STEEL STRESS PATTERNS BETWEEN TWO PRIMARY CRACKS IN CONCRETE

DEDICATED TO THE MEMORY OF PROF. GALLUS REHM (1924-2020)

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https://doi.org/10.32970/CS.2020.1.3

SUMMARY

The national and international codes and standards and the literature consider the stress patterns either in case of a single crack or of equidistant primary cracks which developed at the same load level. Moreover, most FE models consider smeared cracks which do not allow for any insight in the real inner behavior of structural concrete. This paper considers the stochastic character of the concrete tensile strength's distribution along the reinforced concrete element and a realistic local bond stress vs. slip relationship with its hysteretic character when the sign of local slip changes due to the occurrence of a new primary crack.

Keywords: primary crack, local bond stress, slip, hysteretic characteristics, members in tension,

NOTATIONS

a, b, c coefficients of a local bond stress-slip function
a crack spacing
c_s rib spacing on the rebar surface
d_s rebar diameter
f_{cti}, f_{ctk}, $f_{ct\,i+1}$ (β_{bZi}) concrete tensile strengths
f_{ctm} mean concrete tensile strength
f_R relative rib area
ℓ_{bi} transfer length
u perimeter of a rebar
v slip
x_i coordinates along the axis x
w crack width
A_c cross section area of concrete
A_s cross section area of the rebars in the cross section
A_0 uncracked ideal concrete cross section
E_c Young's Modulus of concrete
E_s Young's Modulus of steel (rebar)
N axial tensile force
α_E E_s / E_c
β_w concrete cube strength
ρ geometrical rate of reinforcement
σ_c concrete stress
σ_c concrete tensile stress
σ_s steel (rebar) stress
$\tau_v(x)$ local bond stress
Lower case letters
I, II uncracked and cracked cross sections (state I and II), resp.

1. INTRODUCTION

This paper discusses the steel stress distribution between two primary cracks which develop at different load levels one after another in different distances from each other. This treatise was part of an article which was first published in 1989 in German in a festive collection of articles published on the occasion of Professor Rehm's 65th birthday. Nevertheless, as a German language paper published in a local occasional publication, it was not available to the professional public. Until now no similar results have been published in the literature, while the stress distribution between the two primary cracks provides an important insight into the behavior of reinforced concrete structures. The common FEM models for members in tension and bending consider smeared cracks which develop at the same load level.

In this article a continuous crack theory is explained, which considers

- the stochastic character of the concrete tensile strength's distribution along the reinforced concrete element and
- a realistic local bond stress vs. slip relationship, with the influence of a change in sign of the relative slip between reinforcing steel and concrete.

Note: in this paper Goto-cracks and secondary cracks are not treated. Details see in Windisch (2016, 2017)

2. CONTINUOUS CRACK FORMATION IN MEMBERS IN TENSION

This section describes the development of the crack pattern in a prismatic r.c. member in centric tension with a known distribution of the concrete tensile strength. It is shown that the terms "crack formation stage" and "stabilized cracking stage" and the like always appear in reality side by side and the stochastic character of the crack formation that can be observed in the experiments can be easily explained.

Figure 1a shows a simplified distribution of the concrete tensile strength along an r.c. member.

The tensile force N is increased monotonically. In the non-

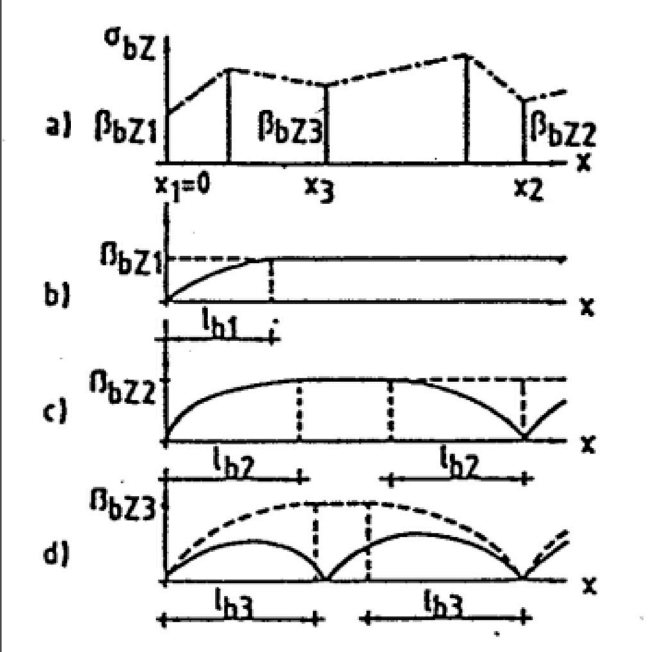


Figure 1: Crack development in a member in tension: Distribution of a) the concrete tensile strength, b) the concrete tensile stresses before and after the development of the first, c) the second, and d) the third primary crack

cracked state, the concrete and steel stresses are constant - apart from the transfer lengths at the member's ends

$\sigma_c = \sigma_{cI} = N / A_0$ $\sigma_s = \alpha_E \cdot \sigma_{cI}$

(Note: in the original figures the concrete tensile stresses are marked as β_{bZi}.)
At the tensile force level:

$$N_1 = A_0 \cdot f_{ct1}$$

the concrete tensile stress reaches the lowest actual concrete tensile strength, then primary cracks appear in one or more cross-sections. The size of the smallest concrete tensile strength and its distribution along the tension rod are random variables. In *Figure 1a* it was assumed that the weakest cross-section is at $x_1 = 0$.

If the concrete tensile strength of this member would be uniform, then infinitely many, infinitely narrow cracks would appear at infinitely small intervals. The cross-sections with the same lowest concrete tensile strength are at different distances from one another: in some areas they can be so close to one another that the transfer lengths extend into one another.

The steel stress in the cracked section is

$\sigma_{sII} = N_1 / A_s = (1 + \alpha_E \cdot \rho) / \rho \cdot f_{ct1}$

The concrete stress in the crack sinks to zero and increases on both sides of the crack, along the transfer lengths according to the bond forces: a crack unloads the concrete on both sides. *Figure 1b* shows the course of the concrete tensile stresses along the bar axis x > 0 (with a dashed line shortly before, with a full line shortly after the occurrence of the first crack in the cross-section $x_1 = 0$). The transfer length is ℓ_{b1}.

If the tensile force is increased further, the stresses in the steel and in the concrete and the transfer lengths increase in the crack/on both sides of the cracks. When the tensile force $N_2 = A_0 \cdot f_{ct2}$ is reached, a new crack will appear in the cross-section at x_2 which can be anywhere outside the transmission length ℓ_{b2}. *Figure 1c* shows the distribution of the concrete stresses before and after the appearance of the 2nd crack.

It is quite possible that cross-sections with a concrete tensile strength

$f_{cti} < f_{ctk} < f_{ct\,i+1}$

initially remain uncracked if they are in the transfer length of a crack. With the increase in the tensile force and the bond forces, it can happen that, due to the relieving influence of the adjacent primary crack, primary cracks will nevertheless occur in some of these cross-sections at a higher tensile force $N > A_0 \cdot f_{ctk}$.

This means that the smallest possible primary crack distance can also be smaller than the transfer length. At the tensile force $N_3 = A_0\ f_{ct3}$ a new crack will appear in cross-section x_3 (see *Figure 1d*).

In our example, the transfer lengths that belong to the tensile force N_3 overlap on both sides of the 3rd crack. Between the cracks No.1 and No.2, along the transfer lengths, there are no areas without any slip between concrete and steel. This still does not mean that the stabilized crack pattern is achieved.

In *Figure 2*, the phases of crack formation shown in *Figure 1* are shown axonometrically as a function of the tensile force N. The relieving effect of the neighboring cracks on the concrete stress, e.g. in cross-sections A and B, or the increase in the transfer lengths during the monotonically increasing load can be followed.

The loading process locates the "weak points" with the lowest concrete tensile strength of the concrete tension member. Of two otherwise identical reinforced concrete tensile members that are reinforced with reinforcing bars of different bond characteristics, the one with the better bond characteristics will have a denser crack pattern: the transfer lengths are shorter, so that shorter zones between the cracks that have already occurred are relieved, higher number of "weak points" will be loaded several beyond their current tensile strength.

3. FORCE- AND DEFORMATION PATTERNS BETWEEN TWO PRIMARY CRACKS

In this section, the patterns of the concrete-, steel- and bond stresses between two primary cracks whose transfer lengths overlap, are examined in more detail.

3.1 Differential equation of the cracked tension member

From the equilibrium condition of a dx-long cracked tension member the relationship

$$d\sigma_s A_s = -d\sigma_c A_c \quad (1)$$

and the differential equation

$$d\sigma_s(x) A_s = \tau_v(x)\ u\ dx \quad (2)$$

can be derived

When deriving Eq. (1) it was assumed that the concrete

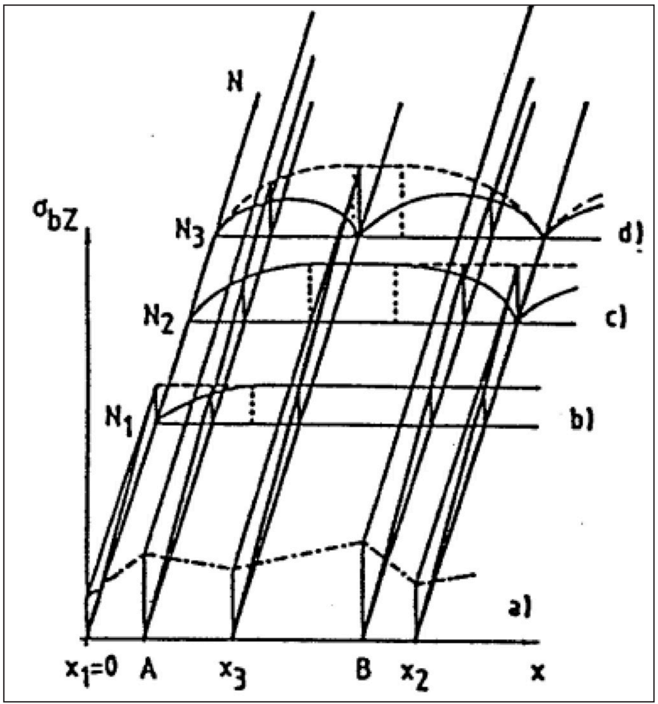


Figure 2: Phases of continuous crack development process

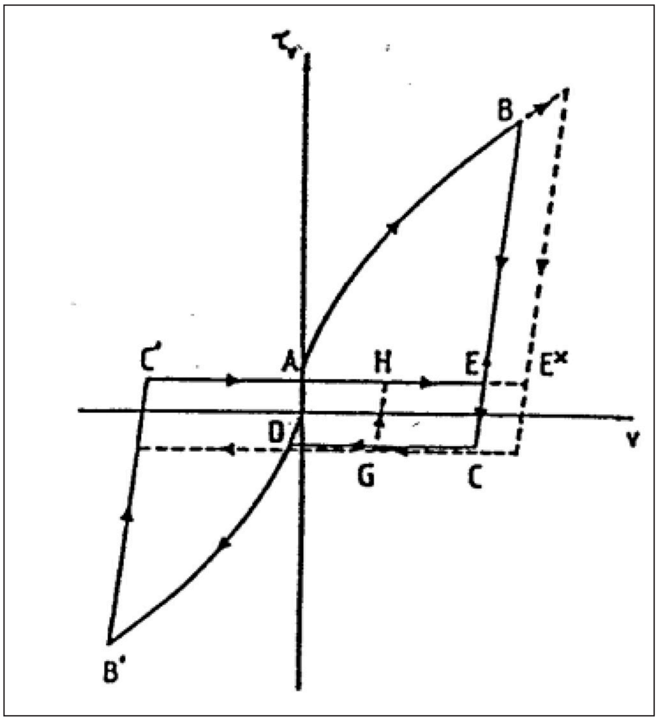


Figure 3: Local bond stress-slip relationship under generalized displacement states

cross-sections would remain plain. This assumption is only permissible in case of limited cross-sectional dimensions - and there, too, only with reservations - because a constant transfer of force into the concrete takes place along the transfer lengths, so these concrete cross-sections are in the de Saint Venant- interference areas, where this assumption is inadmissible.

The Eq. (1) is not valid in the case of beams with flexure, because the eccentric compressive stresses in the cracked cross-sections, too, cause tensile stresses in the concrete tensile zone between the primary cracks.

Eq. (3) describes the compatibility condition:

$$dv/dx = (\sigma_s - \alpha_E \cdot \sigma_c)/E_s \quad (3)$$

This equation is also only valid under the assumption that cross sections remain plane.

From Eqs. (2) and (3) the differential equation

$$dv / d^2x = 4\, \tau_v\, (1 + \alpha_E\, \rho)/(E_s\, d_s) \quad (4)$$

can be derived. When deriving Eq. (4) an elastic behavior of the reinforcing steel was assumed.

The bond-slip relationship is to be used in its most general form, whereby the influence of the hysteresis effect - a reversal of the initial relative displacements - is also taken into account.

3.2 Local bond stress-slip relationship

The local bond stress-slip relationship is of fundamental importance in the cracking process.

Rehm (1961) referred for the first time to the role of the relative displacement (slip) between reinforcing steel and concrete in the interaction of the two building materials and introduced the relationships between the local bond stress and the slip as a material law.

The local bond stress-slip relationship was determined using different embedded lengths and almost always under monotonically increasing tensile forces.

Experiments with cyclic tensile forces by Windisch et al. (1983), Eligehausen et al. (1983) showed that when the sign of the slip is reversed, there are very steep unloading characteristics and very low bond stresses when there are slight opposite slips.

From more than hundred pull-out tests with cyclical loading (Windisch et al. 1983), using a refined evaluation method (Windisch 1985) the simplified local bond stress-slip relationship shown in *Figure 3* was derived.

During the first loading phase (line OAB) the bond behavior can be described with the formula

$$\tau_v = \beta_w \cdot (a + b \cdot v^c) \quad (5)$$

This assumption corresponds to the formulas of Rehm (1961) and Martin (1973).

In the case of unloading, the law follows a straight line (line BC) with an inclination of 250 N/mm³. Hawkins et al. (1982) have given a similar value.

The remaining slip after an unloading can only be canceled by an opposing bond stress with the intensity of

$$\tau_v = -0.1\, \tau_v(B).$$

If the tensile force or the slip is increased in the opposite direction (-v), the law follows the line DB' according to the formula

$$\tau_v = \beta_w \cdot (b \cdot v^c) \quad (6)$$

If the load is further reduced, the relationship between local bond stress and slip follows a straight line again with an incline of 250 N / mm³ (line B'C'). Along the line C'E

$$\tau_v = -0.1\, \tau_v(B')$$

applies.

If the slip reaches the line BE, the local bond force (bond stress) necessary to increase the slip increases according to the line BC. Eq. (5) is valid again beyond point B.

In the case of a new unloading, the bond stress-slip relationship follows the law shown with a dashed line.

If the sign of the slip changes at point G, the bond stress first follows the line GH with an inclination of 250 N/mm³ and then the line HE* etc.

Similar local bond-slip relationships have been proposed by Morita and Kaku (1975) and Tassios (1979), respectively.

In the following calculations, the "deterioration" of the bond in the vicinity of the crack and the scatter of bond strength are not taken into account.

3.3 Solution of the differential equation

When solving the differential equation (4), the corresponding boundary conditions must be taken into account. These boundary conditions change in part during the loading process.

The following applies to all primary cracks

$$\sigma_s = \sigma_{sII} = N / A_s \text{ and } \sigma_{ct} = 0 \quad (7)$$

and in the case of single cracks, at the other end of the transfer length ($x = \ell_b$)

$$\sigma_s = n \cdot \sigma_{ct} \text{ and } v = 0 \quad (8)$$

For tension rod areas between two primary cracks whose distance is smaller than twice the current transfer length, $2\ \ell_b$, only the boundary conditions for both cracks are taken into account.

In view of the very complicated form of the composite law, a closed solution of the differential equation (4) is only possible for individual cracks (Noakowski, 1978, Krips, 1984). For all other cases a numerical solution is sought.

The algorithm is similar to the one used for beams on elastic subgrade.

Starting from a crack, the equilibrium and compatibility conditions are met in the end points of integration sections Δx

$$\sigma_{s,\,i+1} = \sigma_{si} - 4 \cdot \Delta x \cdot \tau_{vi} / d_s \quad (9a)$$

$$v_{i+1} = v_i - \sigma_{s,\,i+1} \cdot \Delta x / E_s \quad (9b)$$

$$\sigma_{c,i+1} = \rho / (1 - \rho) \cdot (N / A_s - \sigma_{s,i+1}) \quad (9c)$$

In the case of the relative displacements (Eq. (9b)), the influence of the concrete deformations was neglected.

The rib spacing c_s is recommended as Δx.

The progressive approximation method was used to solve this algorithm. The principle of this method is that the actual boundary value problem is transformed into an initial value problem, whereby a variable in the starting point - here the relative displacement in crack No. 1 - is increased until the boundary conditions (7) or (8) for the other crack or are met at the other end of the transfer length.

3.4 The behavior of a tension member between two primary cracks

With the method described above, the behavior of a tension rod between two primary cracks was investigated.

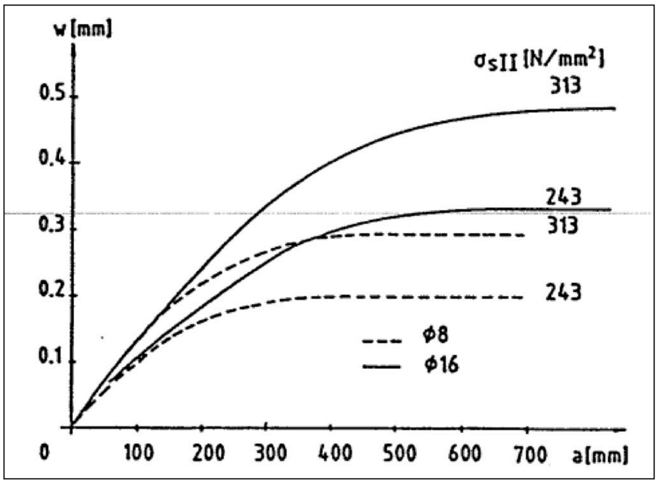


Figure 4: Crack width - crack spacing relationships as a function of the steel stress in the crack and the rebar diameter

As reinforcement, Ø8 mm or Ø16 mm deformed bars (rib spacing 5 mm or 8 mm) with a related rib area of $f_R = 0.075$ were selected.

The two adjacent cracks at distance *a* may have occurred at the same load level or one after the other.

First, the course of the steel stresses and the crack widths as a function of the crack spacing and the steel stress in the crack for the case of two cracks occurring "simultaneously" are examined.

The courses of the concrete- and steel stresses are mirror-symmetrical, those of the bond stresses or the relative displacements (slips) are antimetric to the center of the crack spacing.

On the basis of the crack width - crack spacing relationships (concrete class B25, which roughly corresponds to C20) shown in *Figure 4* the following can be observed:

At the beginning the crack widths increase disproportionately with the crack spacing, but are independent of this above a certain limit distance - it is twice the transfer length for the tensile force in the reinforcement bar in the crack

- for smaller ($a \leq 150$ mm) crack spacings, the crack widths calculated for Ø16 mm rebars are hardly wider than those for Ø8 mm rebars
- The ratio of the largest crack widths for Ø16 mm and Ø8 mm rebars is only about 1.65: 1, i.e. the largest (critical) crack widths do not depend linearly on the rebar diameter.

Between two cracks that occurred at different load levels, the stress or slip curves show completely different properties: there is no symmetry or antimetry more.

Figure 5 a - c show the courses of the steel- and bond stresses and the relative displacements (slips) shortly before (with a dashed line) and immediately after the appearance of the 2nd (right) crack as a function of the crack spacing.

The strong influence of the occurrence of the second primary crack and the crack spacing on all three courses is obvious. The contribution of the concrete increases as the distance between the cracks increases (see *Figure 5a*).

The following can also be assessed from the calculation results:

- When the second crack occurs, the width of the first decreases a bit, the more the "closer" the second crack is to the first,
- If the crack spacing is only slightly greater than the transfer length belonging to the tensile force which caused the second crack, then the width of this second crack is at

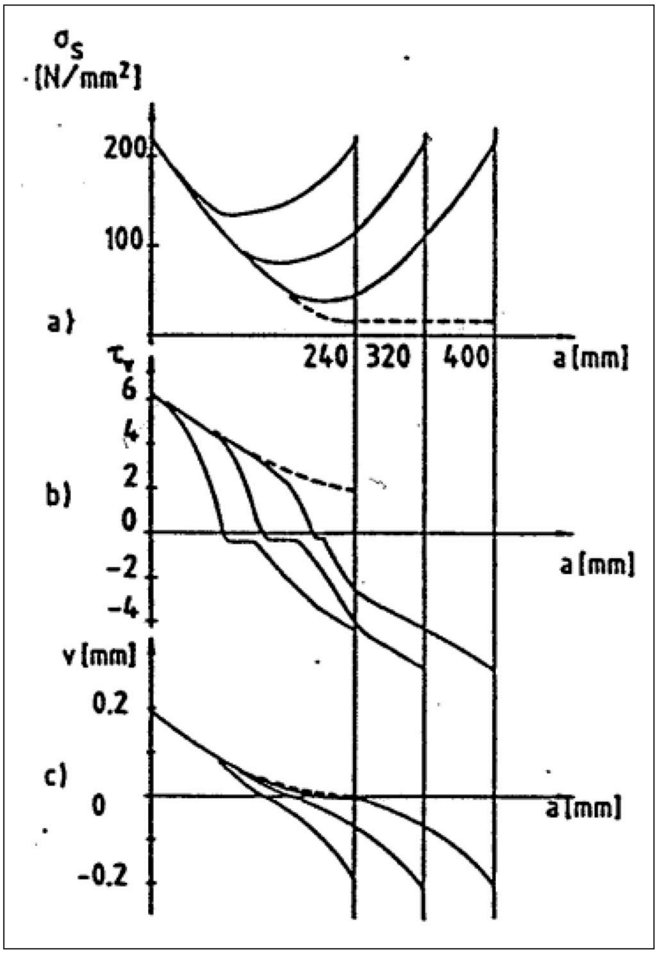


Figure 5: Courses of the steel stresses, slips and bond stresses in the case of cracks that do not occur simultaneously (Ø16 mm, σ_{sII} = 215 N/mm², concrete class B25, which roughly corresponds to C20)

the beginning smaller than that of the first. However, as the tensile force increases, the second crack becomes wider than the first one.

- Neither the steel-, nor the concrete stresses have their extreme (min. and max. resp.) at the middle of the crack distance. From this it follows that the crack spacing does not try to „halve" itself regardless of the distribution of the concrete tensile strength. König and Fehling (1988) came to the same conclusion in a completely different way.
- The „closer" the cracks are to one another, the longer will be the zone with the smallest bond stresses, the flatter are the steel- and concrete stresses around their extreme values.
- The smaller is the concrete tensile strength in the cross section of the second crack, the greater is the likelihood of another crack occurring between the two.
- The further the two cracks are apart, the more the crack width increases with the tensile force.

Figure 6 shows the steel strains measured along a r.c. tension member with Ø20 mm reinforcement according to Scott and Gill (1987). The similarity to the stress curve in *Figure 5a* is obvious.

Figure 7 shows the calculated crack widths as a function of the crack spacing. The straight lines correspond to

$$w = a \cdot \sigma_{sII} / E_s$$

i.e. the pure steel expansion without the contribution of the concrete. The following conclusions can be drawn:

- The relatively small crack distances include several crack

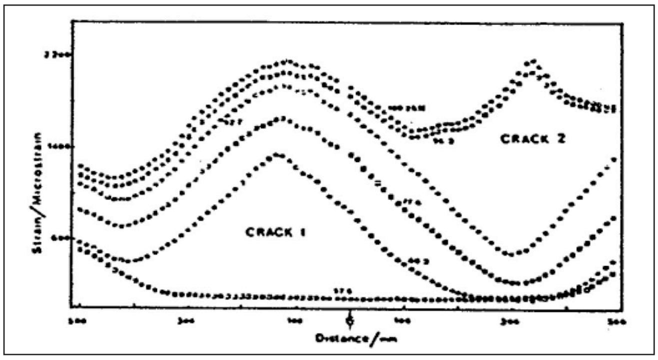


Figure 6: Measured steel strains in tensile reinforcement according to Scott and Gill (1987)

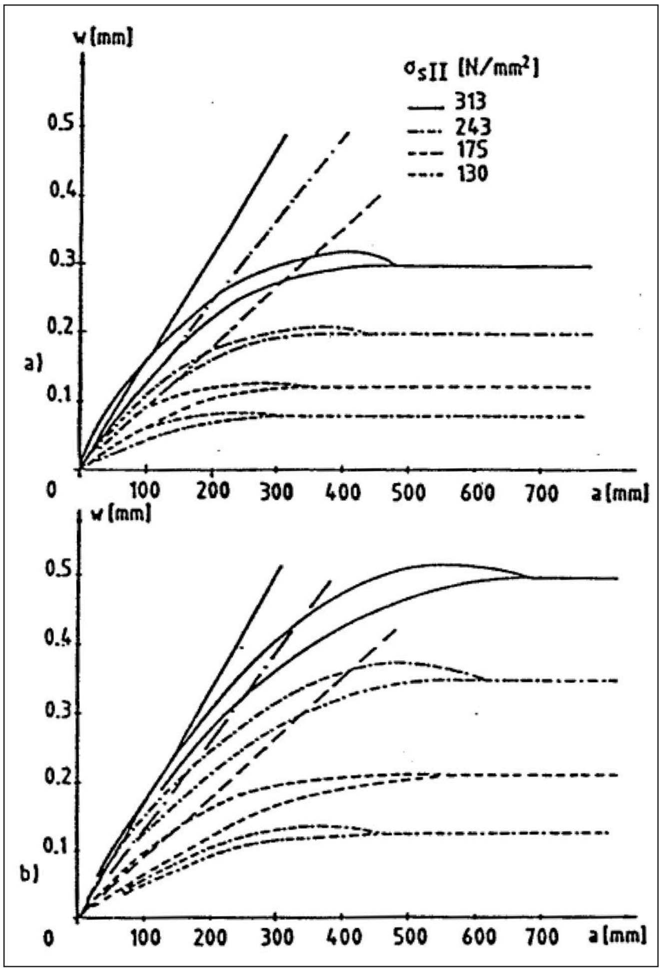


Figure 7: Calculated crack widths for a) Ø8 mm, b) Ø16 mm rebars

widths, the size of which depends on whether the adjacent cracks occurred at the same or different tensile forces.

- The crack widths first increase with the crack spacing, above a crack spacing which corresponds to twice the transfer length for $N = \sigma_{sII} \cdot A_s$, the function becomes monovalent.
- The transfer lengths are not linear, neither on the bar diameter nor on the steel stress.

All calculation results discussed so far have been determined for concrete class B25, (which roughly corresponds to C20).

Figure 8 shows the influence of the concrete quality on the crack width for a tension member with = 1 %, reinforced with Ø8 mm or Ø16 mm rebars. For this numerical example, 300 mm or 480 mm crack spacings and the tensile force at the appearance of the first crack $N = A_0 \cdot f_{ctm}$ were assumed. The following could be concluded:

- The steel stress at the crack development is higher than σ_{sII} = 243 N/mm² for concrete classes B35 (which roughly

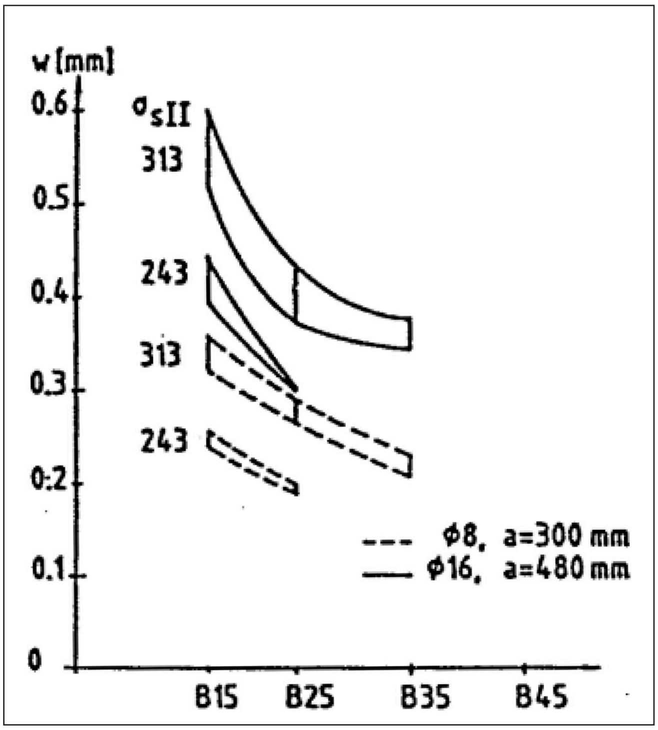


Figure 8: Influence of the concrete strength on the crack width, I.

corresponds to C30) and B45 (which roughly corresponds to C35), and even higher than 313 N/mm² for concrete class B45, so these tension members remain crack-free at these steel stresses.

- An increase in concrete strength reduces the crack width disproportionately.

Figure 9 shows the influence of the concrete strength on the crack width as a function of the crack spacing and the steel stress in the crack, σ_{sII}.

The two straight lines represent the 'naked' elongation of the reinforcing bars. With increasing concrete strength, at constant crack spacing, the increase in the contribution of the concrete can be observed: the higher the concrete strength, the shorter is the transfer length, whereby it approaches the constant crack spacing and accordingly the concrete contribution increases.

4. CONCLUSIONS

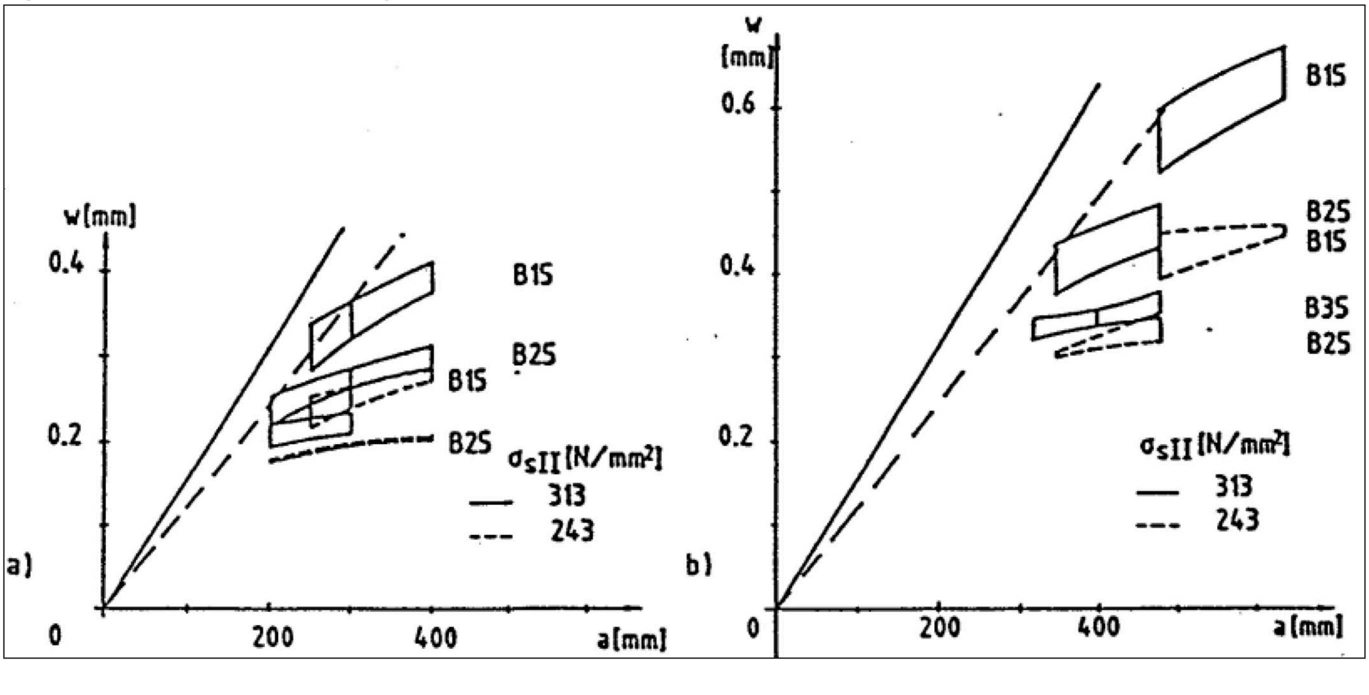
The steel stress distribution between two primary cracks which occur at different load levels and at different distances from each other, is presented. The continuous crack formation is discussed where the stochastic distribution of the actual concrete tensile strength along the axis of the member in axial tension is considered.

Solving the differential equation of the bonded rebar a realistic local bond stress-slip relationship is taken into account where the hysteretic character of the relationship when the sign of local slip changes due to the occurrence of a new primary crack, is considered.

The calculations for Ø8 mm and Ø16 mm rebars and B 25 (which roughly corresponds to C20) concrete grade reveal that

- at the beginning the crack widths increase disproportionately with the crack spacing, but are independent of this above a certain limit distance, it is twice the transfer length for the tensile force in the reinforcement bar in the crack
- for smaller (a ≤ 150 mm) crack spacings, the crack widths calculated for Ø16 mm rebars are hardly wider than those for Ø8 mm rebars
- The ratio of the largest crack widths for Ø16 mm and Ø8 mm rebars is only about 1.65: 1, i.e. the largest (critical) crack widths do not depend linearly on the rebar diameter.
- The strong influence of the occurrence of the second primary crack and the crack spacing on the courses of steel stresses, bond stresses and slips is obvious.
- The contribution of the concrete increases as the distance between the cracks increases
- When the second crack occurs, the width of the first one decreases a bit, the more „closer" the second crack is to the first one,
- If the crack spacing is only slightly greater than the transfer length belonging to the tensile force which caused the second primary crack, then the width of this second crack is at the beginning smaller than that of the first crack. However, as the tensile force increases, the later primary crack becomes wider than the first one.
- Neither the steel-, nor the concrete stresses have their

Figure 9: Influence of the concrete strength on the crack width, II: tensile members reinforced with a) Ø8 mm, b) Ø16 mm rebars



extreme (min. and max. resp.) at the middle of the crack distance. From this it follows that the crack spacing does not try to „halve“ itself regardless of the distribution of the concrete tensile strength.

- The „closer“ the cracks are to one another, the longer will be the zone with the smallest bond stresses, the flatter are the steel- and concrete stresses around their extreme values.
- The smaller is the concrete tensile strength in the cross section of the second crack, the greater is the likelihood of another crack occurring between the two.

These realistic courses of the steel stresses, slips and bond stresses should help to develop mechanically founded crack width calculation models.

5. ACKNOWLEDGEMENT

The author is extremely grateful to Professor Rehm, who has monitored and actively supported his scientific and professional work for more than 40 years.

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