Detailed summary of the results of OTKA grant “Quasi-local observables and the radiative modes in general relativity” (K 67790)

1. Results in connection with the gravitational conserved quantities and the canonical structure of general relativity:

In closed universes the constraint function of Einstein–Yang–Mills–Higgs systems that generates the time evolution (the so-called Hamiltonian constraint) turned out to be a simple Poisson bracket of the volume of the 3-space and a dimensionless functional G on the phase space, where G the sum of the Chern–Simons functional of the Sen connection in the spinor representation (gravitational sector) and another one built from the energy density of the matter fields. This implies that in the canonical quantum theory of gravity the eigenstates of the volume operator are not necessarily physical states. This result is a generalization of our previous one given for the vacuum case. Ref.: [4].

We investigated the extendability of the usual canonical formalism of Einstein's theory (originally developed only for asymptotically flat configurations) to describe the dynamics of gravitational field on finite domains. We showed that the requirement of the mathematical self-consistence of the system requires non-trivial boundary conditions for both the canonical variables and the lapse-shift pairs on the boundary: The induced area 2-form on the boundary 2-surface must be fixed, while the spacetime vector field built from the lapse and the shift must be divergence free with respect to the Sen type connection of the 2-surface. The latter is just one of the three projected parts to the surface of the spacetime Killing equation. The Hamiltonians are shown not to close to a Poisson algebra, rather the boundary terms appearing in the calculation of the Poisson brackets should be interpreted as the flux of the energy-momentum/angular momentum flow between the physical system and the rest of the universe. Ref.: [6].

We showed that any gravitational quasi-local energy expression must tend to the total ADM energy at the spatial infinity as a strictly monotonically decreasing (rather than increasing) set function, otherwise it would not be compatible with the Newtonian limit. We constructed an appropriately behaving expression in static spacetimes, which, for spherically symmetric configurations is positive and is zero precisely in flat spacetimes. Ref.: [13].

A non-negative expression $\mathtt{M}$, built from the L_2 norm of the 3-surface twistor derivative of a spinor field and the integral of the energy-momentum tensor, which on the asymptotically flat/hyperboloidal spacelike hypersurfaces provides a positive lower bound for the ADM/Bondi–Sachs mass. In closed universes the same expression defines a notion of total mass with the properties that i. it is zero precisely in flat spacetimes (in which case the hypersurface is necessarily a 3-torus), ii. Witten's gauge condition admits a non-trivial solution if and only if $\mathtt{M} = 0$, and iii. its is just the smallest eigenvalue of the square of the Sen–Witten operator. Refs.: [1,16].

2. Results in connection with the radiative modes:

We suggested an expression for the total spatial angular momentum in the radiative zone of asymptotically flat spacetimes (i.e. at future null infinity) by means of which the angular momentum flux, carried away by the outgoing gravitational radiation, can be calculated. The formula for the angular momentum reduces to the generally accepted expressions in the known special (axi-symmetric or stationary) cases, and in the absence of outgoing energy flux the angular momentum flux is shown to be also vanishing. We also find a non-negative observable at null infinity whose vanishing is equivalent to the purely electric nature of the asymptotic shear, i.e. a characteristic feature of stationary configurations. Ref.: [3].

We generalized the notion of quasi-local charges, introduced by P. Tod for Yang–Mills fields with unitary groups, to non-Abelian gauge theories with arbitrary gauge group, and calculated its small sphere and large sphere limits both at spatial and null infinity. We showed that for semisimple gauge groups no reasonable definition yield conserved total charges and Newman–Penrose (NP) type quantities at null infinity in generic, radiative configurations. The conditions of their conservation, both in terms of the field configurations and the structure of the gauge group, were also clarified. We also calculated the NP quantities for stationary, asymptotic solutions of the field equations with vanishing magnetic charges, and illustrated these by explicit solutions with various gauge groups. Ref.: [11].

3. Mathematical/technical results:

We investigated the potential global topological obstructions to the tetrad approach to finding the quasi-local conserved quantities associated with closed spacelike 2-surfaces. We found that the only potential obstruction is the non-orientability of the surface and the local time and space-non-orientability of the spacetime. However, on surfaces with non-trivial genus, a choice for the homotopy class of the global tetrad field or for the spin structure in the spinor formalism, must be made as a part of the gauge choice. Ref.: [5].

The boundary conditions that exclude zeros of the solutions of the Witten equation (and hence guarantee the existence of a 3-frame satisfying the so-called special orthonormal frame gauge conditions) were investigated. We determined the general form of the conformally invariant boundary conditions for the Witten equation, and found the boundary conditions that characterize the constant and the conformally constant spinor fields among the solutions of the Witten equations on compact domains in extrinsically and intrinsically flat, and on maximal, intrinsically globally conformally flat spacelike hypersurfaces, respectively. We also provided a number of exact solutions of the Witten equation with various boundary conditions (both at infinity and on inner or outer boundaries) that single out nowhere vanishing spinor fields on the flat, non-extreme Reissner–Nordström and Brill–Lindquist data sets. Our examples show that there is an interplay between the boundary conditions, the global topology of the hypersurface and the existence/non-existence of zeros of the solutions of the Witten equation. Ref.: [12].

We rewrote and updated our review paper (in the 'Living Reviews in Relativity') on the quasi-local conserved quantities in general relativity. Ref.: [7].

4. Perturbative results in cosmological models:

We determined the effect of the rotational perturbations of the homogeneous and isotropic cosmological models on the anisotropy of the cosmic microwave background radiation (CMBR) and the induced temperature fluctuations and their time dependence. We gave an upper bound for the rotation velocity of the universe at the moment of decoupling of the CMBR from the matter. Ref.: [2].

We developed an group theoretical approach to cosmological perturbations. It was shown that the various modes in the traditional approach correspond to the various irreducible representations of SO(3). The new approach is based on the exact SO(1,3) symmetry group of the unperturbed cosmological model, and the corresponding split of the perturbation leads naturally the standard Lifshitz classification of cosmological perturbations. The method could yield a simpler classification of higher order perturbations. Ref.: [15].

4. Results on brane black holes:

We considered curvature corrections to static, axisymmetric Dirac-Nambu-Goto membranes embedded into a spherically symmetric black hole spacetime with arbitrary number of dimensions. Since the next to leading order corrections in the effective brane action are quadratic in the brane thickness l, we adopt a linear perturbation approach in $l2$. The perturbations are general in the sense that they are not restricted to the Rindler zone nor to the near-critical solutions of the unperturbed system. As a result, an unexpected asymmetry in the perturbed system is found. In configurations, where the brane does not cross the black hole horizon, the perturbative approach does not lead to regular solutions if the number of the branes spacetime dimensions D¿3. This condition, however, does not hold for the horizon crossing solutions. Consequently we argue that the presented perturbative approach breaks down for subcritical type solutions near the axis of the system for D¿3. Nevertheless, we can discuss topology-changing phase transitions in cases when D=2 or 3, i.e. when the brane is a one-dimensional string or a two-dimensional sheet, respectively. For the general case, a different, nonperturbative approach should be sought. Based on the energy properties of those branes that are quasistatically evolved from the equatorial configuration, we illustrate the results of the phase transition in the case of a D=3 brane. It is found that small thickness perturbations do not modify the order of the transition, i.e. it remains first order just as in the case of vanishing thickness. Ref.: [8].

We considered static, axisymmetric, thick brane solutions on higher dimensional, spherically symmetric black hole backgrounds. It was found recently that in cases when the thick brane has more than 2 spacelike dimensions, perturbative approaches break down around the corresponding thin solutions for Minkowski type topologies. This behavior is a consequence of the fact that thin solutions are not smooth at the axis, and for a general discussion of possible phase transitions in the system, one needs to use a non-perturbative approach. In the present paper we provide an exact, numerical solution of the problem both for black hole- and Minkowski type topologies with arbitrary number of brane and bulk dimensions. We also illustrate a topology change transition in the system for a 5-dimensional brane embedded in a 6-dimensional bulk. Ref.: [10].

We presented a numerical solution for a topologically flat 2-dimensional thick brane on a higher dimensional, spherically symmetric black hole background. The present solution is the last, missing part of the complete set of solutions for the thickness corrected brane-black hole problem in arbitrary number of dimensions. We show that the 2-dimensional case is special compared to all the higher dimensional solutions in the topologically Minkowskian family as being non-analytic at the axis of the system. We provide the numerical solution in the near horizon region and make a comparison with the infinitely thin case. Ref.: [14].

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