COMMISSIONS G1 AND G4 OF THE IAU
INFORMATION BULLETIN ON VARIABLE STARS
Volume 63 Number 6252 DOI: 10.22444/IBVS.6252

Konkoly Observatory
Budapest
24 August 2018

HU ISSN 0374 – 0676

THE PERIOD ANALYSIS OF THE HIERARCHICAL SYSTEM DI Peg

OZUYAR, D.; ELMASLI, A.; CALISKAN, S.

[1] Ankara University, Faculty of Science, Dept. of Astronomy and Space Sciences, 06100, Tandogan, Ankara / Turkey, e-mail: dozuyar@ankara.edu.tr

Abstract

The existence of an additional body around a binary system can be detected by the help of the light-travel time effect. Due to the motions of the binary and the companion stars around the common mass center of the ternary system, the light-time effect produces an irregularity on the eclipse timings. Monitoring the variations in these timings, sub-stellar or planet companions orbiting around the binary system can be identified. In this paper, additional bodies orbiting the Algol-type binary DI Peg are examined by using the archival eclipse timings including our CCD data observed at the Ankara University Kreiken Observatory. More than five hundred minimum times equivalent to about nine decades are employed to identify the orbital behaviour of the binary system. The best fit to the timings shows that the orbital period of DI Peg has variations due to an integration of two sinusoids with the periods of $P_3 = 49.50 \pm 0.36$ yr and $P_4 = 27.40 \pm 0.24$ yr. The orbital change is thought to be most likely due to the existence of two M-type red dwarf companions with the masses of $M_3 = 0.213 \pm 0.021$ M$_\odot$ and $M_4 = 0.151 \pm 0.008$ M$_\odot$, assuming that the orbits of additional bodies are co-planar with the orbit of the binary system. Also, the residuals of two sinusoidal fits still seem to show another modulation with the period of roughly $P = 19.5$ yr. The origin of this modulation is not clear and more observational data are required to reveal if the periodicity is caused by another object gravitationally bounded to the system.

1 Introduction

Hierarchical multi-body star systems (Evans 1968) form in different ways, such as from interaction/capture in a globular star cluster (van den Berk et al. 2007), from a massive primordial disk involving accretion processes and/or local disk instabilities (Lim and Takakuwa 2006; Marzari et al. 2009) or from a non-hierarchical star system by angular momentum and energy exchange via mutual gravitational interactions (Reipurth 2000). These systems can be basically classified into two groups; circumbinary and circumstellar systems. In circumbinary systems, one or more additional bodies move around a binary star and they are known as companions on P-type orbits (Dvorak 1986). A transiting circumbinary planet, PH1b, around KIC 4862625 which consists of two binary pairs; the quadruple systems HD 98800 (Furlan et al. 2007) and SZ Her (Lee et al. 2012) can be given as examples of such a hierarchy. On the other hand, the systems with companions orbiting one component of a binary pair are the other type of hierarchical systems (circumstellar or S-type configuration; Schwarz et al. 2011). The example of such a system can be found in Neuhäuser et al. (2007) and Chauvin et al. (2007).

A hierarchical circumbinary system can be detected by observing the timings of the mid-eclipse times of the binary companion. The presence of an additional body causes a change in the relative distance of the eclipsing pair to the observer depending on the motion of the third body around the barycenter of the triple system. This binary wobble leads a periodic variation in conjunction times. As a result, the eclipses present lags or advances in the timings of minimum light (Irwin 1952). As known, the light-time effect is a geometrical feature and the third object produces a sinusoidal-like variation in the binary orbital. If the archival database is large and sufficient enough, this variation in eclipse timings provides an opportunity to understand the nature of the multi-body system (Pribulla et al. 2012).

In this sense, space-based missions offer a unique opportunity for the discovery of companions orbiting eclipsing binaries. For example, Kepler provides continuous and highly homogeneous light curves over the time interval of four years. Thus, its photometric observations enable new discoveries of multiple star systems, such as triple, quadruple or even quintuple ones. Indeed, there are a large number of multiple star systems identified from the Kepler observations. Conroy et al. (2014) present a catalog, which includes precise minimum times and third body signals for 1279 close binaries in the latest Kepler Eclipsing Binary Catalog. They find 236 binaries having third body signals. Borkovits et al. (2015) report O–C analysis of 26 compact hierarchical triple stars in the Kepler field. Borkovits et al. (2016) identify the existence of a third body in 222 of 2600 Kepler binaries. The quadruple system KIC 7177553 (Lehmann et al. 2016) consists of two eccentric binaries with a separation of 0.4 arcsec (167 au). The outer orbit's period is in the range of 1000-3000 yr. Another quadruple star system, EPIC 220204960, contains two slightly eccentric binaries with orbital periods of 13.27 and 14.41 days (Rappaport et al. 2017). These binaries are in a quadruple system with an outer period of 1 yr and a physical separation of 30 au. An example for a quintuple star system is EPIC 212651213 and EPIC 212651234 (Rappaport et al. 2016). In this system, EPIC 212651213 hosts two eclipsing binaries with orbital periods of 5.1 and 13.1 days. EPIC 212651234 is a single star with a projected physical separation of about 0.013 pc to EPIC 212651213. It is also stated that EPIC 212651213 and EPIC 212651234 are gravitationally bound to each other.

DI Peg (HIP 116167, GSC 01175-00013, BD+14 5006) was discovered by Morgenroth (1934) and identified to be an Algol type eclipsing binary (F4IV+ K4) by Rucinski (1967) and Lu (1992). From the photographic observations, Jensch (1934) determined the period of the system to be $\sim 0\overset{\mathrm{d}}{.}711811$. Rucinski (1967) analysed the photoelectric observations of Kruszewski (1964) and derived the first orbital solutions. Based on the results, he suggested the existence of a third light which provided 24% contribution to the total light of the system. More photometric studies were performed by Chou and Kitamura (1968), Binnendijk (1973), Chaubey (1982), Lu (1992), and Yang et al. (2014).

Gaposchkin (1953) detected a variation in the orbital period of the star. Ahnert (1974) and Vinkó (1992) proposed a possible light-time effect in the system and they gave periods of ~ 62.4 and ~ 22.1 yr. By using the spectroscopic solutions, Lu (1992) determined the system parameters as $a = 4.14(0.05)$ $R_\odot$, $V_0 = +43.8(2.0)$ $\mathrm{km\,s^{-1}}$, $K_1 = 185.2(2.4)$ $\mathrm{km\,s^{-1}}$, $K_2 = 109.0(2.1)$ $\mathrm{km\,s^{-1}}$, $T_0 =$ HJD 48213.8851(0.0022) and $q_{\mathrm{sp}} = 0.59(0.01)$.

Rafert (1982) derived a downward quadratic ephemeris with a cyclic variation in the O–C diagram. Unlike this, Hanna and Amin (2013) obtained a cyclic modulation with the period of 55 years, superimposed on an upward parabolic variation. The long-term orbital period increase was found to be $dP/dt = 0.17$ s/century and interpreted as a mass transfer from the evolved secondary component to the primary one with the rate of 1.52×10^{-8} $M_\odot$/yr. The cyclic variation was attributed to a low-mass third body with the mass of $M_3 \sim 0.2200 \pm 0.0006$ $M_\odot$. The parameters of the third body were given as

$e_3 = 0.77(7)$ and $w_3 = 300° \pm 10°$.

Recently, Yang et al. (2014) reproduced the photometric models with the help of new multi-color observations and previously published ones in literature. They determined the system parameters as $i = 89^{\circ}\!.02 \pm 0^{\circ}\!.11$, $M_1 = 1.19(2)$ M$_{\odot}$, $M_2 = 0.70(2)$ M$_{\odot}$, $L_1 = 3.70(4)$ L$_{\odot}$, and $L_2 = 0.53(2)$ L$_{\odot}$. According to the results, they stated that the system had a low third light whose fill-out factor for the more massive component was $f_{\rm p} = 78.2(4)$. Their O–C curve also indicated that the orbital period of DI Peg has changed in a complicated mode, such that the period of the star possibly showed two light-time orbits with the modulation periods of $P_3 \sim 54.6(5)$ yr and $P_4 \sim 23.0(6)$ yr, respectively. The masses of the inner and outer sub-stellar objects were given to be $M_{in} \sim 0.095$ M$_{\odot}$ and $M_{out} \sim 0.170$ M$_{\odot}$. On the basis of these results, Yang et al. (2014) suggested that the system has consists of four objects.

The aim of this study is to perform a detailed period analysis of DI Peg for the parameter determination of the additional bodies in the system by using the new and all available archival minimum times. For this purpose, the paper is organized as follows; the observations are presented in Section 2, the analysis is described in Section 3, the results related to the analysis are discussed in Section 4.

2 Observations

We observed DI Peg in V and R filters on the nights of 1 and 2 November 2017 at the Ankara University Kreiken Observatory. Observations were carried out by using an Apogee ALTA U47 + CCD camera (1024 × 1024 pixels) with Johnson $UBVRI$ filters mounted on a 35 cm telescope. In the observing process, BD+14 5004 was chosen as the comparison star (Table 1). Bias, dark, and flat corrections were performed and all images were reduced by using the MaxIm DL software[1]. The individual differential magnitudes were computed by subtracting the variable star from the comparison (V-C). The data covered two minima, the timings of which were determined as Min I $= 2458060.4456 \pm 0.0001$ and Min II $= 2458059.3779 \pm 0.0002$ with the method of Kwee and van Woerden (1956). The values were an average of the minimum times obtained in V and R colors during the same point.

Table 1. Spectral types, brightness, filters and exposure times are given for DI Peg and its comparison star BD+14 5004.

Star		Spectral Type	V (mag)	Filters	Exposure Times (s)
DI Peg	Variable	F4-IV	9.52	R, V	35, 35
BD+14 5004	Comparison	K4	9.83	R, V	35, 35

3 Analysis

The O–C diagram of DI Peg covering a time span of 88 years (Figure 1) was constructed from 85 primary, 14 secondary CCD; 45 primary, 9 secondary photoelectric; 17 primary photographic and 340 visual minimum times. These minima were collected from various observers listed in Table 1. The uncertainties of these values are not given in the table and can be accessed directly from their sources. The light elements of DI Peg were derived from the linear least-square fit applied to the CCD and photoelectric minimum times.

[1] https://diffractionlimited.com/help/maximdl/MaxIm-DL.htm

Table 1: All available minimum times of DI Peg in archives

Min. Time (HJD-2400000)	Typ.	Meth.	Ref.	Min. Time (HJD-2400000)	Typ.	Meth.	Ref.
25644.3150	1	pg	Guthnick & Prager ; AN 258	37193.5400	1	vi	B.Czerlunczakiewic ; AA 17.62
25918.3510	1	vi	A.Jensch ; AN 252.395	37196.3810	1	vis	B.Czerlunczakiewic ; EBC 1-32
26000.2330	1	vi	A.Jensch ; AN 252.395	37196.3830	1	vi	J.Rodzinski ; AA 18.332
26249.3640	1	pg	A.Jensch ; AN 252.395	37196.3910	1	vis	A.Slowik ; EBC 1-32
26266.4440	1	pg	A.Jensch ; AN 252.395	37270.4040	1	vi	F.Gerhart ; AN 288.72
26624.4580	1	pg	A.Jensch ; AN 252.395	37517.4080	1	vi	A.Slowikowna ; AA 17.62
26960.4600	1	vi	A.Jensch ; AN 252.395	37522.3946	1	pe	A.Kruszewski ; AA 17.275
26980.3840	1	vi	A.Jensch ; AN 252.395	37523.4620	2	pe	A.Kruszewski ; AA 17.275
27738.4740	1	vi	R.Szafraniec ; AAC 4.81	37527.3776	1	pe	A.Kruszewski ; AA 17.275
28432.4910	1	vi	W.Opalski ; BBG 1.47	37544.4610	1	pe	A.Kruszewski ; AA 17.275
28434.6270	1	vi	W.Opalski ; BBG 1.47	37556.5410	1	vis	H.Brancewicz ; AA 17.62
28452.4170	1	vi	W.Opalski ; BBG 1.47	37559.4096	1	pe	A.Kruszewski ; AA 17.275
28454.5570	1	vi	W.Opalski ; BBG 1.47	37626.3190	1	vi	R.Gizinski ; BAVM 15
28457.4050	1	vi	W.Opalski ; BBG 1.47	37668.3160	1	pg	H.Huth ; MVS 3.170
28459.5410	1	vi	W.Opalski ; BBG 1.47	37870.4760	1	vi	H.Huth ; MVS 3.170
28460.2510	1	vi	W.Opalski ; BBG 1.47	37907.4920	1	pg	H.Huth ; MVS 3.170
31273.3460	1	vi	W.Zessewitsch ; IODE 4.2.290	37932.3960	1	vi	E.Pohl ; AN 288.72
32441.4410	1	vi	R.Szafraniec ; AAC 4.81	37932.3970	1	vi	F.Gerhart ; AN 288.72
32794.4970	1	vi	R.Szafraniec ; AAC 4.113	37932.4060	1	pg	H.Huth ; MVS 3.170
32809.4430	1	vi	R.Szafraniec ; AAC 4.113	37934.5370	1	vi	K.Klocke ; BAVM 15
33170.3340	1	vi	R.Szafraniec ; AAC 5.5	37944.5060	1	vi	J.Duball ; BAVM 15
33187.4120	1	vi	R.Szafraniec ; AAC 5.5	37947.3540	1	vi	W.Braune ; BAVM 15
33538.3440	1	vi	R.Szafraniec ; AAC 5.7	37956.6032	1	pe	Chou & Kitamura ; JKAS 1
33570.3780	1	vi	R.Szafraniec ; AAC 5.11	37983.6528	1	pe	Chou & Kitamura ; JKAS 1
33871.4780	1	vi	R.Szafraniec ; AAC 5.11	38253.4300	1	vi	P.Flin ; AA 17.62
33913.4740	1	vi	A.Kruszewski ; AA 6.140	38255.5610	1	vi	H.Huth ; MVS 3.170
33916.3240	1	vi	A.Kruszewski ; AA 6.140	38290.4530	1	pg	H.Huth ; MVS 3.170
33918.4510	1	vi	A.Kruszewski ; AA 6.140	38322.4780	1	vi	V.Orlovius ; AN 288.72
33928.4240	1	vi	R.Szafraniec ; AAC 5.11	38399.3620	1	vi	P.Hoffmann ; BAVM 18
34239.4900	1	vi	R.Szafraniec ; AAC 5.53	38591.5270	1	pg	H.Huth ; MVS 3.170
34254.4410	1	vi	R.Szafraniec ; AAC 5.191	39006.5324	1	pe	S.M.Rucinski ; AA 17.275
34580.4550	1	vi	R.Szafraniec ; AAC 5.191	39026.4630	1	vi	W.Braune ; BAVM 18
34664.4400	1	vi	R.Szafraniec ; AAC 5.191	39046.3940	1	vi	W.Braune ; BAVM 18
35010.3850	1	vi	R.Szafraniec ; AAC 5.194	39056.3620	1	vi	W.Braune ; BAVM 18
35341.3830	1	vi	R.Szafraniec ; AA 6.143	39061.3430	1	vi	K.Locher ; ORI 95
35366.3020	1	vi	R.Szafraniec ; AA 6.143	39352.4790	1	vi	W.Braune ; BAVM 23
35699.4320	1	vi	R.Szafraniec ; AA 7.190	39374.5440	1	vi	K.Locher ; ORI 100
35719.3550	1	vi	R.Szafraniec ; AA 7.190	39387.3600	1	vi	W.Braune ; BAVM 23
35731.4490	1	vi	R.Szafraniec ; AA 7.190	39389.4960	1	vi	W.Braune ; BAVM 23
35746.4090	1	vi	R.Szafraniec ; AA 7.190	39407.2890	1	vi	M.Seidl ; BAVM 23
35838.2310	1	pg	H.Huth ; MVS 3.170	39407.2930	1	vi	K.Locher ; ORI 100
36079.5490	1	pg	H.Huth ; MVS 3.170	39419.4010	1	vi	S.Hazer ; AN 291.113
36450.3900	1	vi	R.Szafraniec ; AA 9.49	39683.4680	1	vi	K.Locher ; ORI 103
36455.3779	1	vi	J.Kordylewski ; SAC 30.108	39827.2630	1	vi	K.Locher ; ORI 105
36462.4880	1	pg	H.Huth ; MVS 3.170	40088.4990	1	vi	F.Hromada ; BRNO 9
36818.3880	1	pg	H.Huth ; MVS 3.170	40114.8356	1	pe	L.Binnendijk ; AJ 78.97
40127.6488	1	pe	L.Binnendijk ; AJ 78.97	41928.5370	1	vi	W.Quester ; BAVM 28
40128.3600	1	vi	P.Flin ; IBVS 328	41931.3750	1	vi	R.Germann ; BBS 11
40159.6796	1	pe	L.Binnendijk ; AJ 78.97	41931.3930	1	vi	I.Kohoutek ; BRNO 17
40175.3430	1	vi	W.Braune ; BAVM 23	41941.3530	1	vi	H.Peter ; BBS 11
40424.4746	1	pe	N.Gudur ; IBVS 456	41983.3490	1	pg	P.Ahnert ; MVS 7.38
40471.4540	1	vi	J.Silhan ; BRNO 9	41983.3560	1	vi	J.Hudec ; BRNO 17
40476.4370	1	vi	J.Silhan ; BRNO 9	41988.3210	1	vi	R.Germann ; BBS 12
40483.5590	1	vi	M.Fernandes ; BAVM 26	42008.2630	1	vi	H.Peter ; BBS 12
40500.6394	1	pe	L.Binnendijk ; AJ 78.97	42274.4860	1	vi	J.Hudec ; BRNO 20
40506.3380	1	vi	K.Rausal ; BRNO 12	42289.4270	1	vi	H.Peter ; BBS 17
40512.7402	1	pe	L.Binnendijk ; AJ 78.97	42289.4290	1	pe	O.Demircan ; IBVS 1053
40526.2640	1	vi	K.Locher ; ORI 116	42301.5400	1	vi	J.Hudec ; BRNO 20
40725.5750	1	vi	K.Locher ; ORI 119	42304.3760	1	vi	R.Germann ; BBS 17
40772.5510	1	vi	K.Locher ; ORI 120	42304.3960	1	vi	M.Vlcek ; BRNO 20
40812.4130	1	vi	W.Braune ; BAVM 25	42403.3170	1	vi	K.Locher ; BBS 19
40837.3269	1	pe	O.Demircan ; IBVS 530	42403.3220	1	vi	H.Peter ; BBS 19
40837.3290	1	vi	W.Braune ; BAVM 25	42403.3240	1	vi	R.Diethelm ; BBS 19
40837.3300	1	vi	J.Hubscher ; BAVM 25	42739.2950	1	vi	W.Braune ; BAVM 29
40839.4630	1	vi	R.Diethelm ; ORI 121	42739.3000	1	vi	H.Peter ; BBS 24
40854.4130	1	vi	M.Geseova ; BRNO 12	42754.2470	1	vi	H.Peter ; BBS 25
40856.5400	1	vi	M.Geseova ; BRNO 12	42776.2960	1	vi	R.Germann ; BBS 25
40859.3930	1	pe	C.Endres ; IBVS 530	42786.2710	1	vi	R.Germann ; BBS 26
40859.3960	1	pg	P.Ahnert ; MVS 6.9	42786.2750	1	vi	H.Peter ; BBS 26
40886.4480	1	vi	H.Gese ; BRNO 12	42796.2400	1	vi	H.Peter ; BBS 26
40911.3530	1	vi	K.Locher ; ORI 122	42990.5700	1	vi	K.Locher ; BBS 29
40921.3240	1	vi	K.Locher ; ORI 122	42993.4120	1	vi	K.Locher ; BBS 29
41155.5040	1	vi	L.Frasinski ; IBVS 584	43013.3510	1	vi	R.Germann ; BBS 29
41177.5740	1	vi	K.Locher ; ORI 126	43015.4802	1	pe	J.Ebersberger ; IBVS 1358
41210.3240	1	vi	H.Peter ; ORI 127	43015.4840	1	vi	P.Simecek ; BRNO 21
41232.3940	1	vi	K.Locher ; ORI 127	43034.7010	1	vi	G.Samolyk ; AOEB 2
41247.3320	1	vi	K.Locher ; ORI 129	43040.3980	1	vi	K.Locher ; BBS 30
41267.2632	1	vi	W.Braune ; BAVM 25	43069.5700	1	vi	E.Halbach ; AOEB 2
41513.5560	1	vi	K.Locher ; BBS 4	43069.5830	1	vi	G.Samolyk ; AOEB 2
41550.5620	1	vi	K.Locher ; BBS 5	43071.0029	1	pe	H.D.Kennedy ; IBVS 2118
41563.3810	1	vi	H.Peter ; BBS 5	43112.2910	1	vi	R.Germann ; BBS 31
41565.5120	1	vi	K.Locher ; BBS 5	43134.3600	1	vi	R.Germann ; BBS 31
41580.4600	1	vi	K.Locher ; BBS 5	43154.2880	1	vi	R.Germann ; BBS 32
41595.4070	1	vi	R.Diethelm ; BBS 6	43311.5940	1	vi	K.Locher ; BBS 33
41597.5432	1	vi	W.Quester ; BAVM 26	43341.4850	1	vi	K.Vojtek ; BRNO 21
41605.3720	1	vi	W.Quester ; BAVM 26	43371.3870	1	vi	R.Germann ; BBS 34
41605.3730	1	vi	K.Locher ; BBS 6	43391.3190	1	vi	R.Germann ; BBS 35
41605.3780	1	vi	H.Peter ; BBS 6	43393.4570	1	vi	K.Locher ; BBS 35
41657.3370	1	vi	R.Diethelm ; BBS 7	43393.4730	1	vi	P.Ivan ; BRNO 21
41682.2470	1	vi	J.Hubscher ; BAVM 26	43403.4350	1	vi	P.Ivan ; BRNO 21
41682.2500	1	vi	W.Braune ; BAVM 26	43425.4940	1	vi	K.Vojtek ; BRNO 21
41921.4270	1	vi	Z.Pokorny ; BRNO 17	43433.3230	1	vi	D.Lichtenknecker ; BAVM 31
43434.0295	1	pe	H.D.Kennedy ; IBVS 2118	44517.4160	1	vi	G.Mavrofridis ; BBS 51
43435.4610	1	vi	D.Lichtenknecker ; BAVM 31	44517.4190	1	vi	G.Stefanopoulos ; BBS 52
43455.3900	1	vi	J.Soukopova ; BRNO 21	44524.5340	1	vi	G.Mavrofridis ; BBS 51
43460.3740	1	vi	D.Sasselov ; BRNO 21	44532.3640	1	vi	W.Braune ; BAVM 32
43490.2640	1	vi	J.Mrazek ; BRNO 21	44543.0401	1	pe	H.D.Kennedy ; IBVS 2118
43495.2440	1	vi	R.Germann ; BBS 36	44557.9879	1	pe	H.D.Kennedy ; IBVS 2118
43517.3180	1	vi	R.Germann ; BBS 36	44567.2420	1	vi	H.Peter ; BBS 51
43689.5710	1	vi	K.Locher ; BBS 37	44567.2450	1	vi	R.Germann ; BBS 51
43724.4540	1	vi	P.Simecek ; BRNO 23	44593.5840	1	vi	G.Hanson ; AOEB 2
43725.5179	2	pe	Z.Tufekcioglu ; IBVS 1495	44636.2870	1	vi	R.Germann ; BBS 52
43729.4333	1	pe	Z.Tufekcioglu ; IBVS 1495	44823.4940	1	vi	T.Kaczkowski ; MVS 9.90
43729.4380	1	vi	P.Ivan ; BRNO 23	44823.5000	1	vi	T.Graf ; BRNO 26
43756.4831	1	pe	Z.Tufekcioglu ; IBVS 1495	44843.4272	1	pe	E.Derman et al. ; IBVS 2159
43776.4140	1	vi	D.Lichtenknecker ; BAVM 31	44848.4102	1	pe	E.Derman et al. ; IBVS 2159
43780.3277	2	pe	Z.Tufekcioglu ; IBVS 1495	44853.3920	1	vi	H.Peter ; BBS 57
43791.3540	1	vi	R.Germann ; BBS 39	44853.3950	1	vi	K.Carbol ; BRNO 26
43791.3700	1	vi	H.Peter ; BBS 39	44883.2830	1	vi	N.Stoikidis ; BBS 57
43802.7600	1	vi	G.Samolyk ; AOEB 2	44890.4100	1	vi	H.Peter ; BBS 57
43803.4650	1	vi	H.Peter ; BBS 39	44893.2550	1	vi	N.Stoikidis ; BBS 57
43806.3090	1	vi	R.Germann ; BBS 39	44900.3870	1	vi	G.Mavrofridis ; BBS 57

Table 1 – continued from previous page

Min. Time (HJD-2400000)	Typ.	Meth.	Ref.	Min. Time (HJD-2400000)	Typ.	Meth.	Ref.
43863.2560	1	vi	R.Germann ; BBS 41	44910.3300	1	vi	N.Stoikidis ; BBS 57
43878.2020	1	vi	K.Locher ; BBS 41	44925.2840	1	vi	H.Peter ; BBS 57
44077.5070	1	vi	D.Svelohva ; BRNO 23	45170.8580	1	vi	E.Halbach ; AOEB 2
44092.4600	1	vi	K.Locher ; BBS 44	45196.4870	1	pe	A.Buchtler ; IBVS 2385
44102.4260	1	vi	V.Wagner ; BRNO 23	45201.4690	1	pe	M.Prikryl ; BRNO 26
44117.3690	1	vi	R.Germann ; BBS 44	45201.4720	1	vi	H.Peter ; BBS 62
44117.3770	1	vi	H.Peter ; BBS 44	45228.5220	1	vi	N.Machkova ; BRNO 26
44134.4580	1	vi	H.Peter ; BBS 45	45231.3690	1	vi	G.Mavrofridis ; BBS 63
44143.3560	2	pe	Z.Aslan et al. ; IBVS 1908	45235.6450	1	vi	G.Samolyk ; AOEB 2
44144.4227	1	pe	Z.Aslan et al. ; IBVS 1908	45258.4170	1	vi	H.Bohutinska ; BRNO 26
44164.3545	1	pe	U.S.Chaubey ; ASS 81.283	45554.5250	1	vi	P.Svoboda ; BRNO 26
44166.4920	1	vi	T.Brelstaff ; VSSC 59.19	45579.4470	1	vi	P.Svoboda ; BRNO 26
44189.2670	1	vi	H.Peter ; BBS 45	45609.3400	1	pg	M.Dietrich ; MVS 10.104
44219.1650	1	pe	U.S.Chaubey ; ASS 81.283	45609.3440	1	vi	M.Zejda ; BRNO 26
44435.5650	1	vi	K.Locher ; BBS 49	45624.2920	1	vi	N.Stoikidis ; BBS 69
44440.5400	1	vi	R.Germann ; BBS 49	45671.2750	1	vi	P.Svoboda ; BRNO 26
44445.5250	1	vi	K.Locher ; BBS 49	45915.4230	1	vi	H.Peter ; BBS 73
44455.4900	1	vi	K.Chyzny ; MVS 9.18	45976.6430	1	vi	D.Williams ; AOEB 2
44470.4380	1	vi	P.Kucera ; BRNO 23	45976.6500	1	vi	S.Cook ; AOEB 2
44474.7030	1	vi	G.Samolyk ; AOEB 2	45981.6290	1	vi	S.Cook ; AOEB 2
44490.3640	1	vi	R.Diethelm ; BBS 50	45992.3030	1	vi	A.Paschke ; BBS 74
44490.3660	1	vi	H.Peter ; BBS 50	46002.2610	1	vi	A.Paschke ; BBS 74
44497.4860	1	vi	G.Mavrofridis ; BBS 51	46019.3490	1	vi	S.Krampol ; BRNO 27
44502.4654	1	pe	D.Elias ; BBS 54	46028.6090	1	vi	D.Williams ; AOEB 2
44502.4690	1	vi	D.Mourikis ; BBS 50	46028.6110	1	vi	G.Samolyk ; AOEB 2
44512.4340	1	vi	H.Peter ; BBS 50	46029.3160	1	vi	A.Paschke ; BBS 74
46033.5850	1	vi	S.Cook ; AOEB 2	48148.3950	1	vi	J.Pietz ; BAVM 59
46038.5670	1	vi	D.Williams ; AOEB 2	48205.3360	1	vi	J.Pietz ; BAVM 59
46038.5680	1	vi	G.Samolyk ; AOEB 2	48219.5690	1	vi	G.Samolyk ; AOEB 2
46043.5530	1	vi	D.Williams ; AOEB 2	48266.5520	1	vi	G.Samolyk ; AOEB 2
46290.5420	1	vi	S.Stefanisko ; BRNO 27	48480.8140	1	vi	G.Samolyk ; AOEB 2
46294.1170	1	vi	T.Kato ; VSB 47	48481.5240	1	vi	J.Sojka ; BRNO 31
46305.5010	1	vi	A.Paschke ; BBS 81	48500.0280	1	vis	Hipparcos ; ESA, 2001
46320.4500	1	vi	A.Paschke ; BBS 81	48506.4230	1	vi	L.Honzik ; BRNO 31
46344.6500	1	vi	S.Cook ; AOEB 2	48543.8039	2	CCD	Hipparcos ; ESA, 2001
46350.3450	1	vi	A.Paschke ; BBS 81	48545.5870	1	vi	G.Samolyk ; AOEB 2
46355.3240	1	vi	O.Grugel ; BAVM 43	48554.8375	1	CCD	Hipparcos ; ESA, 2001
46360.3040	1	vi	M.Dietrich ; MVS 11.19	48859.5040	1	vi	J.Chlachula ; BRNO 31
46360.3100	1	vi	O.Grugel ; BAVM 43	48863.7660	1	vi	D.Williams ; AOEB 2
46382.3710	1	vi	M.Dietrich ; MVS 11.19	48873.7330	1	vi	R.Hill ; AOEB 2
46413.6980	1	vi	G.Samolyk ; AOEB 2	48894.3760	1	vi	R.Baule ; BAVM 62
46422.2380	1	vi	A.Paschke ; BBS 81	48935.3002	2	pe	S.ozdemir ; IBVS 4380
46656.4230	1	vi	M.Muller ; BAVM 46	48939.2161	1	pe	S.Selam ; IBVS 4380
46678.4870	1	vi	P.Hajek ; BRNO 28	49215.4130	1	vi	P.Stuchlik ; BRNO 31
46678.4900	1	vi	A.Paschke ; BBS 81	49224.6500	1	vi	S.Cook ; AOEB 2
46738.2760	1	vi	D.Hanzl ; BRNO 28	49241.7350	1	vi	D.Williams ; AOEB 2
46743.2730	1	vi	A.Stuhl ; BRNO 31	49246.3631	2	pe	H.Ak ; IBVS 4380
46759.6390	1	vi	G.Samolyk ; AOEB 2	49248.4963	2	pe	A.Akalin ; IBVS 4380
46769.6070	1	vi	G.Samolyk ; AOEB 2	49276.2546	2	pe	H.Dundar ; IBVS 4380
46774.5910	1	vi	G.Samolyk ; AOEB 2	49277.3259	1	pe	A.Akalin ; IBVS 4380
46779.5640	1	vi	G.Samolyk ; AOEB 2	49333.5600	1	vi	G.Samolyk ; AOEB 2
46999.5200	1	vi	G.Mavrofridis ; BBS 86	49543.5440	1	vi	C.Barani ; BBS 108
47014.4630	1	vi	F.Hroch ; BRNO 30	49543.5500	1	vis	F.Acerbi ; BBS 107
47014.4664	1	vi	E.Wunder ; BAVM 50	49553.5085	1	pe	B.Gurol ; IBVS 4380
47029.4110	1	vi	L.Prokesova ; BRNO 30	49602.6300	1	vi	G.Samolyk ; AOEB 2
47031.5490	1	vi	J.Kolar ; BRNO 30	49743.5640	1	vi	G.Samolyk ; AOEB 8
47034.4000	1	vi	M.Jechumtal ; BRNO 30	49948.5775	1	vi	M.Zibar ; BRNO 32
47039.3790	1	vi	O.Beck ; BRNO 30	49950.7020	1	CCD	S.Cook ; AOEB 8
47054.3330	1	vi	G.Mavrofridis ; BBS 86	50008.3599	1	CCD	W.Kleikamp ; BAVM 90
47066.4290	1	vi	A.Paschke ; BBS 86	50008.3603	1	CCD	M.Wolf ; BBS 110
47091.3460	1	vi	G.Mavrofridis ; BBS 86	50013.3417	1	vi	J.Cechal ; BRNO 32
47107.7180	1	vi	R.Hill ; AOEB 2	50044.6700	1	vi	G.Samolyk ; AOEB 8
47387.4590	1	vi	P.Adamek ; BRNO 30	50050.3564	1	pe	B.Gurol ; IBVS 4380
47387.4610	1	vi	A.Epple ; BAVM 52	50313.7370	1	vi	G.Samolyk ; AOEB 8
47392.4390	1	vi	P.Adamek ; BRNO 30	50318.7140	1	CCD	S.Cook ; AOEB 8
47464.3440	1	vi	G.Samolyk ; AOEB 2	50368.5414	1	vi	A.Dedoch ; BRNO 32
47469.3150	1	vi	G.Samolyk ; AOEB 2	50376.3686	1	CCD	W.Kleikamp ; BAVM 102
47474.3180	1	vi	H.Peter ; BBS 90	50396.3000	1	vi	M.Dietrich ; BAVM 101
47794.6200	1	vi	G.Samolyk ; AOEB 2	50423.3560	1	vi	D.Girrbach ; BAVM 101
47851.5610	1	vi	G.Samolyk ; AOEB 2	50667.4989	1	vi	J.Polak ; BRNO 32
47853.6930	1	vi	M.Smith ; AOEB 2	50672.4793	1	pe	D.Husar ; BAVM 111
48123.4760	1	vi	M.Copikova ; BRNO 31	50672.4805	1	pe	W.Ogloza ; IBVS 4534
50672.4909	1	vi	J.Minar ; BRNO 32	53251.3810	1	vi	R.Obertrifter ; BAVM 202
50712.3428	1	pe	D.Husar ; BAVM 111	53251.3840	1	vi	G.-U.Flechsig ; BAVM 174
50716.6150	1	vi	G.Samolyk ; AOEB 8	53251.3860	1	vi	K.Rutz ; BAVM 174
50717.3278	1	vi	L.Brat ; BRNO 32	53251.3910	1	vi	W.Braune ; BAVM 174
50717.3305	1	vi	P.Sobotka ; BRNO 32	53262.4225	2	CCD	F.Agerer ; BAVM 173
50717.3370	1	pg	M.Dietrich ; BAVM 113	53265.6239	1	vi	W.Ogloza et al. ; IBVS 5843
50719.4618	1	pe	D.Husar ; BAVM 111	53267.7510	1	CCD	W.Ogloza et al. ; IBVS 5843
50754.3480	1	vi	R.Meyer ; BAVM 113	53267.7592	1	CCD	G.Samolyk ; AOEB 11
51035.4000	1	pe	B.Gurol ; IBVS 5069	53272.7416	1	CCD	W.Ogloza et al. ; IBVS 5843
51045.4699	1	vi	M.Vetrovcova ; BRNO 32	53282.7068	1	CCD	W.Ogloza et al. ; IBVS 5843
51076.7940	1	vi	D.Williams ; AOEB 8	53285.5570	1	vi	G.Chaple ; AOEB 11
51079.6400	1	vi	D.Williams ; AOEB 8	53290.5400	1	vi	G.Chaple ; AOEB 11
51084.6290	1	vi	G.Samolyk ; AOEB 8	53292.6790	1	vi	C.Stephan ; AOEB 11
51141.5690	1	vi	G.Samolyk ; AOEB 8	53317.5870	1	pe	G.Lubcke ; JAAVSO 41;328
51422.0120	1	CCD	A.Paschke ; Amateur	53325.4174	1	CCD	W.Quester ; BAVM 173
51432.7010	1	vi	D.Williams ; AOEB 8	53614.4169	1	CCD	V.Bakis et al. ; IBVS 5662
51433.4096	1	CCD	L.Kral ; BRNO 32	53619.3969	1	vi	P.Hejduk ; OEJV 0074
51452.6310	1	vi	G.Samolyk ; AOEB 8	53634.3450	1	CCD	M.Dietrich ; BAVM 178
51467.5790	1	vi	D.Williams ; AOEB 8	53645.0238	1	CCD	Kubotera ; VSB 44
51807.4721	2	CCD	W.Kleikamp ; BAVM 152	53645.7354	1	CCD	G.Samolyk ; AOEB 11
51818.5020	1	CCD	H.Achterberg ; BAVM 152	53671.3609	1	CCD	R.Ehrenberger ; OEJV 0074
51842.7060	1	vi	R.Hill ; AOEB 8	53674.9210	1	vi	Hirosawa ; VSB 44
51868.3321	1	CCD	M.Dietrich ; BAVM 152	53728.3061	1	CCD	J.Coloma ; AOEB 11
52168.7180	1	vi	D.Williams ; AOEB 8	53945.4760	1	CCD	K.Rutz ; BAVM 187
52203.5970	1	vi	D.Williams ; AOEB 8	53967.4772	2	CCD	S.Parimucha et al. ; IBVS 5777
52278.3363	1	CCD	G.Maintz ; BAVM 152	53991.3226	1	vi	S.Dogru et al. ; IBVS 5746
52530.3191	1	CCD	M.Dietrich ; BAVM 158	53992.3940	1	vi	W.Braune ; BAVM 187
52542.7862	2	CCD	Karska & Maciejewski ; IBVS 5380	53993.1031	1	CCD	K.Nagai et al. ; VSB 45
52567.3312	1	CCD	U.Schmidt ; BAVM 158	54016.5920	1	vi	G.Chaple ; AOEB 12
52572.6843	2	CCD	Karska & Maciejewski ; IBVS 5380	54023.7150	1	vi	D.Williams ; AOEB 12
52573.0329	1	CCD	Karska & Maciejewski ; IBVS 5380	54024.4239	1	CCD	F.Agerer ; BAVM 183
52594.3820	1	pe	T.Tanriverdi et al. ; IBVS 5407	54027.2706	1	CCD	R.Ehrenberger ; OEJV 0074
52843.5166	1	CCD	B.Gurol et al. ; IBVS 5791	54032.9670	1	vi	K.Nagai et al. ; VSB 45
52848.5024	1	vi	L.Marcin ; OEJV 0074	54058.5920	1	vi	C.Stephan ; AOEB 12
52848.5081	1	vi	J.Pcola ; OEJV 0074	54059.3020	1	pe	H.V. Senavci et al. ; IBVS 5754
52888.3606	1	CCD	T.Krajci ; IBVS 5592	54063.5720	1	vi	C.Stephan ; AOEB 12
52903.3083	1	CCD	M.Dietrich ; BAVM 172	54070.3254	2	pe	H.V. Senavci et al. ; IBVS 5754
52908.2924	1	CCD	M.Dietrich ; BAVM 172	54096.3177	1	CCD	R.Ehrenberger ; OEJV 0074
52911.1395	1	CCD	Nakajima ; VSB 42	54298.4676	1	vi	M.Mruz ; OEJV 0094
52950.2871	1	CCD	B.Schlereth ; BAVM 172	54309.5089	2	CCD	S.Parimucha et al. ; IBVS 5898
52986.5913	1	CCD	S.Dvorak ; AOEB 11	54335.4878	1	pe	T.Kilicoglu et al. ; IBVS 5801

Table 1 – continued from previous page

Min. Time (HJD-2400000)	Typ.	Meth.	Ref.	Min. Time (HJD-2400000)	Typ.	Meth.	Ref.
52993.7110	1	vi	G.Samolyk ; AOEB 11	54335.4887	1	CCD	L.melcer ; OEJV 0074
53001.5420	1	vi	D.Williams ; AOEB 11	54345.4486	1	CCD	S.Caliskan ; Nat. Ast. Cong., 2008
53236.4399	1	vis	J.Cernu ; OEJV 0074	54351.5003	2	CCD	S.Caliskan ; Nat. Ast. Cong., 2008
53236.4400	1	pe	B.Albayrak et al. ; IBVS 5649	54394.5693	1	CCD	G.Samolyk ; JAAVSO 36(2);171
53236.4476	1	vi	M.Zdvoruk ; OEJV 0074	54416.6361	1	CCD	J.Bialozynski ; JAAVSO 36(2);171
54436.5670	1	CCD	S.Dvorak ; IBVS 5814	56501.5600	1	CCD	K.Rutz ; BAVM 234
54710.6180	1	CCD	G.Samolyk ; JAAVSO 36(2);186	56537.8635	1	CCD	G.Samolyk ; JAAVSO 41;328
54738.3787	1	CCD	S.Parimucha et al. ; IBVS 5898	56557.7934	1	CCD	B.Manske ; JAAVSO 41;328
54774.6840	1	CCD	R.Diethelm ; IBVS 5871	56557.7946	1	CCD	G.Frey ; JAAVSO 42;426
54799.5955	1	CCD	K.Menzies ; JAAVSO 37(1);44	56565.6246	1	CCD	B.Manske ; JAAVSO 41;328
55044.4620	1	CCD	N.Erkan et al. ; IBVS 5924	56567.7599	1	CCD	G.Frey ; JAAVSO 42;426
55064.3929	1	CCD	G.-U.Flechsig ; BAVM 212	56572.7430	1	CCD	G.Frey ; JAAVSO 42;426
55085.7474	1	CCD	G.Samolyk ; JAAVSO 38;120	56577.7255	1	CCD	G.Frey ; JAAVSO 42;426
55116.3557	1	CCD	N.Erkan et al. ; IBVS 5924	56587.6911	1	CCD	G.Frey ; JAAVSO 42;426
55429.5569	1	CCD	S.Dogru et al. ; IBVS 5988	56588.4035	1	CCD	F.Agerer ; BAVM 234
55498.2485	2	CCD	S.Parimucha et al. ; IBVS 5980	56597.6568	1	CCD	G.Frey ; JAAVSO 42;426
55524.9404	1	CCD	K.Hirosawa ; VSB 51	56602.6394	1	CCD	G.Frey ; JAAVSO 42;426
55561.2440	1	CCD	L.melcer ; OEJV 0137	56905.5192	2	CCD	M. Masek ; BRNO 40
55820.3460	1	CCD	A.Paschke ; OEJV 0142	56929.3667	1	CCD	F.Agerer ; BAVM 239
55820.3461	1	CCD	M.Dietrich ; BAVM 225	56930.4362	1	CCD	F.Agerer ; BAVM 239
55867.3270	1	CCD	L.melcer ; OEJV 0160	56953.5685	1	CCD	N.Simmons ; JAAVSO 43-1
55887.2592	1	CCD	D.Buhme ; BAVM 225	56955.7049	1	CCD	G.Frey ; JAAVSO 44-1
56163.4447	1	CCD	S.Parimucha et al. ; IBVS 6044	57251.8222	1	CCD	K.Menzies ; JAAVSO 43-2
56210.0691	2	CCD	Y. Yang ; AJ 147	57267.4823	1	CCD	E. Bahar ; IBVS 6209
56211.1365	1	CCD	Y. Yang ; AJ 147	57278.5163	1	CCD	F.Agerer ; IBVS 6196
56212.2052	2	CCD	Y. Yang ; AJ 147	57308.7680	1	CCD	G.Frey ; JAAVSO 44-1
56219.6785	1	CCD	G.Frey ; JAAVSO 42;426	57327.2750	2	CCD	S.Parimucha ; IBVS 6167
56229.6439	1	CCD	G.Frey ; JAAVSO 42;426	57390.6267	1	CCD	R.Sabo ; JAAVSO 44-1
56231.7796	1	CCD	G.Frey ; JAAVSO 42;426	58059.3779	2	CCD	our study ; –
56256.6934	1	CCD	G.Frey ; JAAVSO 42;426	58060.4456	1	CCD	our study ; –

Thus, the new ephemeris was calculated as;

$$\mathrm{HJD_{MinI}} = 2455867.327300(81) + 0\overset{\mathrm{d}}{.}711816455(19) \times E. \tag{1}$$

The O–C diagram shown in Figure 1 (top panel) displayed two sinusoidal curves superimposed on each other. Of which, the primary curve had an eccentric cyclic change which had almost three maximum and two minima. Also, the residuals from the sinusoidal fit showed another low-amplitude, short-period and eccentric cyclic modulation having three minima and four maxima. Our observational CCD minima were the last two points plotted on the O–C diagram. These points allowed us to determine the turn point of the last maximum of the O–C curve.

We first used the Period04 program (Lenz and Breger 2005) to analyse the weighted data. Then, we extracted the individual frequencies causing the fluctuations. Two frequencies of $f_1 = 0.000041375$ c/E (A_1=0.0082, S/N = 7.84) and $f_2 = 0.000072382$ c/E (A_1=0.0059, S/N = 18.04), shown in Figure 2, were detected. These frequencies corresponded to two periods of 47.10 ± 0.63 and 26.92 ± 0.44 years, respectively. When these two theoretical frequencies were adjusted to the O–C diagram in Figure 1, they were in good agreement with observational data. For the eccentricities seen in the curves, the light-time effect caused by the third and fourth bodies in the system was considered. In order to derive light-time orbits and the parameters of the third and fourth additional bodies, we used the equations given by Irwin (1952). Furthermore, the computer code called OC2LTE30 (Ak et al. 2004) was used to determine the orbital parameters. All of these results are presented in Table 2.

In Figure 1, the orbital parameters of the third and fourth body are presented in the second and the third panels. The sum of these lines, which corresponds to the total theoretical O–C curve, are shown as the continuous line in the first panel. The sum of the least squares of the total residuals is 1.6×10^{-2} day^2. The estimated errors of these parameters arise from the non-linear least-squares method, on which the inverse problem solving method is based. This method does not take into account the error of each observation point and the possible correlations of fitted parameters with each other. Therefore, the standard error values given for the parameters may be smaller than they should be. So, the standard error values given in the table should be considered as the lowest limits.

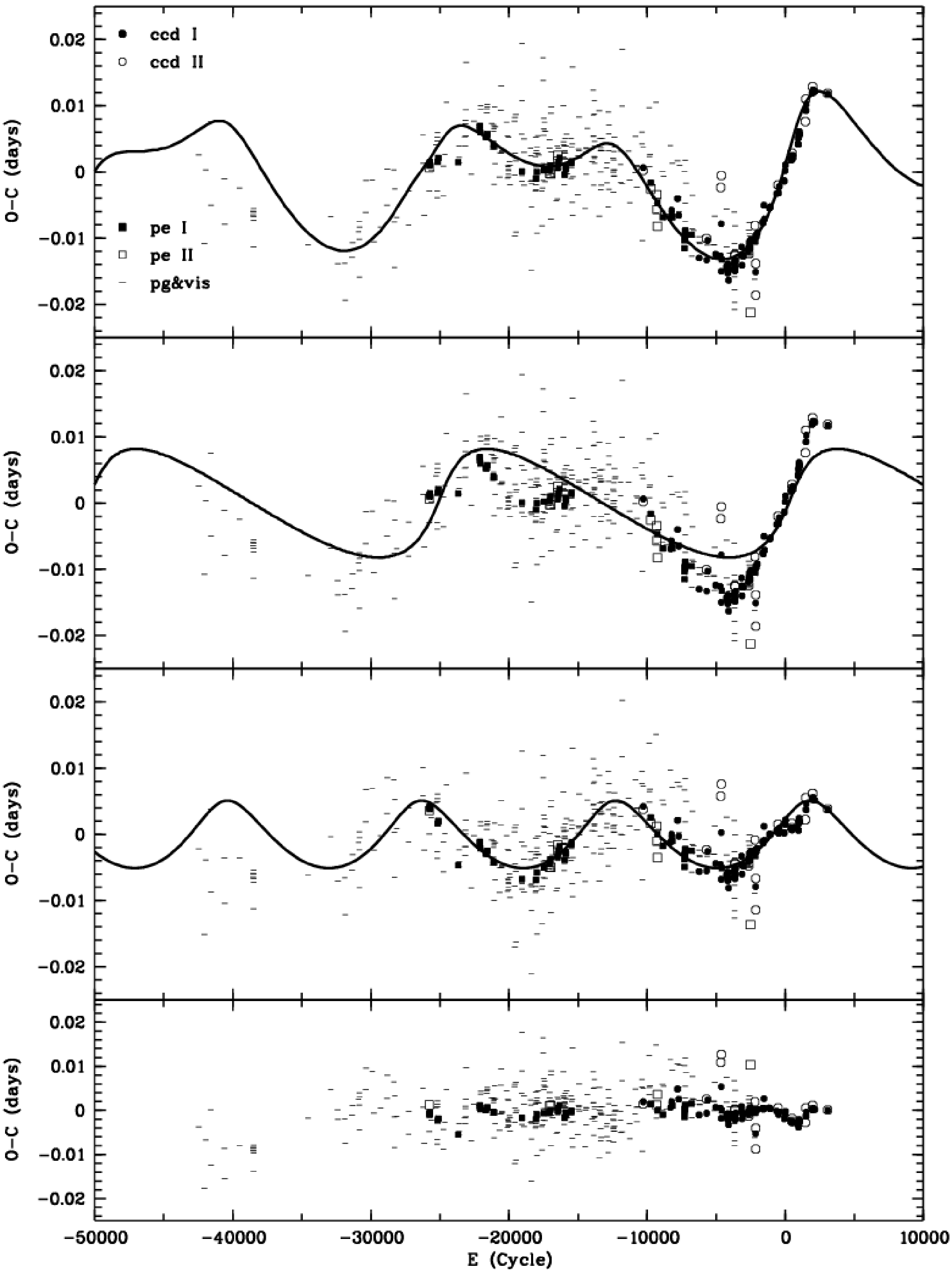

Figure 1. The O–C diagram of DI Peg. The first panel shows the overall data and the total theoretical O–C variation (continuous line). While the second panel presents the primary and highly eccentric sinosoidal variation, the residual data which have another sinusoidal modulation are displayed in the third panel. The final residuals are given in the last panel.

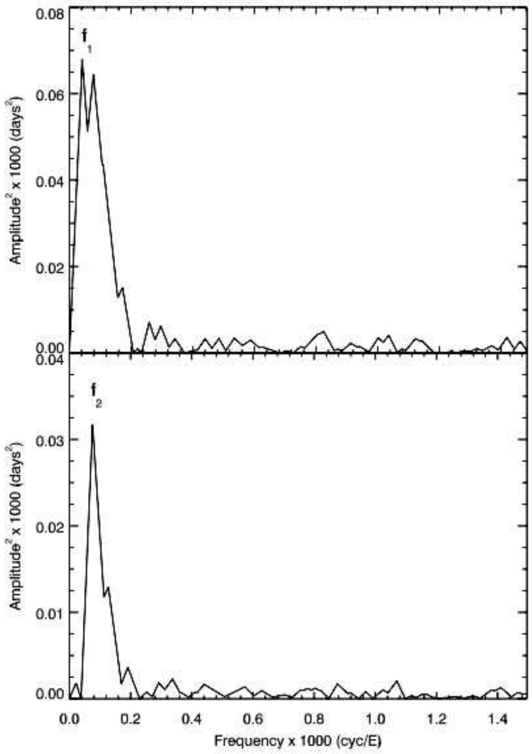

Figure 2. The two frequencies of $f_1 = 0.000041375$ and $f_2 = 0.000072382$ c/E detected by PERIOD04.

Table 2. Parameters and standard errors derived from O–C analysis of each additional body.

Parameters	Third Body	Fourth Body
$P_{3,4}$ [years]	49.50 ± 0.36	27.40 ± 0.24
A [days]	0.0082 ± 0.0002	0.0051 ± 0.0002
$e^{'}$	0.61 ± 0.06	0.30 ± 0.08
$\omega^{'}$ [°]	7.00 ± 1.74	75.00 ± 3.63
$T^{'}$ [HJD]	2456220 ± 261	2456860 ± 150
$f\ (m_{3,4})\ [M_{\odot}]$	0.0023 ± 0.0007	0.0009 ± 0.0001
$m[M\odot]$	0.2135 ± 0.0213	0.1505 ± 0.0075
$L_{\rm Bol}$ [L$_{\odot}$]	0.0061 ± 0.0017	0.0025 ± 0.0003
$M_{\rm Bol}$ [mag]	10.23 ± 0.27	11.22 ± 0.14
$m_{\rm Bol}$ [mag]	18.22 ± 1.38	19.21 ± 1.24
θ [arcsec]	0.0915 ± 0.0277	0.0625 ± 0.0184
$\sum(O-C)^2$ [day^2]	260×10^{-4}	138×10^{-4}

4 Results and Discussion

An O–C diagram is a special plot generally used to determine period changes that are difficult to detect by direct measurements. If there is not any measurable change in period, then the O–C difference generates a straight line. If any variation in period is detected, however, the O–C data generate a structure that displays the characteristic of the mechanism causing this variation. These mechanisms can be arranged as: mass transfer between components or mass loss from the system, spin-orbital interactions, angular momentum loss through stellar winds, gravitational waves, oscillations in rotation, differential rotation, apsidal motion, presence of a third light, and magnetic activity (Mikulasek et al. 2012).

In terms of binarity, orbital period change is quite an important subject since it is related to the formation, structure, and evolution of binary stars. These variables gain and lose mass and angular momentum as specified by Roche geometry. These events are the first proposed mechanisms to explain observed period changes. Both of these mechanisms can increase or decrease the period of the system and generate parabolic structures in the O–C diagram. The mass transfer between components is more effective in changing the orbital period than the mass loss from the system. The most basic case to be considered for exchanging material between components is conservative mass transfer. In this case, the mass lost by one component is gained by the companion star, so the total mass of the system and thus the total orbital angular momentum is preserved.

Among the common mechanisms given above, apsidal motion involves a change in the orientation of the system's major axis, since the potential energy between the components does not exactly obey Newton's gravitational law. In the O–C diagram, the times for secondary and primary minima shift in opposite directions. However, as this mechanism requires large eccentricities, it is rarely observed (Zavala et al. 2002). Alternatively, it is assumed that the cyclic pattern is caused by the presence of a third body in the system. Based on this assumption, the primary and secondary eclipse times are produced by the motion of the binary around the common centre of mass of a triple system. In this case, the periodic pattern arises from the light-time effect (Borkovits and Hegedüs 1996).

Apart from these, another mechanism to cause period variation in binary stars is magnetic activity cycles. In the systems having late-type components, if the shape of the companion star is distorted by tidal and centrifugal forces, changes in the internal rotation associated with a magnetic activity cycle vary the gravitational quadrupole moment. As the quadrupole moment increases, the gravitational field increases leading to a decrease in the period. Otherwise, if the quadrupole moment decreases, the orbital period increases (Applegate 1992). Magnetic activity produces cyclic modulations in the O–C diagram, and their periods are from years to decades.

In Algols, alternate orbital period changes are well known in systems with a late-type secondary star (Zavala et al. 2002). For a binary system, cyclic period variability are generally thought to be caused by either magnetic activity in one or both components (Applegate 1992) or light-time effect due to a third body (Irwin 1952). In terms of magnetic activity, observed oscillations are arisen from the variations of the gravitational quadrupole moment (ΔQ), which is typically around $10^{51} - 10^{52}$ g cm^2 for close binaries and can be calculated from the equation of

$$\frac{\Delta P}{P} = \frac{-9\Delta Q}{Ma^2} \approx \frac{2\pi A_{\rm sin}}{P_{\rm sin}} \tag{2}$$

where M is the mass of the active component (Lanza 2002).

In the case of DI Peg, the O–C diagram shows neither a parabolic change which is an indication of a mass transfer between the components or a mass loss from the system, nor anti-correlation between the primary and secondary minimum timings that is a sign for a change in the orientation of the binary's major axis. On the other hand, it is known that the star has a late-type companion (K4). For this reason, there is a potential that this component may show magnetic activity. In order to search this possibility, we calculate the gravitational quadrupole moment (ΔQ) of the secondary star by using

$\Delta P/P = 3.20 \times 10^{-6}$ which is calculated in this study and by adopting $M_1 = 1.18(3)$ M$_\odot$, $M_2 = 0.70(2)$ M$_\odot$, and $a = 4.14(5)$ R$_\odot$ from Lu (1992). As a result, we find the variation of the quadrupole moment of the star to be $\Delta Q_2 = 4.11 \times 10^{49}$ g cm^2. Since this result is clearly smaller than the typical value and the sinusoidal variations are eccentric, it is unlikely that magnetic activity is responsible for the periodic modulations in DI Peg.

Therefore, two sinusoidal changes can be more likely attributed to the light-time effects due to the presence of two additional bodies. Since the third body is confirmed from the spectroscopic study by Lu (1992), we calculate the specific parameters of the third body under the assumption of the presence of an object gravitationally bound to the system. From the O–C diagram, the period and amplitude of the primary modulation are found to be 49.50±0.36 yr and 0.0082 days. The projected distance of the mass center of the eclipsing pair to the center of mass of the triple system is around 1.78±0.16 au. By using these values the mass function of the third-body is found to be 0.0023(7). If the third-body orbit is co-planar with the orbit of the system (i.e., $i \sim 90°$), its mass would be 0.21(2) M$_\odot$. Also, from the Kepler's third law, the semi-major axis of the orbit is computed as 15.75(7) au. By adopting the parallax of the star from van Leeuwen (2007), we derive the distance of d $\sim$ 191(43) parsecs and hence the maximum angular separation of the third body from the eclipsing pair to be 0.091(28) arcsec. Using the mass-luminosity relation for main-sequence stars given by Demircan and Kahraman (1991), we can estimate the bolometric absolute magnitude of the third body for the given distance to be about $M_{\rm bol} = 10.23(27)$ mag. According to Allen's table (Cox 2000), the spectral type for the third body can be estimated to be M3, which points a red dwarf.

Additionally, as mentioned in the previous section, the residuals from the sine fit show another low-amplitude, short-period and eccentric cyclic modulation. This variation is also interpreted as the existence of a fourth body physically connected to the system by Yang et al. (2014). From the O–C diagram, we calculated the period and amplitude of the secondary modulation as 27.40(24) yr and 0.0051(2) days. The mass function and the mass of the fourth body are $f(m_4) = 0.0009(1)$ and $M_4 = 0.151(75)$ M$_\odot$. Assuming that the object orbits in the same plane as the system and taking the aforementioned distance value into account, we find the angular separation of the fourth body from the eclipsing pair to be 0.0615(183) arcsec. By using the mass-luminosity relation for main-sequence stars given by Demircan and Kahraman (1991), we estimate the bolometric absolute magnitude of the fourth body to be about $M_{bol} = 11.22(14)$ mag. According to Allen's table (Cox 2000), the additional fourth body may be a M4 spectral type red dwarf.

Additionally, from Figure 1, the residuals of two sinusoidal fits still seem to show another modulation. The period and amplitude of this modulation are roughly P = 19.5 years and $A = 0.004$ days. However, it is not possible to attribute this change as another object that is in orbit around the binary system. Therefore, we recommend future photometric and spectroscopic observations to reveal the true nature of DI Peg.

Acknowledgements We thank Ankara University Kreiken Observatory for the support of project number T35_2017_IV_06. This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France, and of NASA's Astrophysics Data System Bibliographic Services.

References:

Ahnert P., 1974, *MitVS*, **6**, 158
Ak, T., Albayrak, B., Selam, S.O., Tanriverdi, T.: 2004, *NewA*, **9**, 265 DOI

Applegate, J.H., 1992, *ApJ* **385**, 621 DOI
Binnendijk, L. 1973, *AJ*, **78**, 97 DOI
Borkovits, T. and Hegedűs, T., 1996, *A&AS* **120**, 63
Borkovits, T., Rappaport, S., Hajdu, T., Sztakovics, J., 2015, *MNRAS* **448**, 946 DOI
Borkovits, T., Hajdu, T., Sztakovics, J., Rappaport, S., Levine, A., Bíró, I.B., Klagyivik, P., 2016, *MNRAS* **455**, 4136 DOI
Chaubey, U. S., 1982, *Ap&SS*, **81**, 283 DOI
Chauvin G., Lagrange A.-M., Udry S., Mayor M., 2007, *A&A*, **475**, 723 DOI
Chou, K. C., and Kitamura, M., 1968, *JKAS*, **1**, 1
Conroy, K.E., Prša, A., Stassun, K.G., Orosz, J.A., Fabrycky, D.C., Welsh, W.F., 2014, *AJ* **147**, 45 DOI
Cox A. N., 2000, *Allan's astrophysical quantities*, New York, AIP Press, Springer
Demircan O., Kahraman G., 1991, *Ap&SS*, **181**, 313 DOI
Dvorak R., 1986, *A&A*, **167**, 379
Evans D. S., 1968, *QJRAS*, **9**, 388
Furlan E., et al., 2007, *ApJ*, **664**, 1176 DOI
Hanna M. A., Amin S. M., 2013, *JKAS*, **46**, 151 DOI
Gaposchkin, S., 1953, *AnHar*, **113**, 67C
Irwin J. B., 1952, *ApJ*, **116**, 211 DOI
Jensch, A., 1934, *AN*, **252**, 393 DOI
Kruszewski, A.: 1964, *AcA*, **14**, 241
Kwee K. K., van Woerden H., 1956, *BAN*, **12**, 327
Lanza, A.F. and Rodonò, M.: 2002, *AN* **323**, 424 DOI
Lee J. W., Lee C.-U., Kim S.-L., Kim H.-I., Park J.-H., 2012, *AJ*, **143**, 34 DOI
Lehmann, H., Borkovits, T., Rappaport, S.A., Ngo, H., Mawet, D., Csizmadia, S., Forgács-Dajka, E., 2016, *ApJ* **819**, 33 DOI
Lim J., Takakuwa S., 2006, *ApJ*, 653, 425 DOI
Lenz, P., and Breger, M.: 2005, *CoAst*, **146**, 53 DOI
Lu, W., 1992, *AcA*, **42**, 73
Marzari F., Scholl H., Thébault P., Baruteau C., 2009, *A&A*, **508**, 1493 DOI
Mikulášek, Z., Zejda, M., Janík, J., 2012, *IAUS*, **282**, 391 DOI
Morgenroth, O., 1934, *AN*, **252**, 389 DOI
Neuhäuser R., Mugrauer M., Fukagawa M., Torres G., Schmidt T., 2007, *A&A*, **462**, 777 DOI
Pribulla T., et al., 2012, *AN*, **333**, 754 DOI
Qian, S., 2001, *MNRAS*, **328**, 914 DOI
Rafert J. B., 1982, *PASP*, **94**, 485 DOI
Rappaport, S., Lehmann, H., Kalomeni, B., Borkovits, et al., 2016, *MNRAS* **462**, 1812 DOI
Rappaport, S., Vanderburg, A., Borkovits, T., et al., 2017, *MNRAS* **467**, 2160 DOI
Reipurth B., 2000, *AJ*, **120**, 3177 DOI
Rucinski, R., 1967, *AcA*, **17**, 271
Schwarz R., Haghighipour N., Eggl S., Pilat-Lohinger E., Funk B., 2011, *MNRAS*, **414**, 2763 DOI
van den Berk J., Portegies Zwart S. F., McMillan S. L. W., 2007, *MNRAS*, **379**, 111 DOI
van Leeuwen F., 2007, *A&A*, **474**, 653 DOI
Vinkó, J., 1992, *IBVS*, **3757**, 1
Yang Y.-G., Yang Y., Li S.-Z., 2014, *AJ*, **147**, 145 DOI
Zavala, R.T., McNamara, B.J., Harrison, T.E., et al., 2002, *AJ* **123**, 450. DOI