Discrete-time Decoupling of Dynamical Subsystems Through Input-Output Blending

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Abstract: This paper presents a subsystem decoupling method for Linear Time Invariant Discrete-time systems. This allows that a selected subsystem can be controlled in a way which does not affect the remaining modes. Decoupling is achieved by suitable input and output blend vectors, such that they maximize the sensitivity of the selected mode, while at the same time they minimize the transfer through the undesired dynamics. The proposed algorithm is based on an optimization problem involving Linear Matrix Inequalities, where the $\mathcal{H}_-$ index of the controlled subsystem is maximized, while the transfer through the undesired dynamics is minimized by a sparsity like criteria. The present approach has the advantage that it is directly applicable to stable and unstable subsystems also. Numerical examples demonstrate the effectiveness of the method. The paper extends earlier continuous time results to discrete time systems over a finite frequency interval.

Keywords: Decoupled subsystems, Linear Parameter Varying systems, Generalized Kalman–Yakubovich–Popov lemma, Linear Matrix Inequality,Minimum Sensitivity, Mode Control

1. INTRODUCTION

It is often desirable to reduce the complexity of the control problem, and many approaches are existing to achieve it. These methods can be categorized into three main groups (Bakule, 2008). Decentralization aims for separate control design for processes and their independent implementation. Decomposition divides the system into certain subsystems, and so reduces the complexity of the control problem. Model reduction lowers the complexity of mathematical models, with the aid of approximate dynamical descriptions.

The present paper focuses on the decoupling (decomposition) of dynamical systems, where we wish to control a given subsystem, without interacting with the remaining dynamics (Gilbert, 1969). This aim points in the same direction as recent trends of systems- and control engineering aiming for the design of structured controllers for complex systems (Apkarian et al., 2015).

A newly developing trend in control design puts an emphasis on the decoupled control of selected modes of a dynamical system. These new approaches are applying input and output blending vectors to decouple modes and convert the design problem into a Single Input Single Output (SISO) one. Pusch (2018) designs the blend vectors based on an $\mathcal{H}_2$ norm criteria, which guarantees the controllability, observability and non-interacting control of selected modes. Pusch and Ossmann (2019) connects the before mentioned method to direct velocity feedback control. Danowsky et al. (2013) isolates the targeted mode by an optimal blend of the measurements, and computes an optimal blend for the inputs to damp the selected mode via a negative optimal feedback, and reduce interactions with other modes.

The paper discusses a novel blending approach for LTI discrete time systems, which make possible the decoupled control of targeted subsystems with simple SISO controllers. The method relies on the $\mathcal{H}_-$ index and a sparsity like criteria. This $\mathcal{H}_-$ index (Liu et al., 2005) is borrowed from the Fault Detection Filtering literature, and it is a minimum sensitivity measure corresponding to the smallest singular value of a dynamical system. If it is maximized, than the system's sensitivity is increased to the highest achievable level. Sparsity criteria can be rendered to optimization problems in order to assure that the result will contain as many zero entries as possible. As an example Polyak et al. (2013) designs a sparse state feedback gain matrix in order to assure as many as possible zero entries in the $u = Kx$ input vector, which leads to the minimization of the necessary actuators for stabilization. In the present paper we apply sparsity criteria in order to assure that the blended inputs and outputs of the subsystem to be decoupled will contain as many as possible small elements. This leads to an approximate decoupling, and a SISO controller will only interact with the targeted subsystem, while not affecting the rest of the dynamics. Our intention is to design the suitable environment (based on input and output transformations) for this controller, but we are not designing any control law.

The present paper extends previous results (Baár and Luspay, 2019) for continuous time, stable LTI systems to discrete time stable and unstable systems. In the previous version of the paper the H_∞ norm of the subsystems to be left unaffected by the control law was minimized. This is replaced by the sparsity like criteria which is directly applicable to stable and unstable subsystems also. Furthermore in the previous version certain frequency filters were added to the subsystems in order to make possible the H_- index calculation of strictly proper systems. The application of frequency filters can be avoided by the use of the Generalized Kalman-Yakubovich-Popov lemma, which guarantees the calculation of the H_- index over a selected frequency range for proper and strictly proper systems also.

The paper is structured as follows. The blending problem is formalized in Section 2, and Section 3 presents the applied mathematical tools. The main contributions are in Section 4, and numerical examples are given in Section 5. The paper is concluded in Section 6.

2. PROBLEM STATEMENT

Take a block diagonalizable discrete-time LTI system in its state space form

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{1}$$

with the standard notations: $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the input vector and $y(t) \in \mathbb{R}^{n_y}$ is the output vector of the system. We assume that the system is given in the following subsystem form, with

$$A = \begin{bmatrix} A_c & 0 \\ 0 & A_d \end{bmatrix}, \quad B = \begin{bmatrix} B_c \\ B_d \end{bmatrix}, \quad C = [C_c \ C_d]. \tag{2}$$

The subsystems to be controlled and to be decoupled are denoted by indexes $\{\cdot\}_c$ and $\{\cdot\}_d$ respectively. We assume that by a corresponding similarity transformation (Kailath, 1980) this form is achievable. Then this state space representation is called as modal form. The matrix A has a block diagonal structure, where each block corresponds to a dynamical mode of the system with real or complex (with real ($\Re$) and imaginary ($\Im$) part) eigenvalues (λ). They determine the structure of A as

$$A_i = \begin{cases} \lambda_i & \text{if } \Im(\lambda_i) = 0 \\ \begin{bmatrix} \Re(\lambda_i) & \Im(\lambda_i) \\ -\Im(\lambda_i) & \Re(\lambda_i) \end{bmatrix} & \text{if } \Im(\lambda_i) \neq 0. \end{cases} \tag{3}$$

Note that the given representation is not decoupled, as (2) shows couplings between the various subsystems through the B, C and D matrices.

The system has a corresponding transfer function representation given as

$$\begin{aligned} \mathcal{G}(e^{j\theta}) &= \sum_{i \in \{c,d\}} \left(C_i (e^{j\theta} I - A_i)^{-1} B_i \right) + D \\ &= \mathcal{G}_c(e^{j\theta}) + \mathcal{G}_d(e^{j\theta}), \quad \forall \theta \in \mathbb{R}, \end{aligned} \tag{4}$$

where $\mathcal{G}_c(e^{j\theta})$ and $\mathcal{G}_d(e^{j\theta})$ are the transfer functions of the subsystems to be controlled and decoupled respectively, and I is the identity matrix.

In the paper we wish to control the $\mathcal{G}_c(z)$ subsystem ($z = e^{j\theta}$), while having least effect on the $\mathcal{G}_d(z)$ one. This is achieved by a suitable input and output transformation. This makes necessary the introduction of $k_u \in \mathbb{R}^{n_u \times 1}$ and

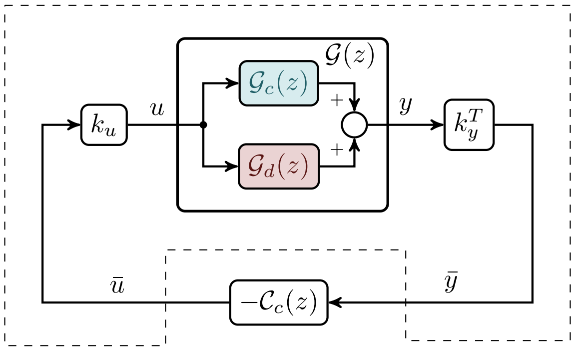

Fig. 1. Closed loop control scheme with input and output blending

$k_y \in \mathbb{R}^{n_y \times 1}$: the normalized input and output blending vectors, respectively. These transform the $u(t)$ and $y(t)$ signal vectors onto single scalars, and so turn the originally MIMO system into a SISO one. In the modified plant the transfer from the blended input to the blended output through $\mathcal{G}_c(z)$ is maximized, while through the other subsystem it is minimized. Figure 1 summarizes the proposed approach, where $\mathcal{G}(z)$ contains both subsystems and $\mathcal{C}_c(z)$ is a SISO controller, designed for $\mathcal{G}_c(z)$. The input of $\mathcal{C}_c(z)$ is $\bar{y} = k_y^T y \in \mathbb{R}$ i.e. the blended output of $\mathcal{G}(z)$. Similarly the output of the controller is the blended input $\bar{u} \in \mathbb{R}$, with $u = k_u \bar{u}$. The corresponding optimization problem is as follows.

Problem 1. Find normalized k_u and k_y vectors such that

$$||k_y^T \mathcal{G}_c(z) k_u||_-^{[\underline{\vartheta},\bar{\vartheta}]} > \beta \tag{5}$$

is maximized, while

$$B_d k_u \rightarrow \text{sparse! and } k_y^T C_d \rightarrow \text{ sparse!} \tag{6}$$

are satisfied.

Here $\underline{\vartheta}$ and $\bar{\vartheta}$ are two scalars denoting the lower and upper boundaries of a selected frequency range. Their values can be calculated based on the conformal mapping $z = e^{sT} = e^{(\sigma + j\omega)T} = e^{\sigma T} e^{j\omega T} = re^{j\omega T}$ between the S-plane and the Z-plane. The $z = e^{sT}$ mapping maps the $[\underline{\omega}, \bar{\omega}]$ frequency range on the imaginary axis to the $[\underline{\vartheta}, \bar{\vartheta}]$ angles on the unit circle in the Z-plane. Furthermore β is a positive constant referring to the minimal sensitivity. If the desired vectors are sparse, that means that they will contain as many as possible small elements, which on the other hand minimizes the transfer through the corresponding subsystem.

3. COMPUTATION OF THE $\mathcal{H}_-$ INDEX

We borrowed the idea of $\mathcal{H}_-$ index from Fault Detection Filtering, where it characterizes the sensitivity of the transfer from faulty inputs to the residual signals (see i.e Wang et al. (2007)). We use the LMI formulation of the $\mathcal{H}_-$ index to describe the minimum sensitivity of the subsystem to be controlled. The following subsection summarizes its main properties and computation for proper discrete-time systems on the $[0, \infty)$ frequency range, based on Li and Liu (2013). A latter subsection extends this computation method to discrete-time over a finite $[\underline{\omega}, \bar{\omega}]$ frequency range.

3.1 *Infinite frequency range*

The $\mathcal{H}_-$ index over the $[0, \infty)$ frequency range can be calculated based on Lemma 1 for the system (1).

Lemma 1. Let $\beta > 0$ be a positive constant scalar. Then $||\mathcal{G}_c(z)||_-^{[0,\infty)} > \beta$, if and only if there exists a P such that $P = P^T$ and

$$\begin{bmatrix} A_c^T P A_c - P + C_c^T C_c & A_c^T P B_c + C_c^T D_c \\ B_c^T P A_c + D_c^T C_c & D_c^T D_c + B_c^T P B_c - \beta^2 I \end{bmatrix} \succ 0. \quad (7)$$

The proof can be found in Li and Liu (2013), and is omitted here. The lemma for strictly proper systems over the complete frequency range yields 0. The $\mathcal{H}_-$ index can be calculated for unstable systems also. In this case the minimum sensitivity yields the lowest value of the singular values of the unstable system. This can be easily seen based on (Rantzer, 2015).

3.2 *Finite frequency range*

In order to compute the minimal sensitivity for strictly proper discrete-time systems over a limited frequency range, Iwasaki and Hara (2005) introduce an LMI based formulation of the $\mathcal{H}_-$ index based on the Generalized Kalman - Yakubovich - Popov (GKYP) lemma (Iwasaki and Hara, 2005). This is summarized in Lemma 2.

Lemma 2. Consider the system given in (1) with transfer function matrix (4). Let $\Pi = \begin{bmatrix} -I & 0 \\ 0 & \beta^2 I \end{bmatrix} \in \mathbb{R}^{(n_x+n_y)\times(n_x+n_y)}$ and $\underline{\vartheta}$, $\bar{\vartheta}$ be given scalars which reflect the investigated frequency range. Then $||\mathcal{G}_c(e^{j\theta})||_- > \beta$ for $\forall\theta \in [\underline{\vartheta},\ \bar{\vartheta}]$, if and only if there exists hermitian P and Q, with $Q \succ 0$ satisfying

$$\begin{bmatrix} A_c & B_c \\ I & 0 \end{bmatrix}^* \Xi \begin{bmatrix} A_c & B_c \\ I & 0 \end{bmatrix} + \begin{bmatrix} C_c & D \\ 0 & I \end{bmatrix}^* \Pi \begin{bmatrix} C_c & D \\ 0 & I \end{bmatrix} \prec 0, \quad (8)$$

where $\Xi = \begin{bmatrix} -P & e^{j((\underline{\vartheta}-\bar{\vartheta})/2)} Q \\ e^{-j((\underline{\vartheta}+\bar{\vartheta})/2)} Q & P - \left(2cos\dfrac{\underline{\vartheta}-\bar{\vartheta}}{2}\right) \end{bmatrix}$ and $\{\cdot\}^*$ denotes the complex conjugate transpose.

The proof is available in (**?**) and omitted here.

4. THE INPUT AND OUTPUT BLEND CALCULATION

The blending algorithm is presented in this Section. A systematic input and output blend calculation is presented in the sequel. We start from the input blend calculation, and then find the output blend.

4.1 *Input blend*

First we are designing k_u, which maximizes the state excitation of the targeted subsystem, and at the same time minimizes the effect on the remaining dynamics. The approach is summarized in Figure 2. The $\bar{u}$ variable is the scalar control input generated by the controller (see Figure 1), and k_u is a unit length vector which maps the single input to the available inputs of the plant. The goal to be achieved in this subsection is given as follows: the minimum sensitivity denoted by the $\mathcal{H}_-$ index from the blended input to the y_c performance output of the selected subsystem should be maximized, while the transfer from $\bar{u}$ to y_d should be minimized. The latter one is achieved by setting $B_d k_u$ as sparse as possible.

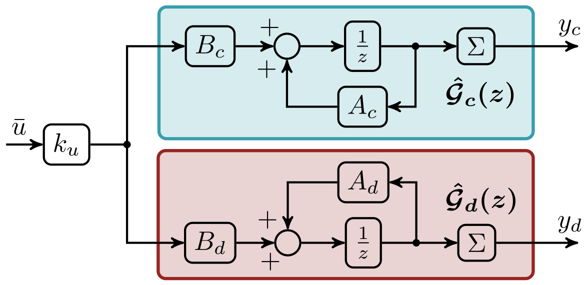

Fig. 2. Problem layout for input blend calculation

Before going into the details, we mention that the input blend calculation uses the dual representation (Kwakernaak and Sivan, 1972), defined by state space matrices

$$\tilde{A} = A^T, \quad \tilde{B} = C^T, \quad \tilde{C} = B^T, \quad \tilde{D} = D^T. \quad (9)$$

This is a necessary step to keep the optimization problem linear in the variables, as explained later. At the same time, note that the $\mathcal{H}_-$ index can only be calculated for tall or square systems (Li and Liu, 2010). However, in case the inputs are blended into a scalar $\bar{u}$ signal, then the dual representation would be a wide system. The problem is converted to a square system, by defining the performance output as the sum of the states as it is shown in Figure 2.

Accordingly, if one writes the LMI (8) for the dual system and then expresses the formula in terms of the original representation, one gets the following

$$\begin{bmatrix} A_c^T & C_c^T \\ I & 0 \end{bmatrix}^* \Xi \begin{bmatrix} A_c^T & C_c \\ I & 0 \end{bmatrix} + \begin{bmatrix} B_c^T & D^T \\ 0 & I \end{bmatrix}^* \Pi \begin{bmatrix} B_c^T & D^T \\ 0 & I \end{bmatrix} \prec 0, \quad (10)$$

where $\Pi = \begin{bmatrix} -K_u & 0 \\ 0 & \beta^2 I \end{bmatrix}$ and Ξ is defined as in Lemma 2.

The blend matrix is defined as the dyadic product of the blend vectors, with $K_u = k_u \cdot k_u^T \in \mathbb{R}^{n_u \times n_u}$.

The introduction of the blend matrix is only possible because of the dual form, otherwise the approach would yield a bilinear (furthermore quadratic) problem. Note that because K_u is a dyadic product, it is a 1 rank matrix. This rank constraint has to be satisfied during the solution process. This is possible by a simple heuristic method proposed by Fazel et al. (2001): the rank minimization of a symmetric positive definite matrix, yields to the minimization of its trace.

As it was stated before the transfer through the subsystem to be decoupled is suppressed by converting the blended input as sparse as possible. This is carried out by the minimization of trace$(B_d K_u B_d^T)$. We term it as a sparsity like criteria because it has a quadratic form, instead of being a linear one. To understand why this criteria work, recall the following property of the Frobenius norm

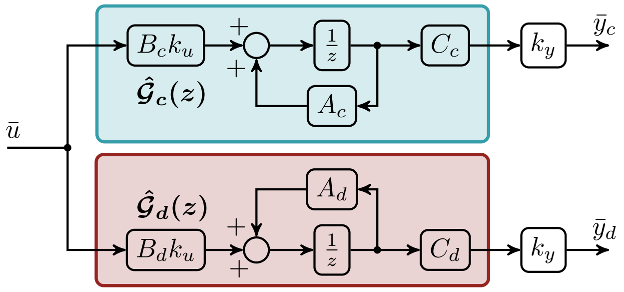

Fig. 3. Problem layout for output blend calculation

$$||Y||_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}|y_{ij}|^2} = \sqrt{\text{trace}(Y^TY)} = \\ = \sqrt{\sum_{i=1}^{min\{m,n\}} \sigma_i^2(Y)}, \tag{11}$$

where σ is the singular value of the $Y \in \mathbb{R}^{m\times n}$ matrix. By substituting $Y = B_d k_u$ into (11) it is obvious that Y^TY has one non-zero singular value, and it can be minimized by minimizing trace(Y^TY). This also means that the effect of the input to the states is also reduced. Note that this approach is directly applicable to stable and unstable modes also.

As a consequence, to find k_u one has to maximize β subject to (10), minimize trace$(B_dK_uB_d^T)$ for the suppression of the undesired dynamics and minimize trace(K_u) to satisfy the rank constraint. The optimization variables are P, Q, K_u and β, where $K_u = K_u^T$. The problem is stated as

$$\begin{aligned} &\text{minimize} \quad -\beta^2 + \text{trace}(K_u) + \text{trace}(B_dK_uB_d^T)W \\ &\text{subject to (10)}, \quad \text{and } 0 \preceq K_u \preceq I,\ Q \succeq 0, \end{aligned} \tag{12}$$

with I being the identity matrix with appropriate dimensions. W is a tuneable weighting factor to emphasize the sparsity criteria.

The before mentioned trace heuristic assures that after the solution of (12), the K_u blend matrix has only one non-zero singular value. We calculate k_u as the singular vector corresponding to this non-zero singular value. The SVD decomposition ensures to find normalized blending vectors.

When k_u is calculated, the inputs are blended yielding $\bar{A}_i = A_i$, $\bar{B}_i = B_ik_u$, $\bar{C}_i = C_i$, $\bar{D} = Dk_u$ for the i^{th} mode. These new matrices are used next.

4.2 Output blend

In this subsection we turn our attention to find a linear combination of the available outputs such that the single scalar measurement will contain as much as possible information about the targeted mode, while the effects of the other mode are suppressed. This means that the use of k_y^T should maximize the sensitivity on the performance output corresponding to the mode to be controlled, while it should yield a minimal transfer on the other one. The method is highly similar to the calculation of k_u, and the solution process is depicted in Figure 3.

The corresponding LMI constraint for the minimum sensitivity maximization of the $\mathcal{G}_c(z)$ subsystem is the following

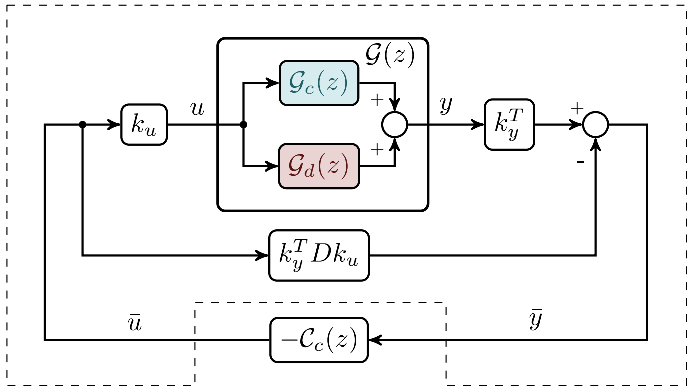

Fig. 4. The proposed control scheme

$$\begin{bmatrix}\bar{A}_c & \bar{B}_c\\ I & 0\end{bmatrix}^* \Xi \begin{bmatrix}\bar{A}_c & \bar{B}_c\\ I & 0\end{bmatrix} + \begin{bmatrix}\bar{C}_c & \bar{D}\\ 0 & I\end{bmatrix}^* \Pi \begin{bmatrix}\bar{C}_c & \bar{D}\\ 0 & I\end{bmatrix} \prec 0, \tag{13}$$

with $\Pi = \begin{bmatrix}-K_y & 0\\ 0 & \beta^2 I\end{bmatrix}$ and Ξ is defined as in Lemma 2. The output blend matrix is defined as $K_y = k_y \cdot k_y^T$.

The blend matrix is the solution of the underlying optimization problem. Find $P = P^T$, $Q = Q^T$, $K_y = K_y^T$ to

$$\begin{aligned} &\text{minimize} \quad -\beta^2 + trace(K_y) + \text{trace}(C_d^TK_yC_d)W \\ &\text{subject to 13}, \ 0 \preceq K_y \preceq I,\ Q \succeq 0. \end{aligned} \tag{14}$$

The Singular Value Decomposition of K_u provides the k_y blend vector.

When the k_y and k_u blends are applied to the subsystems, they will have the form

$$\begin{aligned} x_{c,d}(t+1) &= A_{c,d}x_{c,d}(t) + B_{c,d}k_u\bar{u}(t), \\ \bar{y}_{c,d}(t) &= k_y^TC_{c,d}x_{c,d}(t) + k_y^TDk_u\bar{u}(t). \end{aligned} \tag{15}$$

Note that the direct feedthrough term is not involved into the optimization process. In Figure 4 we propose the control scheme based on input and output blending, where a feedforward term is introduced to compensate the effect of the blended direct feedthrough matrix.

5. NUMERICAL EXAMPLES

The presented examples are involving a flexible wing aircraft to evaluate the decoupling method. The aircraft has been developed in the Flexop project (Consortium et al., 2015) which investigates active control techniques for flutter suppression. Flutter is a dynamic instability, what arises from the coupling of structural and aerodynamic forces. The model has two flutter modes, which describe the symmetric and asymmetric motions of the wing. Flutter speed defines the airspeed over which these modes are becoming unstable. Interested readers can find further details about the flexible modeling in (Luspay et al., 2018b). For the evaluation of the proposed decoupling method the high-fidelity nonlinear model was linearized at certain airspeeds what resulted in a set of linear models. These were then transformed into a parameter varying modal form and a parameter varying model order reduction was performed on them with the method developed by Luspay et al. (2018a). The obtained low order model is given in

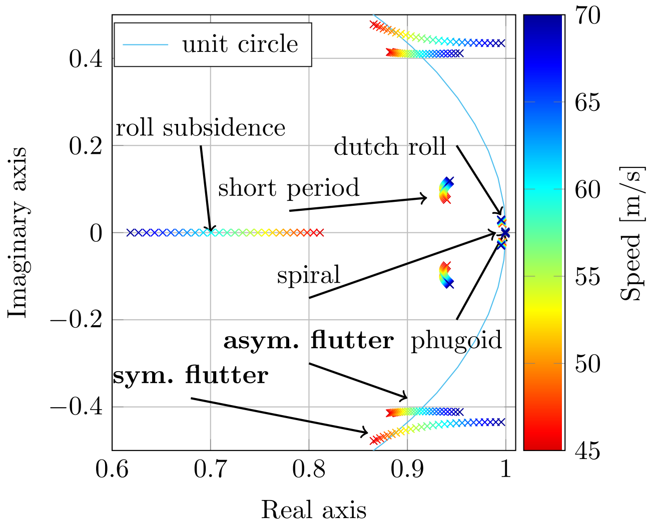

Fig. 5. The pole-zero map of the FLEXOP aircraft (discrete time model)

its modal form and used as the basis for the upcoming examples. The models were discretized by a $T_d = 0.01s$ time constant. The pole-zero map of the discrete-time aircraft model is given in Figure 5.

The aircraft has two ruddervators on each side and eight ailerons (four - four on each wings). These are used as the available inputs to be blended. The acceleration (a_z) and the angular rate (ω_x, ω_y) sensors are placed at the 90% spanwise location on the trailing edges.

The first example is taken at the 64 $\frac{m}{s}$ airspeed, where the flutter modes are unstable. We wish to control the symmetric mode, while minimizing the control impact on the asymmetric one. The frequency interval where the decoupling should be achieved was selected to be between 0 and the natural frequency of the targeted mode(ω_n $\frac{rad}{s}$). This means in (8) $\underline{\vartheta} = 0$ and $\bar{\vartheta} = \omega_n T_d$. The W weighting coefficient was selected to be $W = 100$ for the input and output blend calculations also. The k_u and k_y vectors are the solutions of (12) and (14) respectively. The convex optimization problems were formalized in MATLAB environment based on YALMIP (Löfberg, 2004), and the SeDuMi (Sturm, 1999) solver was used for solution. Figure 6 summarizes the results. The upper subfigure shows the maximum singular value plots for the flutter modes before the blend calculation. Almost in the whole frequency range, the asymmetric flutter mode has higher amplification. However by applying suitable input and output blends, it is possible to decouple the two subsystems, as the lower subfigure shows.

The second example investigates the decoupling of the asymmetric flutter mode from all other modes (rigid body modes + symmetric flutter mode) in the dynamic model at $47\frac{m}{s}$ airspeed. This time the subsystems are stable. The $\underline{\vartheta} = 0$, $\bar{\vartheta} = \omega_n T_d$ and W parameters were selected similarly as in the previous example. Figure 7 presents the results. The above subfigure presents the maximum singular values of the subsystems, which shows significant amplification of the undesired dynamics. The lower subfigure presents singular value plots of the blended subsystems.

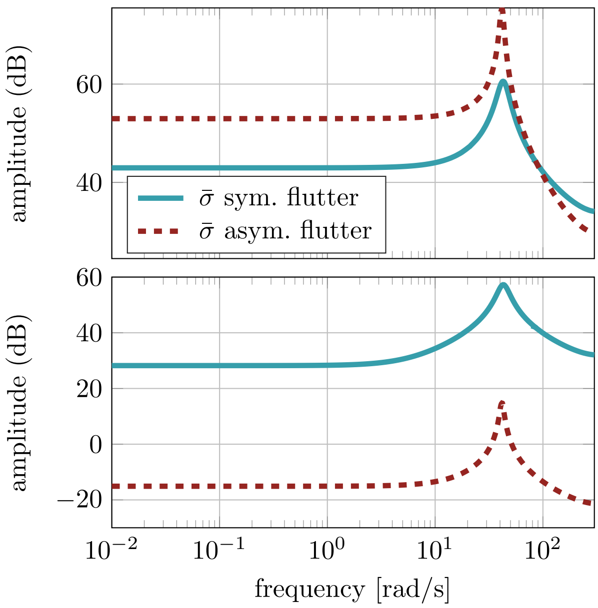

Fig. 6. Above: The maximum singular values of the subsystems. Below: the singular values of the blended subsystems

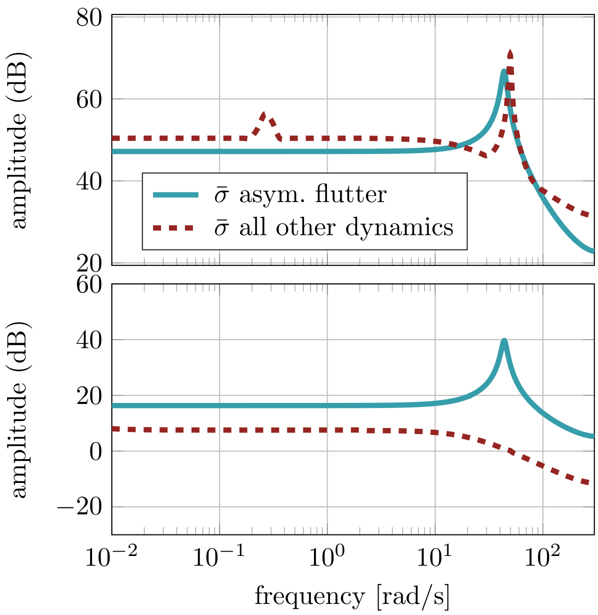

Fig. 7. Above: The maximum singular values of the subsystems. Below: the singular values of the blended subsystems

6. CONCLUSION

The paper presented an approach to decouple stable and unstable discrete time subsystems. The method creates an environment for a SISO controller which is able to control a selected subsystem with reduced interaction with the other subsystems. This environment is designed in two steps. In the first input decoupling is carried out by finding a suitable k_u input blend vector. In the second step a corresponding output blend (k_y) is found. These blend vectors are found by optimization problems consisting of LMIs. During the optimization process the transfer through the subsystem which should be controlled is maximized, while through the other one it is minimized. It has been shown by numerical examples that the method is able to find suitable input and output transformations to successfully decouple the subsystems.

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