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COEFFICIENT ESTIMATES FOR BAZILEVIČ FUNCTIONS OF BI-PRESTARLIKE FUNCTIONS

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Abstract. In the present article, we introduce and study two certain classes of holomorphic prestarlike and bi-univalent functions associated with Bazilevič function. We determinate upper bounds for the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions belonging to these classes. Further we point out certain special cases for our results.

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1. INTRODUCTION

Let $\mathcal{A}$ indicate the collection of all holomorphic functions in the unit disc $U = \{z \in \mathbb{C} : |z| < 1\}$ that have the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k. \tag{1.1}$$

We also denote by S the sub-collection of the set $\mathcal{A}$ containing of functions in U satisfying (1.1) which are univalent in U.

A function $f \in \mathcal{A}$ is called starlike of order δ $(0 \leq \delta < 1)$, if

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \delta, \quad (z \in U).$$

For $f \in \mathcal{A}$ given by (1.1) and $g \in \mathcal{A}$ defined by

$$g(z) = z + \sum_{k=2}^{\infty} b_k z^k,$$

the "Hadamard product" of f and g is defined (as usual) by

$$(f * g)(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k, \quad (z \in U).$$

Ruscheweyh [8] defined and considered the family of "prestarlike functions" of order δ, which are the function f such that $f * I_\delta$ is a starlike function of order δ, where

$$I_\delta(z) = \frac{z}{(1-z)^{2(1-\delta)}}, \quad (0 \le \delta < 1, z \in U).$$

The function I_δ can be written in the form:

$$I_\delta(z) = z + \sum_{k=2}^{\infty} \varphi_k(\delta) z^k,$$

where

$$\varphi_k(\delta) = \frac{\prod_{i=2}^{k} (i - 2\delta)}{(k-1)!}, \quad k \ge 2.$$

We note that $\varphi_k(\delta)$ is a decreasing function in δ and satisfies

$$\lim_{k\to\infty} \varphi_k(\delta) = \begin{cases} \infty, & if\, \delta < \frac{1}{2} \\ 1, & if\, \delta = \frac{1}{2} \\ 0, & if\, \delta > \frac{1}{2} \end{cases} .$$

Singh [9] (also see Kim and Srivastava [4]) introduced and studied the family of Bazilevič functions $f \in \mathcal{A}$ satisfying the condition:

$$Re\left\{\frac{z^{1-\gamma} f'(z)}{(f(z))^{1-\gamma}}\right\} > 0, \quad (z \in U, \gamma \ge 0).$$

According to the Koebe one-quarter theorem (see [3]) "every function $f \in S$ has an inverse f^{-1} which satisfies $f^{-1}(f(z)) = z$, $(z \in U)$ and $f(f^{-1}(w)) = w$, $(|w| < r_0(f), r_0(f) \ge \frac{1}{4})$", where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots. \quad (1.2)$$

For $f \in \mathcal{A}$, if both f and f^{-1} are univalent in U, we say that f bi-univalent function in U. We indicate by Σ the family of bi-univalent functions in U given by (1.1). In fact, Srivastava et al. [18] have actually revived the study of holomorphic and bi-univalent functions in recent years. Some examples of functions in the family Σ are

$$\frac{z}{1-z}, \quad \frac{1}{2}\log\left(\frac{1+z}{1-z}\right) \quad \text{and} \quad -\log(1-z)$$

with the corresponding inverse functions

$$\frac{w}{1+w}, \quad \frac{e^{2w}-1}{e^{2w}+1} \quad \text{and} \quad \frac{e^w - 1}{e^w},$$

respectively. Other common examples of functions is not a member of Σ are

$$z - \frac{z^2}{2} \quad \text{and} \quad \frac{z}{1-z^2}.$$

Recently, many authors introduced various subfamilies of the bi-univalent functions family Σ and investigated upper bounds for the first two coefficients $|a_2|$ and $|a_3|$ in the Taylor-Maclaurin series expansion (1.1) (see, for example [1,5,10–17,19–24]).

We require the following lemma that will be used to prove our main results.

Lemma 1 ([3]). *If $h \in \mathcal{P}$, then $|c_k| \le 2$ for each $k \in \mathbb{N}$, where $\mathcal{P}$ is the family of all functions h holomorphic in U for which*

$$Re\,(h(z)) > 0, \quad (z \in U),$$

where

$$h(z) = 1 + c_1 z + c_2 z^2 + \cdots, \quad (z \in U).$$

2. Coefficient estimates for the functions family $\Omega_\Sigma(\lambda,\gamma,\delta;\alpha)$

Definition 1. A function $f \in \Sigma$ given by (1.1) is called in the family $\Omega_\Sigma(\lambda,\gamma,\delta;\alpha)$ if it fulfils the conditions:

$$\left| arg\left[\frac{1}{2}\left(\frac{z^{1-\gamma}(f*I_\delta)'(z)}{((f*I_\delta)(z))^{1-\gamma}} + \left(\frac{z^{1-\gamma}(f*I_\delta)'(z)}{((f*I_\delta)(z))^{1-\gamma}}\right)^{\frac{1}{\lambda}}\right)\right]\right| < \frac{\alpha\pi}{2}, \quad (z \in U) \tag{2.1}$$

and

$$\left| arg\left[\frac{1}{2}\left(\frac{w^{1-\gamma}(g*I_\delta)'(w)}{((g*I_\delta)(w))^{1-\gamma}} + \left(\frac{w^{1-\gamma}(g*I_\delta)'(w)}{((g*I_\delta)(w))^{1-\gamma}}\right)^{\frac{1}{\lambda}}\right)\right]\right| < \frac{\alpha\pi}{2}, \quad (w \in U), \tag{2.2}$$

$$(0 < \alpha \le 1,\ 0 < \lambda \le 1,\ \gamma \ge 0,\ 0 \le \delta < 1),$$

where the function $g = f^{-1}$ is given by (1.2).

Remark 1. It should be remarked that the family $\Omega_\Sigma(\lambda,\gamma,\delta;\alpha)$ is a generalization of well-known families consider earlier. These families are:

(1) For $\lambda = 1$ and $\delta = \frac{1}{2}$, the family $\Omega_\Sigma(\lambda,\gamma,\delta;\alpha)$ reduce to the family $P_\Sigma(\alpha,\gamma)$ which was introduced by Prema and Keerthi [7];
(2) For $\lambda = 1$, $\gamma = 0$ and $\delta = \frac{1}{2}$, the family $\Omega_\Sigma(\lambda,\gamma,\delta;\alpha)$ reduce to the family $S^*_\Sigma(\alpha)$ which was given by Brannan and Taha [2];
(3) For $\lambda = \gamma = 1$ and $\delta = \frac{1}{2}$, the family $\Omega_\Sigma(\lambda,\gamma,\delta;\alpha)$ reduce to the family $\mathcal{H}^\alpha_\Sigma$ which was investigated by Srivastava et al. [18].

Theorem 1. *Let $f \in \Omega_\Sigma(\lambda,\gamma,\delta;\alpha)$ $(0 < \alpha \le 1, 0 < \lambda \le 1, \gamma \ge 0, 0 \le \delta < 1)$ be given by (1.1). Then*

$$|a_2| \le \frac{2\alpha\lambda}{\sqrt{\alpha\lambda(\gamma+2)(\lambda+1)(1-\delta)\,(2\gamma(1-\delta)+1) + \Upsilon(\alpha,\lambda)\,(\gamma+1)^2\,(1-\delta)^2}}$$

and

$$|a_3| \leq \frac{4\alpha\lambda}{(\lambda+1)(1-\delta)}\left[\frac{\alpha\lambda}{(\gamma+1)^2(\lambda+1)(1-\delta)}+\frac{1}{(\gamma+2)(3-2\delta)}\right],$$

where

$$\Upsilon(\alpha,\lambda) = 2\alpha(1-\lambda)+(1-\alpha)(\lambda+1)^2. \tag{2.3}$$

Proof. It follows from conditions (2.1) and (2.2) that

$$\frac{1}{2}\left(\frac{z^{1-\gamma}(f*I_\delta)'(z)}{((f*I_\delta)(z))^{1-\gamma}}+\left(\frac{z^{1-\gamma}(f*I_\delta)'(z)}{((f*I_\delta)(z))^{1-\gamma}}\right)^{\frac{1}{\lambda}}\right) = [p(z)]^\alpha \tag{2.4}$$

and

$$\frac{1}{2}\left(\frac{w^{1-\gamma}(g*I_\delta)'(w)}{((g*I_\delta)(w))^{1-\gamma}}+\left(\frac{w^{1-\gamma}(g*I_\delta)'(w)}{((g*I_\delta)(w))^{1-\gamma}}\right)^{\frac{1}{\lambda}}\right) = [q(w)]^\alpha, \tag{2.5}$$

where $g = f^{-1}$ and p, q in $\mathcal{P}$ have the following series representations:

$$p(z) = 1+p_1z+p_2z^2+p_3z^3+\cdots \tag{2.6}$$

and

$$q(w) = 1+q_1w+q_2w^2+q_3w^3+\cdots. \tag{2.7}$$

Comparing the corresponding coefficients of (2.4) and (2.5) yields

$$\frac{(\gamma+1)(\lambda+1)(1-\delta)}{\lambda}a_2 = \alpha p_1, \tag{2.8}$$

$$\frac{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}{2\lambda}a_3 + \frac{\left[\lambda(\gamma+2)(\gamma-1)(\lambda+1)+(\gamma+1)^2(1-\lambda)\right](1-\delta)^2}{\lambda^2}a_2^2 = \alpha p_2+\frac{\alpha(\alpha-1)}{2}p_1^2, \tag{2.9}$$

$$-\frac{(\gamma+1)(\lambda+1)(1-\delta)}{\lambda}a_2 = \alpha q_1 \tag{2.10}$$

and

$$\frac{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}{2\lambda}\left(2a_2^2-a_3\right) + \frac{\left[\lambda(\gamma+2)(\gamma-1)(\lambda+1)+(\gamma+1)^2(1-\lambda)\right](1-\delta)^2}{\lambda^2}a_2^2 = \alpha q_2+\frac{\alpha(\alpha-1)}{2}q_1^2. \tag{2.11}$$

In view of (2.8) and (2.10), we conclude that

$$p_1 = -q_1 \tag{2.12}$$

and

$$\frac{2(\gamma+1)^2(\lambda+1)^2(1-\delta)^2}{\lambda^2}a_2^2=\alpha^2(p_1^2+q_1^2). \tag{2.13}$$

Also, by using (2.9) and (2.11), together with (2.13), we find that

$$\left(\frac{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}{\lambda}+\frac{2\left[\lambda(\gamma+2)(\gamma-1)(\lambda+1)+(\gamma+1)^2(1-\lambda)\right](1-\delta)^2}{\lambda^2}\right)a_2^2$$

$$=\alpha(p_2+q_2)+\frac{\alpha(\alpha-1)}{2}\left(p_1^2+q_1^2\right)=\alpha(p_2+q_2)+\frac{(\alpha-1)(\gamma+1)^2(\lambda+1)^2(1-\delta)^2}{\alpha\lambda^2}a_2^2.$$

Further computations show that

$$a_2^2=\frac{\alpha^2\lambda^2(p_2+q_2)}{\alpha\lambda(\gamma+2)(\lambda+1)(1-\delta)(2\gamma(1-\delta)+1)+\Upsilon(\alpha,\lambda)(\gamma+1)^2(1-\delta)^2}, \tag{2.14}$$

where $\Upsilon(\alpha,\lambda)$ is given by (2.3).

By taking the absolute value of (2.14) and applying Lemma 1 for the coefficients p_2 and q_2, we have

$$|a_2|\le\frac{2\alpha\lambda}{\sqrt{\alpha\lambda(\gamma+2)(\lambda+1)(1-\delta)(2\gamma(1-\delta)+1)+\Upsilon(\alpha,\lambda)(\gamma+1)^2(1-\delta)^2}}.$$

To determinate the bound on $|a_3|$, by subtracting (2.11) from (2.9), we get

$$\frac{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}{\lambda}\left(a_3-a_2^2\right)=\alpha(p_2-q_2)+\frac{\alpha(\alpha-1)}{2}\left(p_1^2-q_1^2\right). \tag{2.15}$$

Now, substituting the value of a_2^2 from (2.13) into (2.15) and using (2.12), we deduce that

$$a_3=\frac{\alpha^2\lambda^2\left(p_1^2+q_1^2\right)}{2(\gamma+1)^2(\lambda+1)^2(1-\delta)^2}+\frac{\alpha\lambda(p_2-q_2)}{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}. \tag{2.16}$$

Taking the absolute value of (2.16) and applying Lemma 1 once again for the coefficients p_1, p_2, q_1 and q_2, it follows that

$$|a_3|\le\frac{4\alpha\lambda}{(\lambda+1)(1-\delta)}\left[\frac{\alpha\lambda}{(\gamma+1)^2(\lambda+1)(1-\delta)}+\frac{1}{(\gamma+2)(3-2\delta)}\right],$$

□

Remark 2. In Theorem 1, if we choose

1) $\lambda=1$ and $\delta=\frac{1}{2}$, then we have the results which was given by Prema and Keerthi [7, Theorem 2.2];
(2) $\lambda=1,\gamma=0$ and $\delta=\frac{1}{2}$, then we have the results obtained by Murugusundaramoorthy et al. [6, Corollary 6];

(3) $\lambda=\gamma=1$ and $\delta=\frac{1}{2}$, then we obtain the results obtained by Srivastava et al. [18, Theorem 1].

3. Coefficient estimates for the functions family $\Omega_{\Sigma}^{*}(\lambda,\gamma,\delta;\beta)$

Definition 2. A function $f\in\Sigma$ given by (1.1) is called in the family $\Omega_{\Sigma}^{*}(\lambda,\gamma,\delta;\beta)$ if it fulfills the conditions:

$$Re\left\{\frac{1}{2}\left(\frac{z^{1-\gamma}(f*I_{\delta})'(z)}{((f*I_{\delta})(z))^{1-\gamma}}+\left(\frac{z^{1-\gamma}(f*I_{\delta})'(z)}{((f*I_{\delta})(z))^{1-\gamma}}\right)^{\frac{1}{\lambda}}\right)\right\}>\beta,\quad (z\in U) \tag{3.1}$$

and

$$Re\left\{\frac{1}{2}\left(\frac{w^{1-\gamma}(g*I_{\delta})'(w)}{((g*I_{\delta})(w))^{1-\gamma}}+\left(\frac{w^{1-\gamma}(g*I_{\delta})'(w)}{((g*I_{\delta})(w))^{1-\gamma}}\right)^{\frac{1}{\lambda}}\right)\right\}>\beta,\quad (w\in U), \tag{3.2}$$

$$(0\leq\beta<1,\ 0<\lambda\leq1,\ \gamma\geq0,\ 0\leq\delta<1),$$

where the function $g=f^{-1}$ is given by (1.2).

Remark 3. It should be remarked that the family $\Omega_{\Sigma}^{*}(\lambda,\gamma,\delta;\beta)$ is a generalization of well-known families consider earlier. These families are:

(1) For $\lambda=1$ and $\delta=\frac{1}{2}$, the family $\Omega_{\Sigma}^{*}(\lambda,\gamma,\delta;\beta)$ reduce to the family $P_{\Sigma}(\beta,\gamma)$ which was introduced by Prema and Keerthi [7];

(2) For $\lambda=1$, $\gamma=0$ and $\delta=\frac{1}{2}$, the family $\Omega_{\Sigma}^{*}(\lambda,\gamma,\delta;\beta)$ reduce to the family $S_{\Sigma}^{*}(\beta)$ which was given by Brannan and Taha [2];

(3) For $\lambda=\gamma=1$ and $\delta=\frac{1}{2}$, the family $\Omega_{\Sigma}^{*}(\lambda,\gamma,\delta;\beta)$ reduce to the family $\mathcal{H}_{\Sigma}(\beta)$ which was investigated by Srivastava et al. [18].

Theorem 2. *Let* $f\in\Omega_{\Sigma}^{*}(\lambda,\gamma,\delta;\beta)$ $(0\leq\beta<1,\ 0<\lambda\leq1,\ \gamma\geq0,\ 0\leq\delta<1)$ *be given by (1.1). Then*

$$|a_2|\leq\frac{2\lambda\sqrt{1-\beta}}{\sqrt{\lambda(\gamma+2)(\lambda+1)(1-\delta)\left(2\gamma(1-\delta)+1\right)+2(1-\lambda)\left(\gamma+1\right)^2\left(1-\delta\right)^2}}$$

and

$$|a_3|\leq\frac{4\lambda\left(1-\beta\right)}{(\lambda+1)(1-\delta)}\left[\frac{\lambda\left(1-\beta\right)}{(\gamma+1)^2\left(\lambda+1\right)\left(1-\delta\right)}+\frac{1}{(\gamma+2)(3-2\delta)}\right].$$

Proof. In the light of the conditions (3.1) and (3.2), there are $p,q\in\mathcal{P}$ such that

$$\frac{1}{2}\left(\frac{z^{1-\gamma}(f*I_{\delta})'(z)}{((f*I_{\delta})(z))^{1-\gamma}}+\left(\frac{z^{1-\gamma}(f*I_{\delta})'(z)}{((f*I_{\delta})(z))^{1-\gamma}}\right)^{\frac{1}{\lambda}}\right)=\beta+(1-\beta)p(z) \tag{3.3}$$

and

$$\frac{1}{2}\left(\frac{w^{1-\gamma}(g*I_\delta)'(w)}{((g*I_\delta)(w))^{1-\gamma}}+\left(\frac{w^{1-\gamma}(g*I_\delta)'(w)}{((g*I_\delta)(w))^{1-\gamma}}\right)^{\frac{1}{\lambda}}\right)=\beta+(1-\beta)q(w), \tag{3.4}$$

where $p(z)$ and $q(w)$ have the forms (2.6) and (2.7), respectively. Comparing the corresponding coefficients in (3.3) and (3.4) yields

$$\frac{(\gamma+1)(\lambda+1)(1-\delta)}{\lambda}a_2=(1-\beta)p_1, \tag{3.5}$$

$$\frac{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}{2\lambda}a_3 + \frac{\left[\lambda(\gamma+2)(\gamma-1)(\lambda+1)+(\gamma+1)^2(1-\lambda)\right](1-\delta)^2}{\lambda^2}a_2^2=(1-\beta)p_2, \tag{3.6}$$

$$-\frac{(\gamma+1)(\lambda+1)(1-\delta)}{\lambda}a_2=(1-\beta)q_1 \tag{3.7}$$

and

$$\frac{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}{2\lambda}\left(2a_2^2-a_3\right) + \frac{\left[\lambda(\gamma+2)(\gamma-1)(\lambda+1)+(\gamma+1)^2(1-\lambda)\right](1-\delta)^2}{\lambda^2}a_2^2=(1-\beta)q_2. \tag{3.8}$$

From (3.5) and (3.7), we get

$$p_1=-q_1 \tag{3.9}$$

and

$$\frac{2(\gamma+1)^2(\lambda+1)^2(1-\delta)^2}{\lambda^2}a_2^2=(1-\beta)^2(p_1^2+q_1^2). \tag{3.10}$$

Adding (3.6) and (3.8), we obtain

$$\left(\frac{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}{\lambda} + \frac{2\left[\lambda(\gamma+2)(\gamma-1)(\lambda+1)+(\gamma+1)^2(1-\lambda)\right](1-\delta)^2}{\lambda^2}\right)a_2^2=(1-\beta)(p_2+q_2). \tag{3.11}$$

Hence, we find that

$$a_2^2=\frac{\lambda^2(1-\beta)(p_2+q_2)}{\lambda(\gamma+2)(\lambda+1)(1-\delta)\left(2\gamma(1-\delta)+1\right)+2(1-\lambda)(\gamma+1)^2(1-\delta)^2}.$$

By applying Lemma 1 for the coefficients p_2 and q_2, we deduce that

$$|a_2| \leq \frac{2\lambda\sqrt{1-\beta}}{\sqrt{\lambda(\gamma+2)(\lambda+1)(1-\delta)\left(2\gamma(1-\delta)+1\right)+2(1-\lambda)\left(\gamma+1\right)^2\left(1-\delta\right)^2}}.$$

To determinate the bound on $|a_3|$, by subtracting (3.8) from (3.6), we get

$$\frac{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}{\lambda}\left(a_3-a_2^2\right)=(1-\beta)\left(p_2-q_2\right),$$

or equivalently

$$a_3 = a_2^2 + \frac{\lambda(1-\beta)\left(p_2-q_2\right)}{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}. \tag{3.12}$$

Substituting the value of a_2^2 from (3.10) into (3.12), it follows that

$$a_3 = \frac{\lambda^2\left(1-\beta\right)^2\left(p_1^2+q_1^2\right)}{2\left(\gamma+1\right)^2\left(\lambda+1\right)^2\left(1-\delta\right)^2} + \frac{\lambda\left(1-\beta\right)\left(p_2-q_2\right)}{(\gamma+2)(\lambda+1)(1-\delta)(3-2\delta)}.$$

By applying Lemma 1 once again for the coefficients p_1, p_2, q_1 and q_2, we deduce that

$$|a_3| \leq \frac{4\lambda\left(1-\beta\right)}{(\lambda+1)(1-\delta)}\left[\frac{\lambda\left(1-\beta\right)}{\left(\gamma+1\right)^2\left(\lambda+1\right)\left(1-\delta\right)} + \frac{1}{(\gamma+2)(3-2\delta)}\right].$$

□

Remark 4. In Theorem 2, if we choose

(1) $\lambda = 1$ and $\delta = \frac{1}{2}$, then we have the results which was given by Prema and Keerthi [7, Theorem 3.2];
(2) $\lambda = 1, \gamma = 0$ and $\delta = \frac{1}{2}$, then we have the results obtained by Murugusundaramoorthy et al. [6, Corollary 7];
(3) $\lambda = \gamma = 1$ and $\delta = \frac{1}{2}$, then we obtain the results obtained by Srivastava et al. [18, Theorem 2].

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