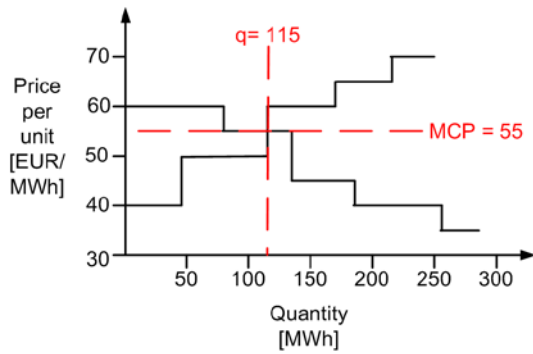
2.3 Demonstrative example

In this subsection we demonstrate the difference of the two clearing approach, considering a simple example (example 1) with 5 demand and 5 supply bids. The bid parameters are described in Table 1.
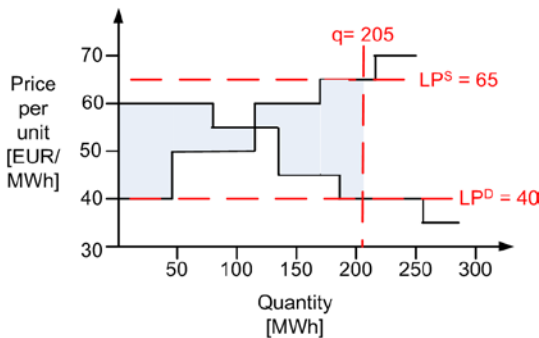
Table 1. Bid parameters of Example 1

Bid ID	quantity (q)	bid price (p)	Bid ID	quantity (q)	bid price (p)
S1	45	40	D1	-80	60
S2	70	50	D2	-55	55
S3	55	60	D3	-50	45
S4	45	65	D4	-70	40
S5	35	70	D5	-30	35

The results of the two clearing approaches are depicted in Figures 1 and 2. Fig. 1 depicts the results of the conventional pay-as-clear method, resulting in the well-known scenario: The MCP is determined by the intersection point of the supply and demand curves, which also determines the traded quantity (115). Bid D2 is partially accepted in this case.

Fig. 1. Result of the clearing of example 1 according to the pay-as-clear method.

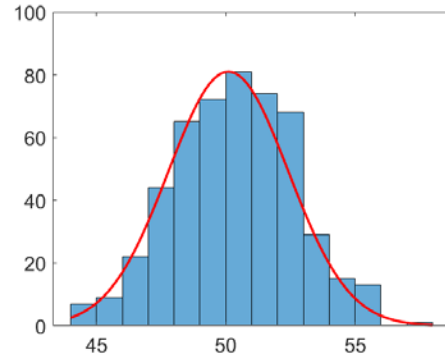**Fig. 2.** Result of the clearing of example 1 according to the price-decoupled pay-as-bid method.

Fig. 2 depicts the results of the price-decoupled pay-as-bid method. In this case bids S4 and D4 are partially accepted. The limit prices for supply and demand ($LP^S= 65$ and $LP^D=40$ respectively) are set by the solver in order to maximize the traded quantity, while keeping the power quantities on the supply and demand side and the incomes/costs in balance as well. The latter is ensured by the equality of the shaded areas to the left and to the right of the intersection points of the two curves (1425 EUR in this case) – in other words, the inequality condition described by eq. (13) holds as equality at the optimum.

3 Market simulation results

In the next step we performed simple market simulations to compare the performance of the two clearing method. In these simulations, bids were randomly generated from the following uniform distributions (denoted by $U([a,b])$). 20-20 bids were generated for the supply and demand side. Quantity of bids was taken from U([20,50]) in the case of supply and from U([-50,-20]) in the case of demand bids. The price of supply bids was taken from *U([40,70])* while the price of demand bids was taken from *U([30,60])*. 500 simulations were run with the above parameters, and in the case of each bid set, the pay-as-clear and the pay-as-bid clearing was also performed.
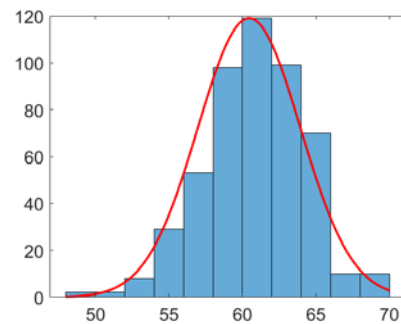
Figure 3. depicts the distribution of the MCP regarding the pay-as-clear clearing results. It can be seen in the figure that the distribution of the MCP values follows a normal distribution – in this case with m=50.1

and σ= 2.32. Let us note here that studies which aim to determine optimal bidding strategies under price uncertainty (see e.g. [14]) usually assume uniform distribution of the MCP. In contrast, market simulations of the current study suggest that assuming normal distribution of the MCP is more realistic.
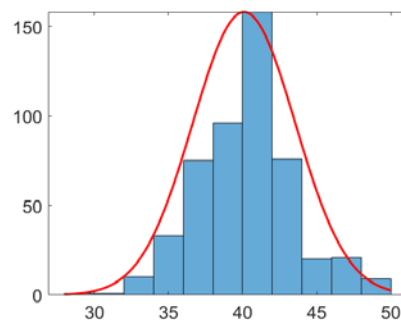

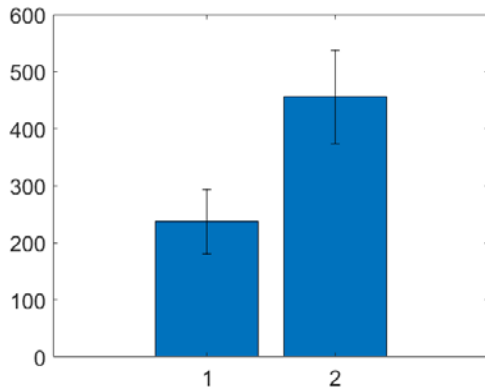**Fig. 3.** The distribution of MCP in pay-as-clear simulations.

Figures 4 and 5 depict the distribution of LPS and LPD respectively, which also follow normal distributions with parameters: m=60.45 and σ= 3.49 (LPS), m=40.94 and σ= 3.46 (LPD). Figure 6 depicts the comparison of the average traded quantity in the case of pay-as-clear and pay-as-bid clearing. As also in the case of the simple example used for the demonstration of the clearing method, we can see that the traded quantity is significantly higher in the case of price-decoupled pay-as-bid clearing.

Fig. 4. The distribution of LPS in price-decoupled pay-as-bid simulations.

Fig. 5. The distribution of LPD in price-decoupled pay-as-bid simulations.

Fig. 6. Traded quantity in the case of pay-as-clear (1) and price decoupled pay-as-bid (2) clearing with standard deviations.

4 Discussuion

In the market simulations we assumed that the set of bids is the same in both cases, and analysed the results according to this. This assumption has been sufficient for our current aim, namely to compare the performance of the proposed price-decoupled pay-as-bid clearing method with the classical pay-as-clear approach in a simple simulation setup, however it may be questionable regarding more realistic scenarios.

Previous studies discuss that in the case of pay-as-bid auctions, bidders have incentives to bid at the highest/lowest price accepted (regarding supply vs. demand) [15], in contrast to pay-as-clear auctions where e.g. suppliers make a rational decision if they bid their real costs (if start-up and variable costs arise in the same time, the situation may be a bit complicated, see the reference [14]). This will lead to 'flattened' supply and demand curves [15]. On the other hand, further studies also establish that the ranking of the uniform-price and pay-as-bid auctions is ambiguous in both revenue and efficiency terms [16], and multiple points of view arise from which the two approaches may be compared.

5 Conclusions and future work

5.1 Conclusions

In this study we introduced a possible price-decoupled pay-as-bid clearing method for two sided multi-unit auctions. The method is based on the concept of limit prices, which determine the acceptance/rejection of the submitted bids, and are defined distinctly for the supply and the demand side (LPS and LPD). The limit prices are determined via the clearing method, which is formulated as a linear optimization problem. The objective of this optimization problem is to maximize the total traded quantity, while considering the power balance and bid acceptance constraints, where the latter are also formulated in linear form, using auxiliary integer variables. The resulting optimization problem is a mixed integer linear problem (MILP), which may be solved efficiently with the available solvers (e.g.

CPLEX). The concept of the formalization is compatible with special products typically considered in day ahead electricity markets (e.g. block orders may be easily included in the proposed formulation via binary variables). Based on market simulations, we have shown that regarding the traded quantity, the price-decoupled pay-as-bid method outperforms the classical pay-as-clear approach in the case when the same bid set is assumed as input for both of the clearing algorithms. This may be a beneficial property in markets where the most efficient utilization of generating capacities is a priority, and may induce more liquid markets as well.

5.2 Future work

As discussed in section 3, the used market simulations may be made more realistic if we assume agents parametrized by demand utility or production costs, and assume that they bid in a rational way considering the applied auction framework as well in addition to their own parameters. This will result in different bid sets for the same agents in the case of the two compared method, and performance evaluation and comparison may be carried out on a more realistic base.

In addition, the formulation of the optimization problem in the proposed approach may be complemented with constraints corresponding to carbon emission, as motivated by the concept of environmental dispatch [17].

The concept of price-decoupling may be applied for conventional pay-as-clear markets as well [11] – in this context the effect of price decoupling on the acceptance of paradoxically rejected block orders [8] could be a research topic of potential interest.

Our further aim is to analyse the applicability of the proposed method in the case of joint energy-reserve markets, where power and reserves are allocated in the same time, considering combined offers as well [18]. The main motivation in this case is that for appropriate clearing of such markets according to the classical pay-as-clear methods, a wide set of prices must be defined and used for energy, reserves and combined bids [19].

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