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The magnetotelluric phase over 2-D structures

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SUMMARY

In 1985 Fischer gave a simple explanation for the behaviour of the E-polarization magnetotelluric phase over a 2-D dike. This phase behaviour is a direct consequence of the rearrangement of the current flow in a 2-D structure as opposed to the flow in a simple uniform half-space. In spite of its usefulness it is shown that the simple phase rule is only qualitatively correct. For B-polarization it seemed at first that the simple phase rule would not remain valid. However, it is found that if the very different current distributions for E- and B-polarization are correctly interpreted, the simple phase rule retains its validity qualitatively. Arguments are given for the very general validity of this rule at the surface of any 2-D structure, even in the presence of complicated topography. The rule is best formulated in terms of two statements: (a) the magnetotelluric phase is a continuous function over any structure, even across outcropping resistivity contrasts; and (b) where the current is drawn to greater depths the phase will rise, and where the current is concentrated near the surface the phase will drop.

Key words: magnetotelluric phase, resistivity contrasts, 2-D profiles.

1 INTRODUCTION

The interpretation, in electromagnetic sounding work, of what is known as the magnetotelluric (MT) phase ϕ—i.e., the argument of the surface impedance Z—has recently acquired much greater importance than before. Our increased understanding of what has become to be known as the 'static shift correction' (*cf.*, e.g., Jones & Savage 1986; Kurtz, De Laurier & Gupta 1986; Sternberg, Washburne & Pellerin 1988; Jones 1988) has revealed that the MT phase appears to be a more stable or more robust parameter than the apparent resistivity. Over a disturbed structure, with outcropping resistivity contrasts, the apparent resistivity can undergo discontinuities of several orders of magnitude where the phase remains perfectly continuous. It is the purpose of this paper to better understand this behaviour of the phase.

In general, the MT phase is referred to the behaviour of the two-layer model. Compared with the 45° value of the uniform half-space, and in the range where the skin-depth δ is larger than about 2/3 of the first layer thickness h (*cf.* Fischer & Schnegg 1980), the well-known MT rule of thumb states that the phase $\phi > 45°$ if the lower layer is a better conductor, whereas the condition $\phi < 45°$ reveals a poorer conductor at depth. A physical explanation of this rule was given only a few years ago (Fischer 1985), by showing why a near-surface current increase leads to a phase reduction and, conversely, why a near-surface current reduction increases the phase. This explanation of the two-layer behaviour was reached as the limiting situation of a very wide 2-D dike under E-polarization induction.

Several attempts have been made to find an equally simple rule for the 2-D dike under B-polarization induction, but none of these was completely successful (see, e.g., Iliceto, Malaguti & Santarato 1986; Szarka 1988).

Recently Szarka & Fischer (1989) were able to give closed formulae for the magnetotelluric (MT) surface parameters in terms of the current distribution at depth. In particular they gave relations concerning the phase according to which $\tan\phi$ is given, for E-polarization, by the ratio of the mean depths of the in-phase and out-of-phase currents. For B-polarization the problem is more complicated because of the surface charges which appear at resistivity contrasts. How do these charges affect the simple phase relationship?

In what follows we shall attempt to answer the following three questions.

(1) How correct is the simple phase rule for E-polarization?

(2) What is the physics of the B-polarization phase behaviour?

(3) Can a formulation of the phase rule be given, which also applies to the B-polarization geometry?

2 THE MT PHASE IN TERMS OF THE CURRENT DISTRIBUTION

2.1 The simple *E*-polarization phase rule

Assuming a dike geometry as shown in Fig. 1 the behaviour of the phase can be understood with the following arguments. Because of the continuity of the electric field component E_x across the narrow conducting graben, strong additional currents j_x will flow in the dike, which are in phase with the local electric field. The sum ΔI_x of these extra currents will produce an anomalous magnetic field around the dike, whose amplitude H_{ay} is to first order proportional to ΔI_x. Thus for a narrow and shallow dike H_{ay} behaves practically like the electric field at the surface, $E_x(z=0)$, i.e.,

$$H_{ay} \cong \alpha E_x(0), \tag{1}$$

with α a real positive constant. In a first approximation the surface impedance over the dike centre thus becomes

$$Z_{xy} = \frac{E_x(0)}{H_y + H_{ay}} \cong \frac{E_x(0)}{H_y + \alpha E_x(0)}, \tag{2}$$

where $E_x(0)$ and H_y are the fields at the surface of the matrix, far away from the graben. Note that we express the impedance as a ratio of electric to magnetic field: E/H. However, we speak of E- and B-polarization since it is these two fields which are measured in actual practice.

The addition to the denominator in equation (2) of a quantity with the same argument as the numerator always reduces the argument of the ratio. Thus the phase ϕ_E of Z_{xy} will be reduced below 45° over the conducting graben. If the graben is more resistive than the matrix, the current in the dike will be reduced and α in equations (1) and (2) will be negative. Subtracting from the denominator a quantity with the argument of the numerator will always increase the argument of the ratio; thus the phase ϕ_E of Z_{xy} will rise above 45° for a resistive graben.

It is well known from the theory for a 1-D two-layer earth that the phase (which in this case is governed entirely by the electric field) is less than 45° for a resistive basement below a good conductor, and greater than 45° vice versa. Since the induced horizontal currents in such models tend to concentrate in the good conductors, the above statement can be rephrased to say that when currents are flowing near the surface the phase is less than 45°, and when they are flowing in deeper regions it becomes greater than 45°. This, as seen in Fig. 2, is precisely the conclusion reached by Fischer (1985), and summarized in the previous paragraph, for a narrow dike, except that in this limit his arguments were based on the phase being determined by the anomalous magnetic field. Between these two limiting configurations, i.e. a wide dike, both the electric and anomalous magnetic fields will play a role in determining the phase, but the phase will still follow the general rule that it has a value less than 45° when the currents are drawn towards the surface (conductive dike) and greater than 45° when they plunge downwards (resistive dike). Thus in terms of current distribution we have, for E-polarization configurations in general, the following rule.

(I) When the currents are concentrated near the surface the phase ϕ will be depressed; on the contrary, if the structure is such as to displace the currents away from the surface to greater depths, the phase ϕ will be seen to rise.

A further property of the E-polarization phase ϕ follows from the continuity of the tangential electric field E_x and of the longitudinal magnetic field H_y, and is expressed here as rule (II).

(II) The phase ϕ is a continuous function over any 2-D profile; in particular it is continuous across outcropping resistivity contrasts.

In the above rules (I) and (II) we have purposefully omitted to specify whether they refer to E- or B-polarization, as we shall find later that their validity is quite general.

2.2 More rigorous expressions for the MT phase

It has recently been shown (Szarka & Fischer 1989) that the electromagnetic fields at the flat surface of a structure—and therefore all the usual MT interpretational parameters as well—can be expressed entirely in terms of the current distribution inside the structure. For B-polarization the surface impedance at a point P $(x, y, z=0)$ is given by the following expression:

$$Z_B = Z_{yx} = \frac{E_y(\omega)}{H_x(\omega)} = -i\omega\mu \frac{\int z j_y \, dz}{\int j_y \, dz} + \frac{\int \partial E_z/\partial y \, dz}{\int j_y \, dz}. \tag{3}$$

The fields $E_y(\omega)$ and $H_x(\omega)$ are components at the surface point P, $\omega/2\pi$ is the frequency, and μ the magnetic permeability, for which we shall take its real and constant vacuum value. The current component j_y and the partial derivative $\partial E_z/\partial y$ refer to a given depth z below P and the integration is from the surface ($z=0$) to infinity ($z=\infty$).

The first term on the right of equation (3) corresponds to a 'complex mean depth' z_B^* of the current distribution inside the conductor, multiplied by $-i\omega\mu$:

$$z_B^* = \frac{\int z j_y(y, z, \omega) \, dz}{\int j_y(y, z, \omega) \, dz}. \tag{4}$$

Since the uniform magnetic field $H_x(\omega)$—which can be

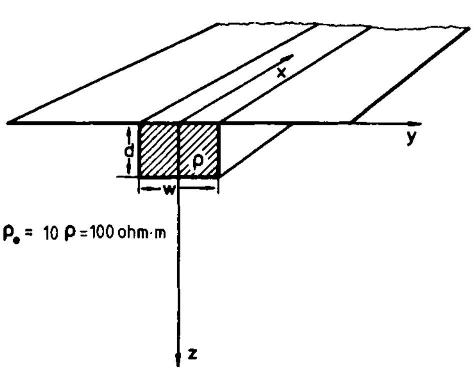

Figure 1. Outcropping conductive dike embedded in a more resistive matrix.

chosen as a *real reference* without loss of generality—is given by

$$H_x(\omega) = -\int_0^\infty j_y(y, z, \omega)\, dz, \tag{5}$$

we can view the real part of z_B^* as giving a *mean depth of the real currents, those which are in phase* with the surface field $H_x(\omega)$, defined similarly to a centre of mass. The imaginary part of z_B^*, on the other hand, gives a *mean depth of the quadrature currents,* i.e., those which are *90° out of phase* with $H_x(\omega)$, normalized again by the real total current (the total imaginary current vanishes). In Fig. 3 we give examples of the behaviour of z_B^* for two typical two-layer structures.

If for the last term in equation (3) we write

$$f(E_z) = \frac{\int \partial E_z/\partial y\, dz}{\int j_y\, dz}, \tag{6}$$

we can express the B-polarization MT phase as

$$\phi = -\tan^{-1} \frac{\mathcal{Re}\,(z_B^*) - \mathcal{Im}\,[f(E_z)/\omega\mu]}{\mathcal{Im}\,(z_B^*) + \mathcal{Re}\,[f(E_z)/\omega\mu]}. \tag{7}$$

This equation clearly shows that the B-polarization MT phase depends both on the distribution of the currents and on the vertical component E_z inside the conductor. Since the occurrence of vertical electric fields is strongly linked with the charges appearing on the various resistivity interfaces, the terms involving E_z can be considered as describing the effects of these charges on the B-polarization MT phase, as has been shown elsewhere (Szarka & Fischer 1989).

In contrast, for 1-D structures, we have

$$\phi_{1-D} = -\tan^{-1} \frac{\mathcal{Re}\,(z_{1-D}^*)}{\mathcal{Im}\,(z_{1-D}^*)}, \tag{8}$$

and similarly for the 2-D geometry with E-polarization induction,

$$\phi_E = -\tan^{-1} \frac{\mathcal{Re}\,(z_E^*)}{\mathcal{Im}\,(z_E^*)}. \tag{9}$$

However, in equation (9) the current j_E which must be used to compute z_E^*—with integrals similar to those of equation (4)—is given by (*cf.* Szarka & Fischer 1989):

$$j_E = j_x - \partial H_z/\partial y. \tag{10}$$

The behaviour of the phase expressed by equations (8) and (9) is not identical with the simple phase rule (Fischer 1985), which considers only changes of the in-phase currents. As we have seen, both the in-phase and quadrature currents determine the phase. Why the more rudimentary rule was nevertheless very successful is easy to understand. In a two-layer structure with the better conductor at the top, Fig. 3 shows that the in-phase currents are pulled toward the surface and the quadrature currents are pushed deeper. The simple phase rule ignores the

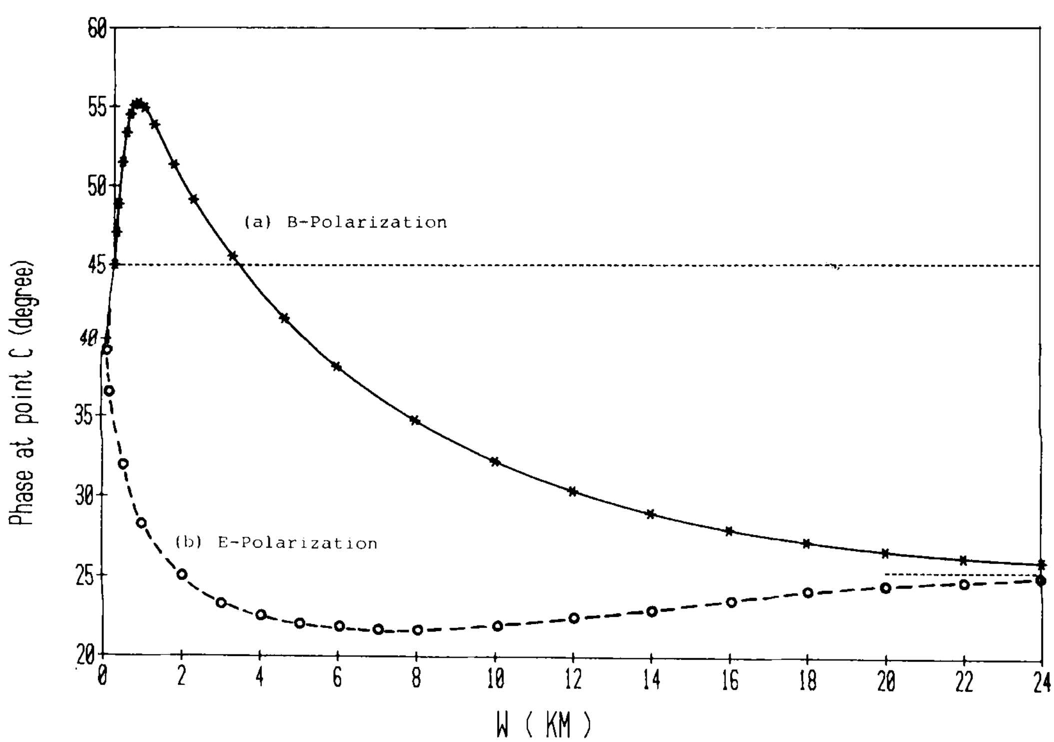

Figure 2. Magnetotelluric phase at the centre C of the dike of Fig. 1, as a function of a changing dike width w. The depth $d = 600$ m and the inducing frequency is 1 Hz. Curve (a) is for B-polarization induction and curve (b) for E-polarization. When the dike is very wide the phase is close to the two-layer value. At intermediate and narrow widths it is seen to obey rule (I) for both polarizations. These curves can be computed with the numerical routines of Wannamaker, Stodt & Rijo (1987), Brewitt-Taylor & Weaver (1976), or Weaver *et al.* (1985), all of which yield essentially the same results.

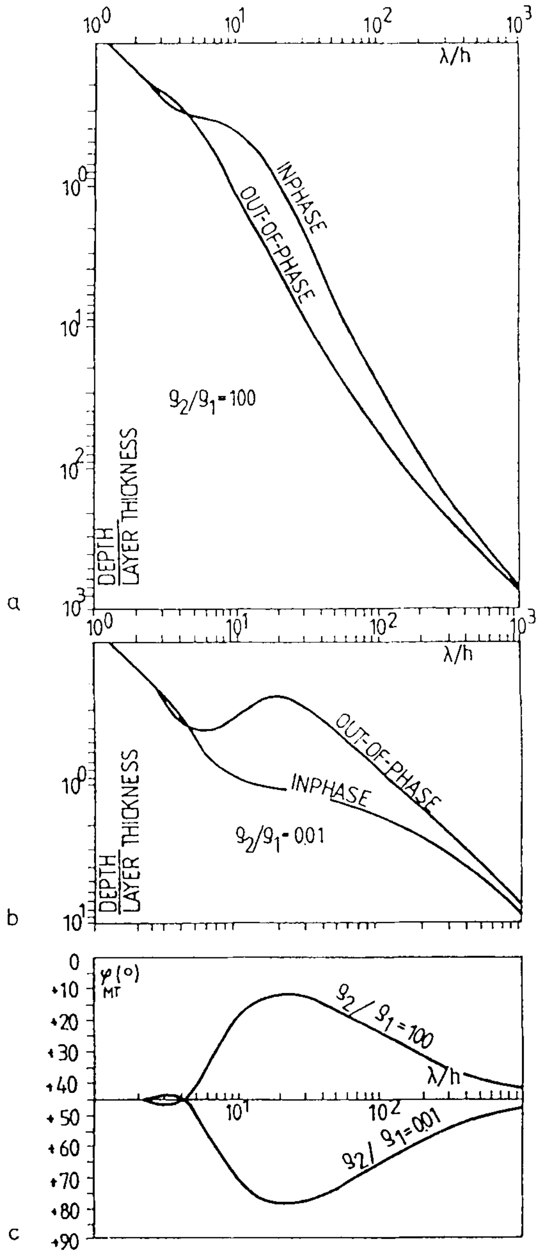

Figure 3. Behaviour of the mean depths of the currents which are in phase or in quadrature (90° out of phase) with the surface magnetic field for two-layer structures with (a) $\rho_2/\rho_1 = 100$ and (b) $\rho_2/\rho_1 = 0.01$. In (c) the phase of the magnetotelluric impedance is shown for these two structures. The abscissa expresses the ratio of the wavelength λ_1 in the upper layer to the thickness of this layer (note that $\lambda_1 = 2\pi\delta_1$, with δ_1 the skin-depth in material with the resistivity of the upper layer).

deflection of the quadrature currents and only says that the in-phase currents rise toward the surface. Qualitatively this is a correct description, but the simple rule would not permit a quantitative derivation of the true change of the phase. Exactly the opposite happens for a more resistive top layer. In truth the in-phase currents are pulled down and the quadrature currents rise, whereas the phase rule only states that the in-phase currents are pulled into the lower conductor.

While equation (9) tells us that the E-polarization phase is determined entirely by the average depths of the in-phase and quadrature currents, it is evident from equation (7) that the situation is more complicated for B-polarization, with the involvement of the vertical electric field.

3 CONTINUITY OF THE MT PHASE AT THE SURFACE OF CONDUCTING STRUCTURES

Under B-polarization induction it is well known that whenever a resistivity interface crops to the surface under any finite angle the apparent resistivity suffers a sharp discontinuity (*cf.* Fischer, Le Quang & Müller 1983). This is a consequence of the necessary continuity of the electric current density $\mathbf{J}(y, z)$. Let us call E_1 and E_2 the horizontal electric field components on both sides of a resistivity contrast at the surface of a conducting structure with level top, as in the example of Fig. 4. Since displacement currents can be neglected, the continuity of the current density requires that

$$E_1/\rho_1 = E_2/\rho_2. \tag{11}$$

The continuity of the current density is achieved thanks to the appearance of surface charges on the interface which, in turn, render the electric field discontinuous. From equation (11) it immediately follows that the apparent resistivity suffers a discontinuity, given by the square of the ratio of ρ_2/ρ_1, since

$$\rho_B(\omega) = \frac{1}{\omega\mu_0}\left|\frac{E_y(\omega)}{H_x(\omega)}\right|^2, \tag{12}$$

where $E_y(\omega)$ and $H_x(\omega)$ both refer to field components along the top surface, with $H_x(\omega)$ real and of uniform amplitude (*cf.* equation 5).

Whereas the apparent resistivity $\rho_B(\omega)$ suffers a discontinuity at every outcropping resistivity interface, the B-polarization phase $\phi_B(\omega)$ remains perfectly continuous. This also follows at once from equation (11) and from the uniformity of the magnetic field at the surface: since H_x is uniform across the top surface the MT phase on the two sides of the interface is given by the arguments of E_1 and E_2; but with equation (11) and ρ_1 and ρ_2 real, these two arguments are identical. *The B-polarization MT phase $\phi_B(\omega)$ is therefore continuous over the entire 2-D profile.* In fact the phase will also be continuous over the top surface when this surface is not level, i.e. exhibits topographical features, since continuity of the current flow along the free surface requires equation (11) to hold along the entire surface profile. This has recently been demonstrated in numerical calculations over a steeply embanked valley (Fischer 1989), where it can be seen that both E- and B-polarization phases are indeed continuous across the entire 2-D profile, although they do seem to exhibit strong discontinuities in their slopes. This means that rule (II) applies equally well to the B-polarization MT phase ϕ_B as it does to ϕ_E, as was claimed in Section 2.1, and it is therefore continuous at the surface of any conducting 2-D structure.

The continuity of the phase at the surface of any conducting 2-D structure can also be demonstrated with the following arguments. For B-polarization the phase ϕ_B is

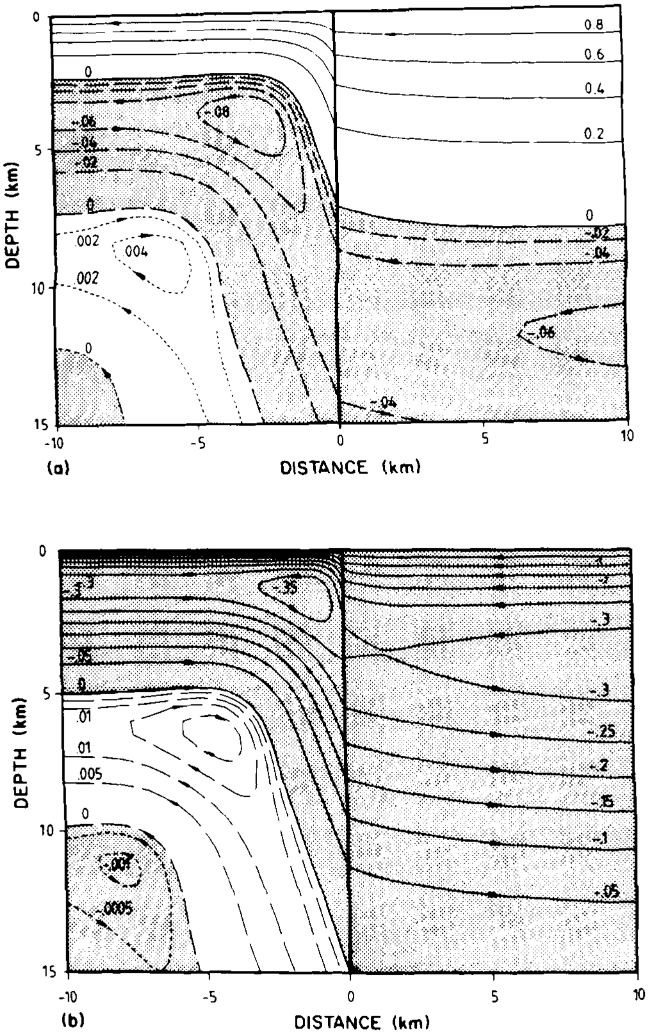

Figure 4. Lines, or surfaces, of uniform magnetic field [(a) real, (b) imaginary] for B-polarization induction in a quarter-space model with $\rho_1 = 10\,\Omega$ m on the left, $\rho_2 = 10\rho_1$ on the right, and a 1 Hz inducing frequency. The lines can also be viewed as streamlines of electric current. The magnetic field is positive and into the plane of the paper in the unshaded regions, and negative and out of the paper in the shaded regions. The corresponding directions of current flow are indicated by arrows. The three types of lines in the figure are drawn to different density scales; the changes in magnetic field between adjacent solid, broken and dotted lines are in the ratio 100:10:1. In (a) the separation of the solid lines corresponds to a change of $0.2B_0$, where B_0 is the 'normal' magnetic field at the surface. In (b) solid lines are separated by $0.05B_0$.

continuous because $H_x(y)$ is uniform and along the 2-D profile the electric field parallel to the surface satisfies equation (11), even if the surface is not level, for example across a ridge. With E-polarization $E_x(y)$ is continuous and the transverse magnetic field $\mathbf{H} = (0, H_y, H_z)$ is also continuous. ϕ_E will therefore be continuous *if the magnetic field used to compute the impedance Z_{xy} is always measured in the same direction.* Indeed, the field practice is to systematically utilize the *horizontal* magnetic field component H_y to compute Z_{xy}, even over rough terrain. Under these circumstances the phase will invariably be found to be a continuous function over the entire 2-D surface.

The property of the MT phase to remain continuous over any 2-D structure while the apparent resistivity undergoes severe discontinuities at outcropping resistivity contrasts is at the basis of the arguments which are presently advanced to justify the so-called 'static shift' corrections (*cf.*, e.g., Jones & Savage 1986; Kurtz *et al.* 1986; Sternberg *et al.* 1988; Jones 1988).

4 E- AND B-POLARIZATION MT PHASES AT VERTICAL RESISTIVITY INTERFACES EXTENDING TO GREAT DEPTHS

Here we consider a structure with a level top consisting of two quarter-spaces, as pictured in Fig. 4 for B-polarization and Fig. 5 for E-polarization. Far away from the vertical boundary the impedances on the left and on the right are identical with those of uniform half-spaces with resistivities ρ_1 and ρ_2, and both phases are therefore equal to 45°. The MT phase anomalies when going across the interface are shown in Fig. 6. The B- and E-polarization anomalies are seen to be almost mirror images of each other with respect to the interface, but the very different behaviour observed for the two polarizations is in perfect accord with the respective current patterns.

The rapid change of ϕ_E across the contact zone is caused by the fast variations of both the electric and magnetic fields. In the better conductor Figs 5(a) and (b) show how the required continuity of the electrical field E_x leads to an increase of the current density near the boundary and thus creates a magnetic vortex which increases the H_y field amplitude. In the poorer conductor the same continuity requirements reduce the current density near the boundary, thereby creating an opposite vortex which decreases the magnetic field amplitude. The discontinuous current density across the $y = 0$ plane thus produces two magnetic vortices which combine in the vicinity of the contact zone into a strong vertical magnetic field. At the surface this anomalous magnetic field rotates or fans out from a direction parallel to the normal inducing field in the good conductor, to the opposite direction in the poor conductor. An interesting observation is that across the $y = z = 0$ line of contact the phase equals 45°, a result which we have confirmed with the analytical method of Weaver, Le Quang & Fischer (1986).

We now turn to B-polarization induction and assume first that one is at sufficient distance from the vertical contact that the effect of the charges on the interface is not felt. Because $\rho_2 > \rho_1$, when the interface is approached from the left the electric field at the surface experiences a marked decrease in amplitude; this indicates that the current flow lines plunge to greater depth (*cf.* Figs 4a, b). Under these circumstances phase rule (I) predicts that the phase will rise above 45°, and this is effectively seen to occur in Fig. 6. If we approach the interface from large distances on the right the electric field amplitude increases and the current flow lines rise toward the surface, as depicted in Figs 4(a) and (b); according to phase rule (I) this means that the phase will be depressed below 45°. This is again confirmed by the numerical computations of Fig. 6. As one comes closer to the line of contact $y = z = 0$, the effect of the charges on the interface increases and also influences the behaviour of the phase. However, at the contact line itself analytical

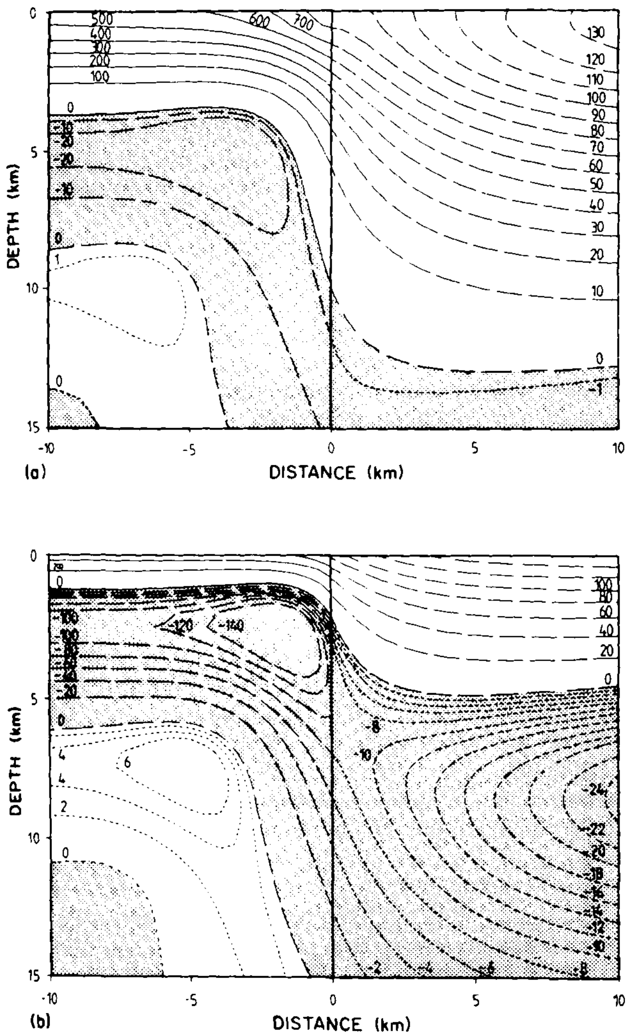

Figure 5. Lines, or surfaces, of uniform current density [(a) real, (b) imaginary] for *E-polarization induction in a quarter-space* model with $\rho_1 = 10\,\Omega$ m on the left, $\rho_2 = 10\rho_1$ on the right, and a 1 Hz inducing frequency. Positive current flow is into the plane of the paper in unshaded regions; regions of negative flow and out of the paper are shown shaded. The density scale for solid, broken and dotted lines are in the ratio 100:10:1. In (a) the separation of the solid lines corresponds to a change in current density of $10^{-7}B_0$ A m^{-2}, where B_0 (measured in nT) is the 'normal' magnetic field on the surface at infinity. In (b) solid lines are separated by $2\times10^{-7}B_0$ A m^{-2}. These diagrams also represent lines, or surfaces, of uniform electric field amplitude, but whereas the electric field remains continuous, the current density suffers a 10:1 discontinuity across the contact. The curves are also 'magnetic field lines' (compare with Fig. 4), where a $\mathcal{Re}\,(j_x)$ = const. curve is a field line of $\mathcal{Im}\,(B)$, and vice versa.

calculations (Weaver, Le Quang & Fischer 1985) again confirm a continuous phase, with $\phi = 45°$ as shown in Fig. 6, which was returned by the numerical computations.

Whereas under E-polarization induction we have across the $y = 0$ plane a discontinuous current density which results in a rotating magnetic field at the surface, we have under B-polarization excitation a vertical sheet of surface charges which produces a similarly rotating electric field at the top of the structure. However, the superposition of the normal and

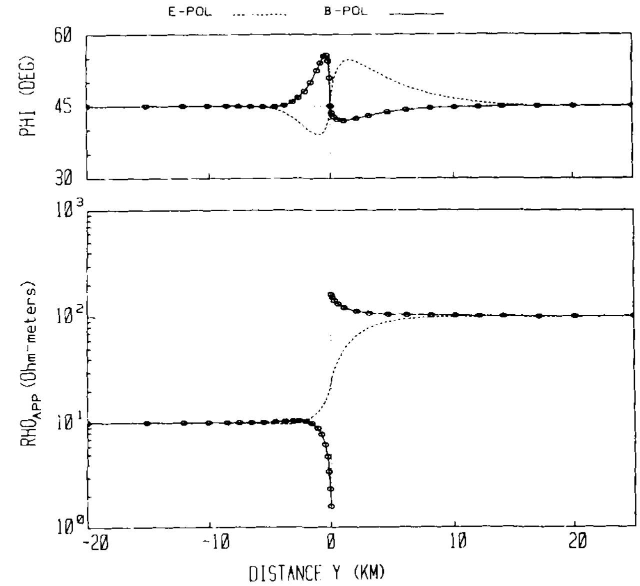

Figure 6. E- and B-polarization apparent resistivity and phase over the resistivity interfaces of Figs 4 and 5. Note that the *B-polarization phase obeys rules* (I) *and* (II)*: it varies strongly but* remains continuous; however, the negative excursion is much smaller than the positive one. With E-polarization both apparent resistivity and phase are continuous. The behaviour of the phase can be understood with phase rule (I): coming from the left $E_x(y)$ rises toward the interface, as if there was a poorer conductor at depth; coming from the right, $E_x(y)$ decreases toward the interface, as if a better conductor was below. But arguments like those of Section 2 can also be invoked: because of the continuity of the electric field there is a positive magnetic vortex on the left and a negative one on the right of the interface (*cf.* Fig. 4a, b); this decreases the phase on the left and increases it on the right. These curves were computed with the 2-D finite element modelling programme of Wannamaker, Stodt & Rijo (1987).

anomalous electric and magnetic fields in these two configurations are oppositely directed. Note that for this particular example the derivatives of both $\phi_E(y)$ and $\phi_B(y)$ seem to remain continuous.

What we have just seen is that phase rule (I) remains equally valid for E- and for B-polarization induction, even though the field patterns are entirely different for the two polarization modes. *The two phase rules* (I) *and* (II) *are thus applicable to both polarization modes* and they are probably of quite general validity. But it is important to interpret the phase rules with the appropriate pattern of current flow. For the vertically outcropping resistivity contrast extending to great depths we express these results as corollaries (CE) and (CB) to the phase rules, as follows.

(CE) On the more conducting side of a vertically outcropping resistivity contrast the E-polarization phase ϕ_E undergoes a negative excursion, and a positive one on the more resistive side.

(CB) At a vertically outcropping resistivity contrast the B-polarization phase ϕ_B undergoes a positive excursion on the more conducting side of the interface and a negative excursion on the more resistive side.

Fig. 6 illustrates these statements.

5 B-POLARIZATION PHASE OVER A CONDUCTING DIKE OF VARYING WIDTH

Let us choose a shallow conducting dike similar to the one shown in Fig. 1, but with the following parameters: for the dike we take a resistivity of 10 Ω m and 100 Ω m for the matrix. The dike depth is only 600 m which, at the period of $T = 1$ s, is small compared with the matrix skin-depth of approximately 5 km. The dike width w is assumed to be variable.

First we compute apparent resistivity and phase across the entire structure for a dike of $w = 6$ km. As expected we see in Fig. 7 that the apparent resistivity jumps discontinuously by a factor of $(\rho_0/\rho)^2 = 100$ at the two dike walls. The phase remains continuous however, but with lower values on the matrix side and higher values inside the dike. Because the dike is fairly wide the phase drops towards the dike centre; it tries to adjust to the two-layer value. At the dike centre C the apparent resistivity is also close to the corresponding value of a two-layer structure.

Next we compute the phase at the central point C of the dike as a function of the dike width w. In Fig. 2(a) we see that when the dike is very wide this phase reaches the limiting two-layer value of about 25.4°. This value, in effect, represents the contribution to the MT phase of the current redistribution alone. Here the effect of the surface charges reaches to larger distances because there are also such charges at the bottom of the dike. As the dike width decreases the effects of the two sets of surface charges on the dike walls gain in importance and for a dike width of 600 m a maximum effect is reached, with an MT phase of about 56°. Upon further reduction of the width the charge effect diminishes. This decrease is easy to understand. It is a consequence of the diminishing horizontal components of the electric fields caused by the two sets of surface charges: as the dike becomes narrower the central point C moves closer to the resistivity interfaces, where the phase is at an intermediate value, tending toward 45° when the dike disappears. This is what we observe in Fig. 2(a).

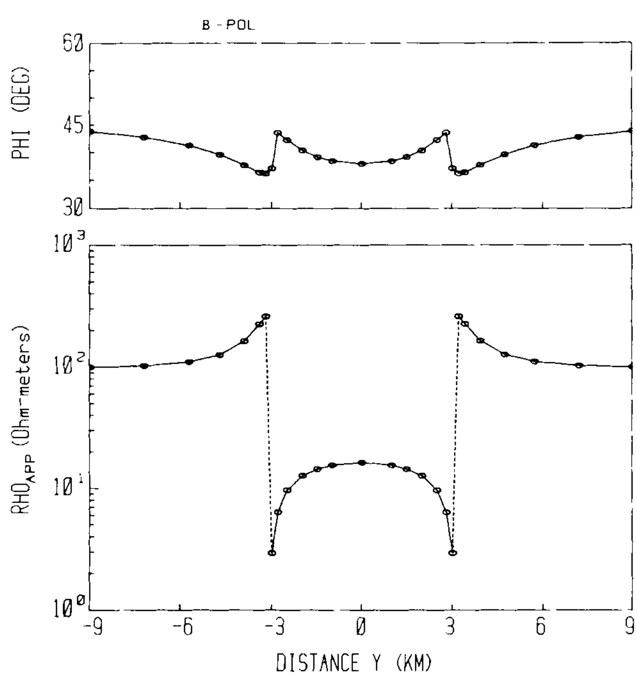

Figure 7. *B*-polarization apparent resistivity and phase over the dike of Fig. 1, with the following parameters: $d = 600$ m, $w = 6$ km, $\rho = 10\ \Omega$ m, $\rho_0 = 100\ \Omega$ m, $T = 1$ s. At the two vertical dike faces the phase remains continuous, but it is slightly below 45° throughout because of the current redistribution entailed by the dike.

However, the behaviour of the phase is perfectly explained by phase rules (I) and (II). As the conducting dike is approached from the left or right the current rises, as if there were a poorer conductor at depth and the phase drops. Approaching the dike walls from inside the dike, the current plunges, as if there were a better conductor at depth and the phase rises. At the dike walls the phase swings around but remains perfectly continuous.

It is of interest to study in more detail the behaviour of the phase at one of the boundaries of a very wide dike. We choose the left dike boundary and a dike width w which we increase to infinity; we also shift the origin of coordinate y to this dike wall. Since the other parameters have not been changed we have, in effect, the contact between a uniform quarter-space of resistivity $\rho = 100\ \Omega$ m with a two-layer quarter-space with $\rho_1 = 10\ \Omega$ m, $\rho_2 = 100\ \Omega$ m and $h =$ 600 m. The inducing frequency is still chosen as 1 Hz. In Fig. 8 we see the behaviour of the phase over this contact in great detail. The *B*-polarization phase is obviously continuous, but it shows a discontinuity in its derivative. Also apparent is that phase rules (I), (II) and (CB) are obeyed as one moves toward the boundary from large distances. On the left the current flow lines rise toward the conducting top layer and the phase therefore falls. Coming from the right the current flow dips toward the interface and the phase is seen to rise.

Close to the interface the effect of the surface charges is to ensure continuity of the phase, so there is a rapid reversal from the behaviour at larger distances, as the charge-generated field fans through 180° over the outcropping resistivity interface (*cf.* Fig. 4a, b).

Under *E*-polarization induction the phase rules (I), (II) and (CE) are also obeyed, but we note that the derivative of the phase remains continuous.

6 THE RANGE OF VALIDITY OF THE PHASE RULES

In the Introduction we stated that the phase rule is applicable to the 1-D two-layer structure, provided the skin-depth δ is larger than about 2/3 of the first layer thickness h. What is meant by this statement is that the period T should exceed the largest finite value T_c where the phase equals 45°. As is well known, for $T < T_c$ the phase undergoes small oscillations, whereas with $T > T_c$ it experiences its single large excursion. The precise relation between the critical values of δ_c and h is (*cf.*, Fischer *et al.* 1981)

$$\delta_c = \frac{2}{\pi} h. \tag{13}$$

From this relation we derive for the critical period T_c the

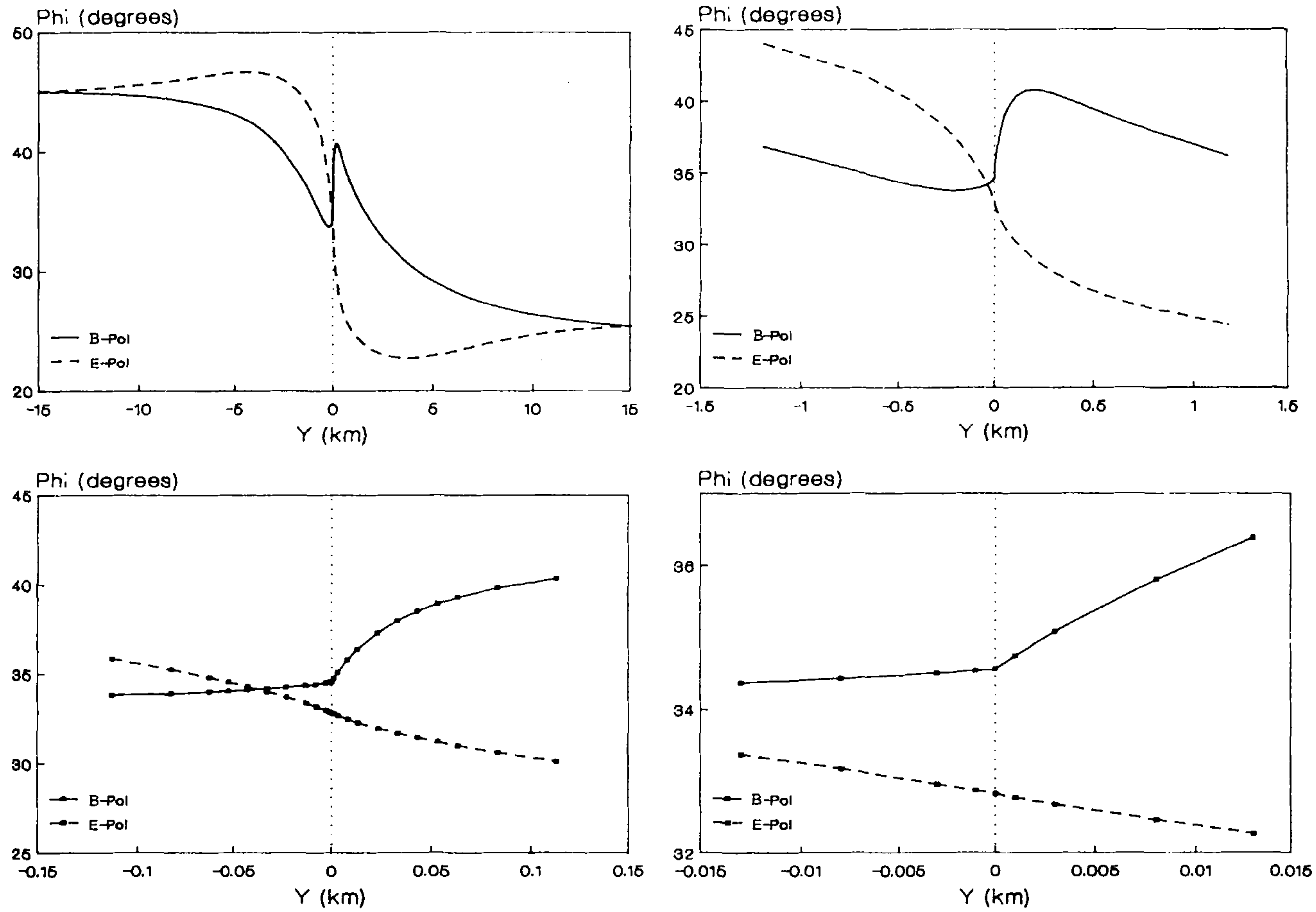

Figure 8. Detailed study of the phases $\phi_H(y)$ and $\phi_E(y)$ for a vertical interface between a uniform quarter-space on the left ($\rho = 100\,\Omega$ m) and a two-layer system on the right ($\rho = 10\,\Omega$ m, $\rho_2 = 100\,\Omega$ m, $h = 600$ m) at an exciting frequency of 1 Hz. On the top we see that the phase follows rules (I) and (II) as well as the corollaries (CB) and (CE) which concern outcropping resistivity interfaces. The two lower diagrams show the behaviour of the phases $\phi_B(y)$ and $\phi_E(y)$ over the outcropping interface in greater detail. Both phases are continuous, but only $\phi_E(y)$ also has a continuous slope.

condition

$$T_c = \frac{4\mu_0}{\pi} h^2/\rho_1 = \pi\mu_0\delta_c^2/\rho_1, \tag{14}$$

where ρ_1 is the top layer resistivity.

While T_c sets a lower limit for the phase rule validity over a two-layer structure, there is no such limit of validity for a vertically outcropping resistivity contrast extending to infinite depth, as sketched in Figs 4 and 5. Indeed the responses of this model, shown in Fig. 6, apply to all situations where $\rho_2 = 10\rho_1$, provided the ordinate is normalized by ρ_1, rather than $1\,\Omega$ m, and the abscissa by $\sqrt{\rho_1 T/10}$, rather than by 1 km.

For two-layer structures we have seen that there is a critical shortest period T_c; but there is no critical longest period. For three-layer structures, however, there is also an upper limiting period T_u, and it is worth discussing this situation in some detail to appreciate the significance of this long period T_u. To this end we consider a structure consisting of an intermediate good conductor with $\rho_2 = 1\,\Omega$ m and a thickness $h_2 = 1$ km, embedded at a depth of 0.5 km in a $500\,\Omega$ m matrix. Here the lowest critical period T_c is given by equation (14) as $T_c = 0.0008$ s. When the period increases the fields progressively penetrate more deeply into the second good conductor. At periods around 1 s the thin resistive overburden is no longer felt by the fields, which in fact begin to pierce through the good conductor and penetrate into the resistive basement. The structure then behaves as a new two-layer model, now with a good conductor on top. The former positive phase excursion must therefore turn into a negative one at some upper period T_u, which in fact is the lowest critical period for this new two-layer system:

$$T_u = \frac{4\mu_0}{\pi} h_2^2/\rho_2 = 1.6\text{ s}. \tag{15}$$

The above example also illustrates the behaviour which is to be expected with a buried good conducting prism under E-polarization induction: at low periods there will be a positive phase anomaly (the current is drawn to greater depths), but at very long periods the phase anomaly will reverse and become negative (the prism will draw a large part of the current closer to the surface). Examples of this behaviour have been given by, e.g., Adam (1987).

As shown by the foregoing discussions, the phase rules retain their validity at all periods longer than the critical shortest T_c, but they will manifest themselves with a behaviour which is dependent on the period, according to the way resistivity varies with depth. Their correct interpretation, therefore, simply provides an alternative means of investigating the structure of the Earth's crust.

7 CONCLUSIONS

The principles governing the behaviour of the magnetotelluric phase over 2-D structures have been clarified. It has been shown that a simple qualitative rule first proposed by Fischer (1985) for E-polarization induction retains its validity for B-polarization induction as well if the differing current flow patterns for these two modes of induction are properly considered. This simple phase rule is best formulated in two statements: the MT phase is continuous at the surface of any conducting structure; wherever currents are drawn to greater depths the phase rises, and wherever the currents are concentrated near the surface the phase drops. Across vertical 2-D resistivity contrasts the rule predicts, on the side of the better conductor, positive and negative phase excursion for B- and E-polarization inductions respectively. The opposite behaviour occurs on the more resistive side of the interface.

The reasons for the success of this qualitative rule are analysed and more correct mathematical expressions are given which allow a quantitive evaluation of the MT phase. These formulae clearly show that the E-polarization phase is determined by the depth distribution of the currents which are in phase and in quadrature with the inducing magnetic field. For B-polarization the charges appearing at resistivity contrasts are also involved.

Both E- and B-polarization effects have been demonstrated by means of numerical models and an understanding of them should prove useful in the interpretation of magnetotelluric soundings. In particular, this study has clearly shown the reasons for the stability of the phase with regard to the effects of static shift.

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