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AKADÉMIAI KIADÓ

The impact of budget deficit, public debt and education expenditures on economic growth in Poland

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ABSTRACT

This paper investigates the relationship between economic growth in Poland and a few metrics of fiscal policy: budget deficit relative to GDP, the structure of public debt, education expenditures, and public consumption. We prove that with constant values of parameters of fiscal policy, over time the economy converges to the balanced growth path which is unique and globally asymptotically stable.

Having calibrated the model with statistical data, we demonstrate that in the period of 2000–2016 economic growth in Poland was driven primarily by rapid improvement in the level of human capital (at a rate of 5.4% per annum), and secondarily due to the accumulation of capital (2.7% annually). If recent trends in fiscal policy are continued, the Polish economy will converge to the balanced growth path with GDP growing at 3.7%. This rate may be boosted, if fiscal policy is appropriately adjusted, for example by permanent reduction in budget deficit. We also analyse the effects of changes in the financing structure of public debt. Finally, we present several scenarios of increasing public and private spending on education.

KEYWORDS

optimal fiscal policy, economic growth, human capital, budget deficit, public debt

JEL CLASSIFICATION INDICES

E13, E62, F43, H6, H52

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1. INTRODUCTION

Konopczyński (2014) developed a simple exogenous growth model capable of simulating the long-run effects of changes in tax rates as well as adjustments in private and public spending on education. Following standard approach in the theoretical literature (e.g. Lee – Gordon 2005; Dhont – Heylen 2009; Turnovsky 2009), we assumed that the government runs balanced budget. This assumption is theoretically justified by the Ricardian equivalence, at least in case of closed economies (Elmendorf – Mankiw 1998). However, from an empirical point of view, it is obviously unrealistic. Therefore, the main purpose of this paper is to augment our previous model by allowing the government to run deficit financed by domestic as well as foreign lenders. The modified model allows for simulating the long-run effects of changes in the level of budget deficit as well as the structure of financing of public debt. Last, but not least, we also analyse growth effects of increased investment in education.

The literature on the long-run (growth) implications of public deficit and debt is abundant. Konopczyński's (2015) book provides a detailed overview. Theoretical literature is almost uniformly based on endogenous growth models, with all their strengths and weaknesses. By contrast, our present analysis is based on exogenous growth theory, which dates back to the 1990s. There are quite a few arguments in favour of such an approach – they have been exposed in Konopczyński (2014). In short, the Central and Eastern European countries (including Poland) over the last 2–3 decades have experienced so many deep, structural changes, that it would be unjustified to apply strong assumptions of endogenous growth theory. It is not reasonable to assume that economic agents are continuously optimizing, smoothly adjusting their economic decisions to evolving conditions, because these conditions change frequently, deeply, and often unexpectedly. The expansion of the EU in 2004 was probably the greatest of many deep, structural changes, which profoundly affect economic conditions that consumers and businesses face. Moreover, according to some researchers, the post-socialist past of Poland can also be an influencing factor even today (Mihalyi – Banasz 2016).

For these reasons, our model is deliberately constructed in a much simpler way, with many elements taken directly from the Mankiw et al. (1992) growth model. Therefore, human capital is a stock, which depreciates and requires investment just like other factors of production. Furthermore, the rates of savings and investment are set exogenously.

Mathematical rules describing the public sector are mostly borrowed from the literature on the so-called optimal fiscal policy (e.g. Agenor 2007; Lee – Gordon 2005; Dhont – Heylen 2009). There are five types of taxes: on capital, labour, human capital, interest on government bonds held by domestic residents, and consumption. Public expenditures are divided into three broad categories: public consumption, education and financial transfers. The government chooses (fixes) the size of the budget deficit relative to GDP and controls the financing structure of public debt.

The paper is organized as follows. Section 1 presents the details of the model. Section 2 contains a qualitative sensitivity analysis. In Section 3, the model is calibrated for the Polish economy over the period of 2000–2016. Section 4 outlines the baseline scenario. In section 5, we search for the optimal level of budget deficit, whereas in section 6 we analyse the optimal financing structure of public debt. Section 7 presents several scenarios of increased educational expenditures by the government and by the private sector. In section 8, we search for the optimal structure of private investment. Summary synthesizes the main results and offers some further comments. Mathematical proofs are included in the Appendix.

2. THE MODEL

The aggregate output of the country is described by the following production function:

$$Y = aK^{\alpha}H^{1-\alpha-\beta}(EL)^{\beta}, \quad 0<\alpha,\ \beta<1, \tag{1}$$

where K represents the stock of physical capital, H represents the stock of human capital, and L is raw labour. Following Barro – Martin (2004), we assume positive externalities related to learning-by-doing and spillover-effects, embedded in the labour-augmenting technology index $E = xK/L$, where $x =$ const.>0. It follows that the production function can be written as:

$$Y = AK^{\alpha+\beta}H^{1-\alpha-\beta}, \tag{2}$$

where $A = ax^{\beta} = const>0$. The labour supply in the country is growing exponentially: $L = L_0e^{nt}$, where $L_0>0$ denotes the initial stock of labour (at $t = 0$), whereas $t \geq 0$ is a continuous time index. Demand for all factors of production results from the rational decisions of firms maximizing profits in perfectly competitive markets. Let w_K and w_H denote the real rental price of physical capital and human capital, respectively, and let w denote the real wage rate. In the profit maximizing equilibrium, all factors are paid their marginal products, i.e.,

$$MPK = \partial Y/\partial K = \alpha Y/K = w_K = r + \delta_K, \tag{3}$$

$$MPH = \partial Y/\partial H = (1-\alpha-\beta)Y/H = w_H, \tag{4}$$

$$MPL = \partial Y/\partial L = \beta Y/L = w, \tag{5}$$

where δ_K represents the rate of depreciation of capital. Note that the variables w, w_H and $w_K = r + \delta_K$ represent gross rates (before taxation), i.e., the unit costs of labour, human capital and physical capital from the perspective of the representative firm.

2.1. The public sector

The government levies income and consumption taxes. Let τ_L, τ_H and τ_K denote the average tax rates on labour, human capital and physical capital, respectively. Taxes on labour and human capital are levied on gross wage rates, i.e., the government collects $\tau_L w$ and $\tau_H w_H$. To the contrary, the income tax on capital is levied on net capital income, defined as gross income minus a depreciation allowance, i.e. the tax bill is calculated as follows: $\tau_K(w_K - \delta_K) = \tau_K r$. In addition, the interest on government bonds held by domestic residents is taxed with the rate equal to τ_D. The sum of all income taxes is expressed as:

$$T_1 = \tau_L wL + \tau_H w_H H + \tau_K rK + \tau_D r_D D_D, \tag{6}$$

where D_D denotes the domestic debt of the government. The consumption tax is equal to:

$$T_2 = \tau_C C, \tag{7}$$

where τ_C is the average tax rate on aggregate consumption C. Total government revenue is $T = T_1 + T_2$. The real deficit of public sector J is the difference between total government spending (including debt servicing) and tax revenue, i.e.

$$J = G + r_D D - T, \tag{8}$$

where G denotes government spending, and D is the total public debt. We assume that the budget deficit is fixed in relation to GDP, i.e.

$$J = \xi Y, \tag{9}$$

where $\xi = const>0$. Using (8), the budgetary rule (9) can be written as follows:

$$G = T - r_D D + \xi Y. \tag{10}$$

Public debt is accumulated according to the equation: $\dot{D} = \xi Y$. Certain part (ω) of bonds is sold to the foreign creditors, and the rest to the domestic lenders:

$$\dot{D}_F = \omega \dot{D} = \omega \xi Y \quad \text{with} \quad 0 \leq \omega \leq 1, \tag{11}$$

$$\dot{D}_D = (1 - \omega)\dot{D} = (1 - \omega)\xi Y, \tag{12}$$

where D_F is the foreign debt of the government. Obviously, at any moment of time, $D = D_D + D_F$. Public expenditures include three components:

$$G = G_T + G_E + G_C, \tag{13}$$

where G_T denotes cash transfers to the private sector (mainly social transfers such as pensions, various benefits, etc.), G_E represents public spending on education, and G_C is public consumption (primarily health care, national defence and public safety). By assumption, public consumption and education expenditures are a fixed share of GDP:

$$G_C = \gamma_C Y, \quad \text{where} \quad 0<\gamma_c<1. \tag{14}$$

$$G_E = \gamma_E Y, \quad \text{where} \quad 0<\gamma_E<1. \tag{15}$$

Obviously, $\gamma_C + \gamma_E<1$. Eqs. (10) and (13) determine the real size of the cash transfers:

$$G_T = G - G_C - G_E = T + \xi Y - r_D D - (\gamma_C + \gamma_E)Y. \tag{16}$$

Therefore, the tax revenue augmented by the scheduled budget deficit are used to service the public debt, public consumption and education expenditures as planned by the government. Whatever remains is transferred to the private sector.

2.2. The private sector

On the one hand, we assume that all factors of production are owned by domestic residents (closed economy), the aggregate production function exhibits constant returns to scale. It follows that households' disposable income Y_d is equal to GDP net of taxes, plus the interest on government bonds *held by domestic lenders*, plus transfers from the government. Certain fraction of that income is saved, and the remainder is consumed; hence the budget constraint of the private sector is expressed as follows:

$$Y_d = Y - T_1 - T_2 + r_D D_D + G_T = C + S. \tag{17}$$

We assume a constant and exogenous rate of private savings:

$$S = \gamma Y_d = \gamma(Y - T_1 - T_2 + r_D D_D + G_T). \tag{18}$$

Savings are used to either purchase government bonds, or finance investment: $S = I + \dot{D}_D$. There are two types of investment: in physical and human capital, with a fixed share coefficient $0<\psi<1$:

$$I_K = (1 - \psi)I, \tag{19}$$

$$I_H = \psi I, \tag{20}$$

From (17), it follows that private consumption is equal to:

$$C = Y_d - S = Y - T_1 - T_2 + r_D D_D + G_T - S. \tag{21}$$

Notice that Eqs. (18) and (21) are interconnected because of (7). According to (18), savings depend on consumption, and simultaneously, according to (21) consumption depends on savings. It is convenient to solve this system of equations. Substituting (16) into (18) and (21), and using (7), after simple transformation yields:

$$C = (1 - \gamma)[(1 + \xi - \gamma_C - \gamma_E)Y - r_D D_F] \tag{22}$$

$$S = \gamma[(1 + \xi - \gamma_C - \gamma_E)Y - r_D D_F] \tag{23}$$

From Eqs. (12), (18), (19), (20), (23), and $S = I + \dot{D}_D$, it follows that:

$$I_K = (1 - \psi)[\gamma[(1 + \xi - \gamma_C - \gamma_E)Y - r_D D_F] - (1 - \omega)\xi Y]. \tag{24}$$

$$I_H = \psi[\gamma[(1 + \xi - \gamma_C - \gamma_E)Y - r_D D_F] - (1 - \omega)\xi Y], \tag{25}$$

The accumulation of private capital and human capital is described as follows:

$$\dot{K} = I_K - \delta_K K, \quad 0<\delta_K<1, \tag{26}$$

$$\dot{H} = G_E + I_H - \delta_H H, \quad 0<\delta_H<1. \tag{27}$$

where δ_K and δ_H denote depreciation rates. These equations can easily be transformed to yield the following growth rates:

$$\widehat{K} = \frac{\dot{K}}{K} = \frac{I_K}{K} - \delta_K, \tag{28}$$

$$\widehat{H} = \frac{\dot{H}}{H} = \frac{G_E + I_H}{H} - \delta_H, \tag{29}$$

Substituting (24), Eq. (28) can be transformed into the following form:

$$\widehat{K} = (1 - \psi)\left[\gamma(1 + \xi - \gamma_C - \gamma_E) - (1 - \omega)\xi - \gamma r_D \frac{D_F}{Y}\right] \frac{Y}{K} - \delta_K, \tag{30}$$

Similarly, using (15) and (25) in Eq. (29) yields:

$$\widehat{H} = \psi\left[\gamma(1 + \xi - \gamma_C - \gamma_E) - (1 - \omega)\xi - \gamma r_D \frac{D_F}{Y} + \frac{\gamma_E}{\psi}\right] \frac{Y}{H} - \delta_H, \tag{31}$$

Finally, using (2), the growth rates (30) and (31) can be written as:

$$\widehat{K} = (1-\psi)A\left[\gamma(1+\xi-\gamma_C-\gamma_E)-(1-\omega)\xi-\gamma r_D\frac{D_F}{Y}\right]\left(\frac{K}{H}\right)^{\alpha+\beta-1}-\delta_K. \tag{32}$$

$$\widehat{H} = \psi A\left[\gamma(1+\xi-\gamma_C-\gamma_E)-(1-\omega)\xi-\gamma r_D\frac{D_F}{Y}+\frac{\gamma_E}{\psi}\right]\left(\frac{K}{H}\right)^{\alpha+\beta}-\delta_H. \tag{33}$$

For brevity, let us write Eqs. (32) and (33) in the following form:

$$\widehat{K} = (1-\psi)AE_1\left(\frac{K}{H}\right)^{\alpha+\beta-1}-\delta_K, \tag{34}$$

$$\widehat{H} = \psi A\left[E_1+\frac{\gamma_E}{\psi}\right]\left(\frac{K}{H}\right)^{\alpha+\beta}-\delta_H, \tag{35}$$

where

$$E_1 = \gamma(1+\xi-\gamma_C-\gamma_E)-(1-\omega)\xi-\gamma r_D\frac{D_F}{Y}. \tag{36}$$

Note that it is necessary (though not sufficient) to assume that $E_1>0$. Otherwise $\widehat{K}<0$ or $\widehat{H}<0$ (or even both), so that GDP is shrinking to zero over time. This uninteresting (unwanted) situation may happen if γ is very low (the private sector is saving too little), and/or if the government behaves irresponsibly. To be more precise, E_1 may turn negative if, for example, the foreign-debt-to-GDP ratio is above some critical level (implying very high cost of servicing foreign debt on the economy), or if the expression $(1-\omega)\xi$ is high enough, which means that the government is running very high deficit covered too extensively by domestic borrowing.

Technically, the 'laws of motion' (34) and (35) are similar to their counterparts (36) and (37) in Konopczyński (2014). However, augmenting the model by adding public deficit and debt has significantly complicated the dynamics of the model. Finding the balanced-growth equilibrium in Konopczyński (2014) was relatively simple – it boiled down to equating the right-hand sides of Eqs. (36) and (37) therein and solving (numerically) the resulting nonlinear equation in one unknown (the ratio of K/H). Now it is more complicated, because the 'laws of motion' include an additional "variable", the foreign-debt-to-GDP ratio $d_F = D/Y$, which evolves over time according to the following equation:

$$\widehat{d}_F = \widehat{D}_F - \widehat{Y}. \tag{37}$$

Fortunately, the following proposition can easily be proved.

Proposition 1 (proof in the Appendix)
Over time, $d_F \rightarrow \omega\xi/\widehat{Y}$, regardless of whether $\widehat{Y}$ is constant or changing over time.

Intuitively speaking, it means that the foreign-debt-to-GDP ratio is continuously converging to the ratio of public deficit covered by foreign sources (as percentage of GDP) and the rate of growth of GDP, regardless of any changes (increase or decrease) in the latter. This proposition leads to:

Proposition 2 (proof in the Appendix)
In the long run, the economy converges towards the balanced growth path (hereafter denoted by overbar), with K, H, D and Y growing at the same, constant rate (the balanced growth rate, BGR).

This balanced growth equilibrium is unique and globally asymptotically stable. The steady-state foreign-debt-to-GDP ratio $\overline{d}_F$ is equal to

$$\overline{d}_F = \omega\xi/\mathrm{BGR} = const.>0 \tag{38}$$

The easiest way to find the balanced growth path is to equate the right-hand sides of Eqs. (32) and (33), incorporating Eq. (38). This results in the following system of 2 equations in 2 unknowns, K/H and d_F:

$$(1-\psi)AE_1\left(\frac{K}{H}\right)^{\alpha+\beta-1} - \delta_K = \psi A\left[E_1 + \frac{\gamma_E}{\psi}\right]\left(\frac{K}{H}\right)^{\alpha+\beta} - \delta_H, \tag{39}$$

$$d_F = \frac{\omega\xi}{(1-\psi)AE_1\left(\frac{K}{H}\right)^{\alpha+\beta-1} - \delta_K} \tag{40}$$

where E_1 is a function of d_F given by Eq. (36). The BGR can then be calculated by substituting the obtained value of K/H into either (32) or (33).

Obviously, it is not possible to derive an explicit formula for the BGR as a function of the parameters of the model. It can only be calculated numerically, after substituting some values for all parameters. Nevertheless, it is perfectly possible (and worthwhile) to perform a qualitative sensitivity analysis in order to determine the relationships between the parameters of the model and the BGR.

3. QUALITATIVE SENSITIVITY ANALYSIS

In this section, we investigate how changes in the parameter values influence the BGR. The analysis is performed in 3 stages. First, we investigate whether an increase in the value of a selected parameter (ξ, ω, etc.) increases or reduces the value of E_1. Second, using formulas (34) and (35), we investigate whether the graphs of functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ shift up or down. Third, based on these observations, we conclude whether the intersection of these curves, which determines the BGR (see Appendix, Fig. A1), moves up or down.

On the face of it, these stages are identical as in the model without budget deficit and public debt – see section 3 in Konopczyński (2014). However, stage 2 is far more complex than therein, because functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ depend on d_F. In fact, this stage must be decomposed into 3 steps. First, we investigate how the aforementioned graphs shift under an artificial assumption that the steady-state value of d_F is not affected. Second, we investigate how the steady-state value of d_F changes, and how it shifts the graphs of $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$. Third, we investigate the combined effects of these two shifts.

As an example of this procedure, let us present in some detail the analysis of the effects of an increase in the deficit-to-GDP rate ξ. It is useful to follow all steps in Fig. 1, starting from the initial graphs of functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$, labelled $\widehat{K}^{old}$ and $\widehat{H}^{old}$. The intersection of these curves determines the initial value of BGR labelled as BGR^{old}.

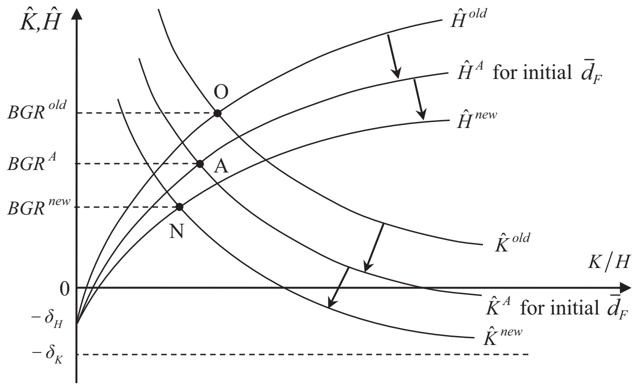

Fig. 1. **The effects of an increase in the rate of government deficit ξ in case A**

Note that $\frac{\partial E_1}{\partial \xi} = \gamma - (1-\omega)$, which may be negative, positive or equal to zero. Therefore, we must distinguish 3 cases:

A. $(1-\omega)>\gamma$,
B. $(1-\omega)<\gamma$,
C. $(1-\omega) = \gamma$.

First, consider case A. Note that $\frac{\partial E_1}{\partial \xi} = \gamma - (1-\omega)<0$. It follows from Eqs. (34) and (35) that an increase in ξ shifts both functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ down to their new positions, which in Fig. 1 are labelled as $\widehat{K}^{A}$ for initial $\overline{d}_F$ and $\widehat{H}^{A}$ for initial $\overline{d}_F$, respectively. Therefore, the BGR *instantly* falls. We may call it an *immediate effect*. If the steady-state value of $\overline{d}_F$ would remain at this initial level, then the new BGR would simply fall down to the level labelled BGR^{A}, and our analysis would be complete. However, $\overline{d}_F$ will not remain at its initial level, as lower BGR coupled with higher value of ξ implicates higher value of $\overline{d}_F$, which follows directly from Eq. (39). This, in turn, in accordance with Eqs. (34) and (35) shifts both functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ *further downwards* to positions labelled as $\widehat{K}^{new}$ and $\widehat{H}^{new}$. We may call it the *gradual effect*, because it is obviously spread over time: lower GDP rate of growth coupled with higher deficit-to-GDP ratio cause gradual accumulation of public debt, which *gradually* (quarter after quarter) increases the burden of public debt weighing on the economy, thus *gradually* reducing both rates of growth, $\widehat{K}$ and $\widehat{H}$.

Therefore, it is clear, that in case A higher government deficit reduces the BGR. To provide an intuitive explanation of a negative relationship between ξ and the BGR, let us consider a stylized example. Recall that in this case $(1-\omega)>\gamma$, which means that the share of domestic lenders in financing government deficit (and ultimately public debt) is higher than their average rate of savings (out of their disposable income). Let $(1-\omega) = 60\%$ and $\gamma = 30\%$ (As we will see below, these parameters are similar in Poland over the last 2 decades or so). Imagine that the government decides to increase its deficit by some specified amount of money, say, €100. Thus, it must borrow additional funds from domestic and foreign lenders. As $(1-\omega) = 60\%$, it borrows €60 from domestic lenders (and €40 from foreigners). An increase in public deficit can be implemented with either a reduction in taxes, or an increase in financial transfers to the private sector G_T, or an appropriate combination of both[1]. Irrespective of the chosen method, the €100 increase in public deficit results in an immediate increase in private sector's disposable income Y_d equal to €100. This extra income is partly saved, according to Eq. (18). As $\gamma = 30\%$, private savings increase by €30. However, since $(1-\omega) = 60\%$, the private sector meanwhile purchases €60 worth of government bonds[2], so that the net change in private savings is in fact negative and equal to minus €30. Consequently, private investment in both types of capital falls, reducing the GDP rate of growth. (All of these are elements of what we call an *immediate effect*; we will not elaborate on the *gradual effect*, because it has been clarified above.)

Let us now consider case B, with $\frac{\partial E_1}{\partial \xi} = \gamma - (1-\omega) > 0$. It follows from Eqs. (34) and (35) that an increase in ξ shifts both functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ upwards to their new positions, which in Fig. 2 are labelled as $\widehat{K}^A$ for initial $\overline{d}_F$ and $\widehat{H}^A$ for initial $\overline{d}_F$, respectively. Therefore, contrary to case A, the *immediate effect* is positive: the BGR *instantly* rises. If the steady-state value of $\overline{d}_F$ would remain at this initial level, then the new BGR would simply rise to the level labelled as BGR^A, and this analysis would be complete. However, $\overline{d}_F$ will not remain at its initial level, as higher BGR coupled with higher value of ξ implicates a *different* value of $\overline{d}_F$, which follows directly from Eq. (38). This, in turn, in accordance with Eqs. (34) and (35) shifts both functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ *further*. However, without additional assumptions, we cannot determine the direction of this secondary effect – we cannot be sure whether it is an upward or downward shift. Moreover, we cannot exclude a possibility of a negative *gradual effect* being big enough to finally offset (or even more than offset) the positive immediate effect. To see why, let us consider the boundary case C, where $(1-\omega) = \gamma$, so that $\frac{\partial E_1}{\partial \xi} = \gamma - (1-\omega) = 0$. Obviously, in this case there is no immediate effect: both curves $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ *initially* remain unchanged, so that *initially* the BGR is not affected. However, there is a negative secondary effect, because higher value of ξ raises the steady-state value of $\overline{d}_F$. This, in turn, in accordance with Eqs. (34) and (35) shifts both functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ *downwards*. This *gradual effect* is spread over time: falling GDP rate of growth coupled with higher deficit-to-GDP ratio causes a gradual accumulation of public debt, which *gradually* increases the burden of public debt, thus *gradually* reducing both rates of growth, $\widehat{K}$ and $\widehat{H}$.

Therefore, it is clear, that in case B, negative gradual effect may well be big enough (in absolute terms) to more than offset the initial positive effect, so that the BGR may finally be lower than its initial value, i.e., $\mathrm{BGR}^{new} < \mathrm{BGR}^{old}$. This will happen, if $(1-\omega) > \gamma$, but the difference is small enough.

[1]Strictly speaking, in our model, it is an increase in financial transfers, according to Eq. (16). However, in practice, there are 3 options.

[2]We assume that the private sector (somewhat passively) purchases any amount of bonds that the government supplies. In practice, it could be realistic in such countries, where the public sector is relatively large, and the government has significant share in the banking sector and/or direct control over some large state-owned companies. We claim that Poland has recently been such a case, and in fact is recently moving more and more towards such a situation.

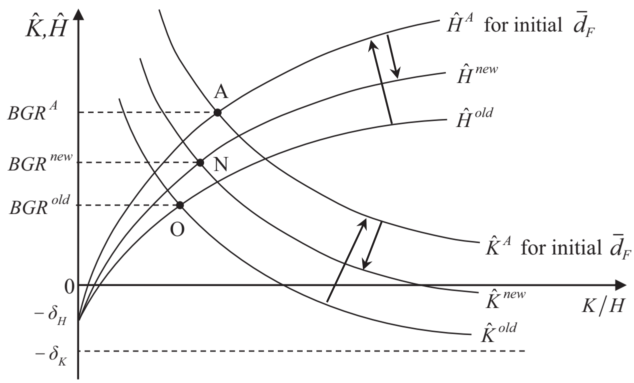

Fig. 2. **The effects of an increase in the rate of government deficit ξ in case B**

The effects of an increase in all other parameters can be traced down in a similar way. In some cases, the initial change (increase/decrease) in the BGR is partially, entirely, or even more than entirely offset by the secondary effect: a gradual change (decrease/increase) in $\bar{d}_F$. The results are summarized in Table 1.

Some conclusions are intuitively clear, but others require explanation. First, let us note, that all tax rates in our model are neutral, which is a common result in theoretical literature, and is a direct outcome of strict fiscal rules embedded in the model. Recall that two out of three components of public expenditures are fixed in relation to GDP, just like the size of the budget deficit. Therefore, any change in the tax revenue translates into an identical (equal in size) change in the third component of public spending, which happens to be transfers to the private sector. Putting simply, any additional taxes imposed on private sector immediately return to that sector as financial transfers. Thus, the disposable income of private sector is "immune" to any changes in tax rates, which implies that the equilibrium (balanced growth path) is also entirely insensitive to the changes in the tax rates. However, these are the only parameters of the model that are neutral: all remaining parameters do influence the BGR.

For example, unsurprisingly, increasing the rate of private savings γ speeds up the accumulation of both types of capital (technically speaking, it shifts both curves $\widehat{K}(K/H)$ and

Table 1. **Qualitative sensitivity analysis**

	$\tau_K\uparrow$	$\tau_H\uparrow$	$\tau_L\uparrow$	$\tau_C\uparrow$	$\gamma\uparrow$	$\gamma_C\uparrow$	$\gamma_E\uparrow$	$\psi\uparrow$	$\xi\uparrow$	$\omega\uparrow$
E_1	=	=	=	=	↑	↓	↓	=	?	?
graph of $\widehat{K}(K/H)$	=	=	=	=	↑	↓	↓	↓	?	?
graph of $\widehat{H}(K/H)$	=	=	=	=	↑	↓	?	↑	?	?
BGR	=	=	=	=	↑	↓	?	?	?	?
$\bar{d}_F$	=	=	=	=	↓	↑	?	?	?	?
$\bar{K}/\bar{H}$	=	=	=	=	?	?	?	↓	?	?

$\widehat{H}(K/H)$ upwards), thus raising the BGR. To the contrary, raising γ_C, i.e., increasing public consumption in relation to GDP, reduces the BGR. This is also the result of applied fiscal rules: for a given level of tax revenue and public spending on education (both in relation to GDP), an increase in γ_C requires an appropriate reduction in public transfers to the private sector, in order to keep the level of budget deficit unchanged (as a percentage of GDP). Thus, an increase in public consumption has a detrimental effect on the disposable income of private sector, and this, in turn, leads to a lower level of private investment in both productive capital and education.

Remarkably, the relationship between γ_E and the BGR is ambiguous. Recall that γ_E represents the size of public spending on education in relation to GDP. One might expect that raising γ_E should lead to a higher rate of growth of human capital, and therefore higher BGR. This is, however, not always true. For clarity of explanation, let us now focus on the *immediate effect* (i.e., neglect *gradual effect*).

On the one hand, $\frac{\partial E_1}{\partial \gamma_E} = -\gamma < 0$, so it follows from Eq. (34) that an increase in γ_E shifts the curve $\widehat{K}(K/H)$ downwards. It follows directly from strict budgetary rules that the government must obey given the fixed deficit-to-GDP ratio, fixed tax revenue and fixed public consumption, raising γ_E automatically reduces cash transfers G_T to the private sector, thus reducing its disposable income and savings, which reduces both types of private investment.

On the other hand, $\frac{\partial(E_1+\gamma_E/\psi)}{\partial \gamma_E} = \frac{1}{\psi} - \gamma$, which may be positive, negative or zero – depending on the factual values of parameters γ and ψ. Thus, it follows from Eq. (35) that an increase in γ_E shifts $\widehat{H}(K/H)$ upwards, downwards, or leaves it unchanged. Therefore, an increase in γ_E raises the BGR, if the $\widehat{H}(K/H)$ curve not only shifts upwards, but it shifts strongly enough to offset the negative effect of the downward shift of the $\widehat{K}(K/H)$ curve. Note that it requires sufficiently low value of ψ coupled with sufficiently low value of γ. This conclusion has a clear-cut, intuitive interpretation: low values of γ and ψ mean that the private sector saves little and, moreover, spends most of these small savings on productive capital rather than education. In such a case, an increase in public spending on education is beneficial: although it slightly reduces private savings and investment in productive capital (due to strict fiscal rules explained above), it boosts the rate of growth of human capital significantly, so that the net effect is positive: the GDP grows faster.

Let us now turn to ψ, which represents the share of private savings invested in education. Accordingly, raising ψ increases the rate of human capital accumulation and reduces the rate of growth of physical capital. Thus, the graph of $\widehat{H}(K/H)$ shifts up, whereas the graph of $\widehat{K}(K/H)$ shifts down. The intersection of these curves unambiguously moves to the left, but it is unclear whether it moves up or down. Therefore, a higher value of ψ reduces the balanced growth ratio of $\overline{K}/\overline{H}$ – there is more human capital per each unit of physical capital. Nonetheless, the relationship between ψ and the BGR is ambiguous.

The relationship between ω and BGR is also ambiguous. To be more precise, it is positive for sufficiently low levels of r_D, but negative if r_D exceeds certain critical value. To see why, let us first consider a trivial case, where $r_D = 0$. It follows from (35) that $\frac{\partial E_1}{\partial \omega} = \xi > 0$, so that an increase in ω shifts both functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ up, unambiguously raising the BGR. An intuition behind this result is simple: if the government can borrow from abroad for free (at zero cost), it makes perfect sense to finance the entire deficit (and ultimately public debt) from foreign sources. Such policy leaves more money in the hands of domestic private sector, facilitating higher level of savings and investment, which is beneficial for economic growth.

Let us now turn to a more realistic situation, with r_D>0. In that case we should use the same 3-stages procedure which we applied above to ξ. To keep our analysis more concise (and avoid repeating some explanations and graphs), we will, however, not present all details. Instead, we will focus on intuition. Note that an increase in ω produces a positive *immediate effect*: regardless of the value of r_D, an increase in ω instantaneously raises the value of E_1, shifting both curves $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ upwards, which boosts economic growth. Over time, however, bigger share of foreign lending in public debt results in a steadily climbing value of d_F, and this, in turn, gradually shifts both curves $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$ downwards. Whether this negative, *gradual effect* will be big enough to eventually offset the initial, positive effects depends on the value of r_D: for sufficiently high value of r_D, the burden of foreign debt will, over time, rise so substantially, that the GDP rate of growth will ultimately decline below the initial level.

Intuitively, this conclusion is perfectly comprehendible: if the cost of borrowing by the government exceeds certain critical level, the government should borrow domestically rather than abroad – it's better to pay high interest to domestic lenders (the funds will remain in the country as investment or consumption) rather than to foreigners. However, if the cost of borrowing is sufficiently low, this advice turns upside down: it is better to finance public debt by borrowing from abroad rather than domestically: in such a case, additional funds borrowed from abroad and (in certain part) invested domestically yield positive effects (which may be measured by the rate of return on capital and education) which outweigh the cost of borrowing abroad.

These *qualitative* results, though interesting *per se*, only enhance our desire for *quantitative* results. Moreover, as the BGR cannot be determined analytically, it is not possible to determine *how strongly* changes in the values of parameters influence the BGR. In other words, we already know the *direction* of the effect, but we know nothing of the *size* of the effect. Answering these questions requires calibrating the model and performing numerical analyses. In what follows, we calibrate the model for Poland and numerically analyse optimal fiscal policy, as well as optimal private sector parameters. The calibration is based on macroeconomic data for Poland for the period of 2000–2016, published by the Eurostat, IMF, OECD, and the Kiel Institute for the World Economy: 'real total net capital stock as a percentage of real GDP' (see Kamps 2004, for more details).

4. MODEL CALIBRATION FOR POLAND[3]

4.1. Public sector

The deficit of the public sector in Poland calculated in accordance with Eurostat methodology is presented in Fig. 3. It fluctuated wildly between 2 and 7.3%, with the minimum shortly before, and the maximum during the Great Recession. On average, budgetary deficit was equal to 4.3% of GDP, thus we set $\xi = 4.3\%$.

Table 2 presents the level and structure of public debt in Poland. The best statistical measure of net foreign debt of the government D_F is the (inverse of) net international investment position (NIIP) of the public sector. Subtracting it from the total public debt yields domestic debt of the government D_D.

[3]Source of the data: https://ec.europa.eu/eurostat/data/database.

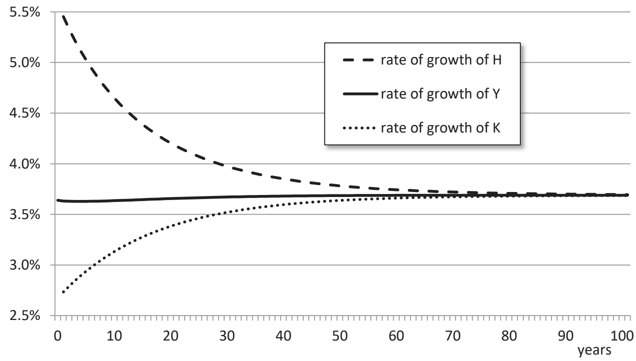

Fig. 3. Baseline scenario: Convergence to the balanced growth path

The share of foreign debt in total public debt fluctuated between roughly 35 and 55%, with a sharp but temporary drop in 2008 (due to rapid fluctuations in the value of Polish zloty induced by financial crisis around the world). For our calculations we will adopt the average value over this period, which is equal to $\omega = 0.435$. Likewise, the average value of public debt over the same period will serve as the initial value (endowment), i.e. we set $D/Y(t = 0) = 47.8\%$. The real rate of return on 10-year government bonds in Poland over the period of 2000–2016 was, on average, equal to 3.3% (source: NBP), thus we will assume that $r_D = 3.30\%$.

Public expenditures on education in Poland during the period of 2000–2015 (the latest available data) were on average equal to 5.62% of GDP (Eurostat); hence based on formula (15), we set $\gamma_E = 5.6\%$. The value of γ_C is calibrated on the basis of statistics regarding 'final consumption expenditure of general government', which over the period of 2000–2016 amounted to 18.4% of the GDP (and was very stable). This statistical aggregate includes, in particular, public spending on education, which in this model is singled out as a separate component of public expenditures. Subtracting $\gamma_E = 5.6\%$ yields $\gamma_C = 12.8\%$.

4.2. Private sector

4.2.1. Technological parameters. The elasticities of the production function (1) have been estimated in many papers (e.g. Mankiw et al. 1992; Manuelli – Seshadri 2005; studies focussing on Poland include Cichy 2008 and Próchniak 2013). The estimated values are typically close to 1/3; hence we set: $\alpha = \beta = 1 - \alpha - \beta = 1/3$. As we argued in Konopczyński (2014), the rate of physical capital depreciation is difficult to estimate, due to rapid economic transformation which resulted in huge amount of obsolete machinery, infrastructure, etc. inherited from the centrally 'planned' economy. In various research papers regarding OECD countries, physical capital depreciation varies from approximately 3.5%–7%. As the focus of our analysis is on the long run, we set the depreciation rate at a rather low level of $\delta_K = 4\%$, which is similar to Nehru – Dhareshwar (1993). The rate of human capital depreciation has been estimated by Manuelli – Seshadri (2005), Arrazola – de Hevia (2004) and others. Following these authors, we set $\delta_H = 1.5\%$.

Next, we must assess the real rate of return on capital (r). From (3), it follows that $r = \alpha Y/K - \delta_K$. The ratio of Y/K is very difficult to estimate for Poland – we have exposed

Table 2. Public debt in Poland as a percentage of GDP

Year	Total public debt D	NIIP of the public sector $-D_F$	Domestic debt of the government $D_D = D - D_F$
2000	36.5	−17.7	18.8
2001	37.3	−14.6	22.7
2002	41.8	−15.4	26.4
2003	46.6	−17.7	28.9
2004	45.0	−19.7	25.3
2005	46.4	−19.3	27.1
2006	46.9	−18.1	28.8
2007	44.2	−16.4	27.8
2008	46.3	−12.5	33.8
2009	49.4	−18.4	31.0
2010	53.1	−22.0	31.1
2011	54.1	−23.1	31.0
2012	53.7	−28.9	24.8
2013	55.7	−27.7	28.0
2014	50.2	−28.5	21.7
2015	51.1	−28.4	22.7
2016	54.1	−27.5	26.6

Source: National Bank of Poland (NBP) and own calculations.

major problems in Konopczyński (2014). Most importantly, the data available for Poland only reflect a fraction of all productive capital – namely the "gross value of fixed assets". Therefore, in Konopczyński (2014), we applied the average ratio from the entire sample of OECD countries in Kiel database, i.e. we set $Y/K = 1/3$. Substituting this value into (3) yields the real rate of return on private capital equal to $r = 1/3 \cdot 1/3 - 0.04 = 7.11\%$. This outcome is very close to most long-run empirical estimates for OECD countries. For example, Campbell et al. (2001) report that the average real rate of return on stocks in the U.S. over the period 1900–1995 was 7%. In our opinion, analogous indicators for the Polish stock market are irrelevant, because this stock market is still too young and volatile, and presumably does not reflect the long-run equilibrium.

4.2.2. Social transfers and the rates of savings and investment. The average rate of savings can be calibrated on the basis of Eqs. (17)−(20), which can be transformed into the following formula: $\gamma = \frac{S}{Y_d} = \frac{I_K + I_H + \dot{D}_D}{Y - T + r_D D_D + G_T}$. Substituting (16) yields:

$$\gamma = \frac{I_k/Y + I_H/Y + \dot{D}_D/Y}{1 + \xi - \gamma_C - \gamma_E - r_D D_F/Y}. \tag{41}$$

In order to calibrate γ, we must first establish all ratios on the right-hand side. Equation (12) implies that $\dot{D}_D/Y = (1-\omega)\xi = (1-0.435)\cdot 4.3\% = 2.43\%$. The initial ratio of foreign debt to GDP is $D_F/Y = \omega D/Y = 0.435\cdot 47.8\% = 20.8\%$. According to Eurostat, gross fixed capital formation in Poland in the period of 2000–2016 was on average 20.2% of GDP. Private spending on education in the period of 2000–2011 (the latest available data) was on average 0.65% of GDP. Substituting all of these numbers into (41) yields

$$\gamma = \frac{20.2\% + 0.65\% + 2.43\%}{1 + 4.3\% - 12.8\% - 5.6\% - 3.3\%\cdot 0.208} = 27.32\%.$$

The share parameter ψ can be calculated directly from Eq. (20): $\psi = \frac{I_H}{I} = \frac{I_H/Y}{I_K/Y + I_H/Y} = \frac{0.65\%}{20.2\% + 0.65\%} = 3.12\%$. Thus, in Poland, a mere 3.1% of total private investment is invested in education. However, private spending on education is probably underestimated in the official statistics – Eurostat takes into account only "school fees; materials such as textbooks and teaching equipment; transport to school (if organized by the school); meals (if provided by the school); boarding fees; and expenditure by employers on initial vocational training". All other private expenses related to education are classified as consumption, e.g. the cost of accommodation, travel, books, etc.

4.3. Average tax rates

Eurostat reports 'implicit tax rates' on capital, labour and consumption. In Poland during the period of 2000–2015 (the latest available data), these rates were on average equal to: $\tau_K = 20.1\%$, $\tau_L = 32.0\%$, and $\tau_C = 19.4\%$, respectively. The average tax rate on interest income was $\tau_D = 19.0\%$.

Note that the implicit tax rate on labour is defined as the "*Ratio of taxes and social security contributions on employed labour income to total compensation of employees*". To the best of our knowledge, there are no data on the average tax rates on human capital. As we wrote in Konopczyński (2014), some researchers suggest that for obvious reasons in countries with highly progressive taxes on personal income, tax rates on human capital must be higher than tax rates on (raw) labour. However, in Poland, the size of tax wedge on labour is nearly independent of the level of income, i.e. effective tax rates on wages are nearly linear. Thus, it is reasonable to assume that the average tax rates on human capital and raw labour in Poland are identical, i.e. $\tau_H = \tau_L$.

Recall that according to Eurostat, $\tau_L = 32.0\%$. However, if we set $\tau_H = \tau_L = 32.0\%$, and complete the rest of calibration as follows, the model significantly overestimates the total revenue from income taxes (by approximately 4.3% of GDP)[4]. This problem arises because our concepts of human capital and raw labour are wider than the Eurostat definitions. In particular, Eurostat classifies "taxes on income and social contributions of the self-employed" as part of the

[4]In the period of 2000–2016 'total receipts from taxes and social contributions to GDP' amounted to 33.5% (and this ratio was very stable), whereas consumption taxes were equal to 11.9% of GDP. Thus, the ratio of income taxes to GDP was equal to 21.6%.

capital income tax – a detailed explanation can be found in the methodological publication by Eurostat (2010), Annex B. However, the self-employed entrepreneurs definitely correspond to our concept of human capital (as well as part of raw labour). Self-employment is very popular in Poland – not only are there millions of small, family businesses, but very often individuals operate single-person firms and provide services for larger enterprises. Note that the tax rate on capital income published by Eurostat is much lower (20.1%) than the tax rate on labour (32.0%). Therefore, in our model, the tax rate on human capital and labour should be somewhere between these two numbers. As there are no additional statistics, we calibrate both rates at such a level, for which the model yields a total share of taxes in GDP that is consistent with the statistics (33.5%, see above). In doing so, we obtain $\tau_H = \tau_L = 25.55\%$ i.e. rates that are roughly 20% lower than those reported by Eurostat.

4.4. Closing steps

The next step in the calibration is computing the initial value of $E_1>0$ from Eq. (36). Substituting all calibrated parameters into Eq. (36) yields: $E_1 = 0.2085$. Knowing these values, and using formula (30), we compute the average capital growth rate during the period of 2000–2016: $\widehat{K} = (1-\psi)E_1 Y/K - \delta_K = 2.73\%$.

The average GDP growth rate in Poland during the period of 2000–2016 was 3.64%. Knowing this, we can estimate the human capital growth rate, on the basis of Eq. (2), which implies that $\widehat{H} = \frac{\widehat{Y}-(\alpha+\beta)\widehat{K}}{(1-\alpha-\beta)} = \frac{3.64\% - 2/3 \cdot 2.73\%}{1/3} = 5.45\%$.

These results imply that in the period of 2000–2016, economic growth in Poland was primarily driven by rapid growth in the stock of human capital, and only secondarily by the accumulation of productive capital. An impressive increase in human capital in Poland is a well-known 'stylized fact' confirmed by a sharp increase in the number of students, PhDs, etc.

For simulations it is necessary to set the value of the parameter A. First, from Eq. (31), we calculate the proportion $Y/H = \frac{\widehat{H}+\delta_H}{\psi\left(E_1+\frac{\gamma_E}{\psi}\right)} = 1.1125$. As $Y/K = 1/3$, we get $K/H = \frac{Y}{H}\frac{K}{Y} = 3.3376$. Transforming Eq. (2) and substituting the above ratios yields $A = \frac{Y}{K^{\alpha+\beta}H^{1-\alpha-\beta}} = \frac{Y}{K}\left(\frac{K}{H}\right)^{1-\alpha-\beta} = 0.4981$. To perform the simulations, we should also assume certain initial (endowment) values of K, H and L. Two of these values (K and L) can be set completely freely, provided that we confine our interest to the rates of growth and relationships (the proportions) among variables. Therefore, we set $L(0) = 1$ and $K(0) = 300$. This particular choice is convenient, as the initial level of GDP is then equal to 100, so that the initial values of all the other variables are identical to their percentage shares of GDP. Given the ratio $K/H = 3.3376$, it follows that $H(0) = 89.88$.

In summary, we have the following base set of parameters and endowments:

$$\begin{aligned}&A = 0.4981,\ \alpha = 1/3,\ \beta = 1/3,\ \delta_K = 4.0\%,\ \delta_H = 1.5\%,\ \gamma = 27.32\%,\ \psi = 3.12\%,\ \omega = 0.435,\\ &\gamma_E = 5.60\%,\ \gamma_C = 12.8\%,\ \tau_K = 20.1\%,\ \tau_C = 19.4\%,\ \tau_D = 19\%,\ \tau_H = \tau_L = 25.55\%,\\ &L(0) = 1,\ K(0) = 300,\ H(0) = 89.88,\ D(0) = 4.78.\end{aligned} \tag{42}$$

5. THE BASELINE SCENARIO

Obviously, the baseline scenario with the set of parameters (42) reproduces actual statistics on the Polish economy during the period of 2000–2016, in particular it reproduces factual (average) ratios of the following variables to GDP: C, I_K, I_H, T_1, T_2, G_C, G_E, as well as the (average) rate of GDP growth. The rates of growth for $t = 0$ generated by the model in the baseline scenario are equal to:

$$\widehat{Y} = 3.64\%,\ \widehat{K} = 2.73\%,\ \widehat{H} = 5.45\%.$$

These rates are not identical, which implies that the Polish economy is not yet on the balanced growth path. Using the procedure described at the end of Section 2, we can numerically obtain the BGR in the baseline scenario, which is equal to 3.69% – marginally higher than the average growth rate during the period of 2000–2016. The process of convergence towards the balanced growth path is presented in Fig. 3, which illustrates the trajectories of several growth rates in the baseline scenario.

Although the baseline scenario looks rather benign in terms of the BGR, note that the level of public debt in this scenario rises to a dangerously high level, reaching as much as 116% of GDP (see Table 3). It seems that the Polish government cannot continue running such a high level of budget deficit (equal to 4.3% of GDP, on average).

We are now ready to simulate the effects of changes in fiscal policy, including budget deficit, the way public debt is financed, and education expenditures.

Table 3. Permanent reduction in budget deficit equal to 1 or 2 percentage points of GDP

The BGR and structural indicators (%)	Baseline scenario $\xi = 4.3\%$	Scenario A1 $\xi = 3.3\%$	Scenario A2 $\xi = 2.3\%$
the BGR	3.69	3.78 (the cumulated effect after 30 years $= +2.3\%$ of GDP)	3.87 (the cumulated effect after 30 years $= +4.6\%$ of GDP)
C/Y	61.2	60.8	60.4
T/Y	33.9	33.7	33.5
S/Y	23.0	22.9	22.7
I_K/Y	19.9	20.3	20.7
G_C/Y	12.8	12.8	12.8
G_E/Y	5.6	5.6	5.6
G_T/Y	16.0	15.8	15.5
I_H/Y	0.64	0.65	0.67
K/Y	2.59	2.61	2.63
D/Y	**116.6**	87.3	**59.4**

Note: The assumptions of scenarios A1 and A2 are presented in section.

6. THE LEVEL OF BUDGET DEFICIT

Let us first present the results of scenario A1 and A2, where the government reduces budget deficit by 1 or 2 percentage points of GDP. Thus, in scenario A1 $\xi = 3.3\%$, whereas in scenario A2 $\xi = 2.3\%$ (from $t = 0$ onwards). Table 3 presents structural characteristics of the new balanced growth paths in both scenarios.

The baseline scenario clearly indicates that Poland cannot afford to maintain budget deficit equal to 4.3% of GDP, because over time the debt-to-GDP ratio would more than double and reach almost 117%. Thus, the baseline scenario is unacceptable from the point of view of the Polish constitution, which sets the 60% ceiling on public debt. Moreover, continuing current unbalanced fiscal policy is simply harmful for economic growth, as both alternative scenarios prove. In both scenarios the GDP grows faster (along the balanced growth path) than in the baseline scenario: reducing the size of budget deficit by 1 percentage point of GDP rises the BGR by approximately 0.1 percentage point. It seems to be negligible but remember that economic effects are accumulated exponentially over time. To better visualize these long-term (welfare) effects, Table 3 includes numbers indicating by how many percent GDP exceeds the baseline GDP after 30 years (in Table 3, numbers in parentheses). These indicators are calculated as follows:

$$\text{gain after 30 years} = \frac{Y(t = 30) \text{ in selected scenario}}{Y(t = 30) \text{ in the baseline scenario}} - 1.$$

Let us now focus on scenario A2. Note that reducing budget deficit to 2.3% of GDP would keep the debt-to-GDP ratio within the constitutional constraints: over time it should stabilize at 59.4%. After 30 years, GDP would be 4.6% higher than under the baseline scenario, which is not negligible at all. A precise (mathematical) as well as an intuitive (stylized) explanation of this fact has already been presented in Section 2. Recall that we had to distinguish 3 cases, and statistical data implicates that Poland falls into case A, as $(1 - \omega) = 0.565 > \gamma = 27.3\%$. As we explained in Section 2, a reduction in ξ (from $t = 0$ onwards) produces the *immediate effect* (raising the GDP rate of growth instantly) as well as the *gradual effect*, which is slowly (year after year) raising the GDP rate of growth further. Both effects are clearly visible in Fig. 4, which contains the trajectories of rates of growth in scenario A2.

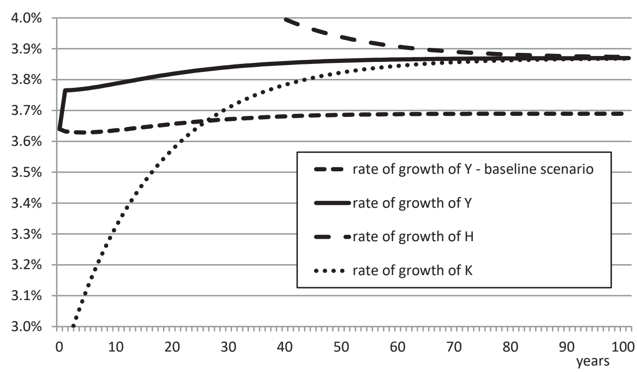

Fig. 4. Scenario A2 in comparison with the baseline scenario

Scenario A2 entails some noteworthy structural changes. A permanent reduction of budget deficit by 2 percentage points of GDP in the long run leads to an approximately twice smaller cost of servicing public debt, which results in a significantly lower overall tax burden (it falls from 33.9% to 33.5% of GDP). In the meantime, however, in order to keep public spending on education and public consumption at the same level as before (5.6% and 12.8% of GDP, respectively), the government decreases financial transfers to the private sector by 0.5 percentage points of GDP. Nonetheless, the lower tax burden coupled with lower borrowing needs of the public sector induces a beneficial change in the structure of private spending: an increase in investment (from 19.9% to 20.7% of GDP) coupled with a slight rise in private expenditures on education. The accelerated accumulation of both physical and human capital shifts the economy towards a higher balanced growth path. As a result, the BGR increases by 0.18 percentage points.

Finally, let us generalize these results further by assuming that the government can choose any value of ξ (as long as it is not negative). Fig. 5 presents the relationship between the BGR and ξ. Clearly, the optimal level of deficit is zero, which means that in order to maximize the GDP rate of growth, the Polish government should aim at balanced budget (at least on average over the long-run).

7. THE OPTIMAL STRUCTURE OF PUBLIC DEBT

Let us now return to the baseline scenario and see how it depends on the value of the parameter ω. Recall that in the baseline scenario it is equal to 0.435, because recently 43.5% of public debt was in the hands of foreign lenders. We may rightly wonder whether this is an optimal value. The solid-line curve on Fig. 6 presents the relationship between the BGR and ω, assuming that all other parameters are taken from the baseline scenario. Clearly, the bigger the foreigners' share of public debt, the higher is the GDP rate of growth. This is the result of recent statistics: recall that over the period of 2001–2016 the real rate of return on productive capital K was equal to 7.11%, whereas the real rate of return on 10-year government bonds in Poland was only 3.3%, which reflects a much lower (perceived) risk of lending to the government. The baseline scenario is a straightforward extrapolation of these statistics, so obviously such a significant discrepancy is an opportunity (a specific type of arbitrage): for the country as a whole, it pays to borrow from abroad, and invest the proceeds in productive capital (as well as education). It may, however, easily change, if the cost of borrowing by the government rises significantly and/or the rate of

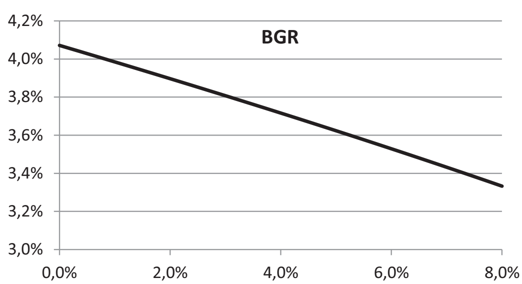

Fig. 5. BGR as a function of ξ

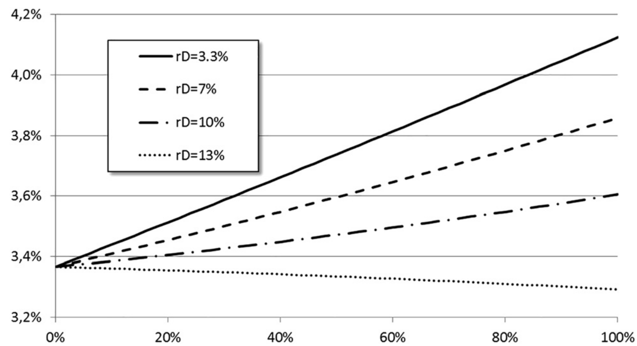

Fig. 6. The relationships between BGR and ω as r_D is rising

return on productive capital decreases. For example, as r_D rises, the relationship between the BGR and ω flattens out (see Fig. 6), and for sufficiently high values of r_D it may even become negative. Thus, if the cost of borrowing by the government is sufficiently high (in our simulations 12.3%), then our conclusion turns upside down: it pays to reduce the foreigners' share down to zero. In fact, in that case, the best long-run strategy is to reduce all of public debt to zero and run a balanced budget.

8. INCREASING PUBLIC AND PRIVATE SPENDING ON EDUCATION

This section contains four scenarios of increased investment in education:

B1. The government increases public spending on education by 1 percentage point of GDP at the expense of public consumption.

B2. The government increases public spending on education by 1 percentage point of GDP at the expense of financial transfers to the private sector.

C1. Private savings increase by 1 percentage point of GDP (at the expense of consumption), with an unchanged structure of investment expenditures (i.e. the same value of ψ). As a result, private investment in physical and human capital increases by a total of 1 percentage point of GDP.

C2. Private sector savings increase by 1 percentage point of GDP (at the expense of consumption), however additional savings are spent exclusively on education. (For this purpose, the value of ψ has been appropriately adjusted). Put simply, private spending on education increases by 1 percentage point of GDP at the expense of private consumption.

Table 4 presents the results.

Conclusions are similar to the results obtained in Konopczyński (2014). All three scenarios considerably outperform the baseline scenario. Importantly, scenarios B1 and C2 are better than B2 and C1, which means that the best option is to increase expenditures on education at the cost of public (scenario B1) or private (scenario C2) consumption. Both cases require a change in preferences of consumers: they must be willing to accept lower consumption (either public or

Table 4. Increasing public and private spending on education

The BGR and structural indicators (%)	Baseline scenario $\gamma_E = 5.6\%$ $\gamma_C = 12.8\%$ $\gamma = 27.32\%$ $\psi = 3.12\%$	B1 Increase in public spending on education by 1 pp of GDP $\gamma_E = 6.6\%$ $\gamma_C = 11.8\%$	B2 Increase in public spending on education by 1 pp of GDP $\gamma_E = 6.6\%$	C1 Increase in private savings by 1 pp of GDP $\gamma = 28.47\%$ $\psi = 3.12\%$	C2 Increase in private spending on education by 1 pp of GDP $\gamma = 28.47\%$ $\psi = 7.6\%$
BGR	**3.69**	**4.03 GDP effect after 30 years +10.2%**	**3.97 GDP effect after 30 years +8.1%**	**3.91 GDP effect after 30 years +6.8%**	**4.03 GDP effect after 30 years +10.1%**
C/Y	61.2	61.3	60.6	60.3	60.3
T/Y	33.9	34.0	33.9	33.7	33.8
S/Y	23.0	23.0	22.8	24.0	24.0
I_K/Y	19.9	20.0	19.7	20.9	19.9
G_C/Y	12.8	11.8	12.8	12.8	12.8
G_E/Y	5.6	6.6	6.6	5.6	5.6
G_T/Y	16.0	16.4	15.2	16.0	16.2
I_H/Y	0.64	0.64	0.63	0.67	1.64
K/Y	2.59	2.49	2.47	2.64	2.49
H/Y	1.20	1.31	1.32	1.16	1.31
D/Y	**116.6**	**106.6**	**108.4**	**109.9**	**106.7**

private) today (for some period of time) in exchange for higher consumption in the future. Scenario B2 is slightly worse than B1, because funds for additional public investment in education come from a reduction in public transfers to private sector (pensions etc.) rather than a reduction in public consumption. Therefore, in scenario B2, the disposable income of private sector, as well as its savings and investment are negatively affected. Finally, scenario C1 is worse than C2, because in scenario C1 extra savings of private sector (by assumption equal to 1 percentage point of GDP) are spent primarily on investments in physical capital (97%), and only 3% are invested in education, whereas in scenario C2 all extra savings are invested in education. If follows that, given the current conditions in Poland, it is much better to invest additional savings into education rather than into physical capital.

9. THE OPTIMAL STRUCTURE OF PRIVATE INVESTMENT

Recall that ψ represents the share of private savings devoted to education, and it has been very low in Poland – in the baseline scenario it is a mere 3.1%. As the previous section implies that

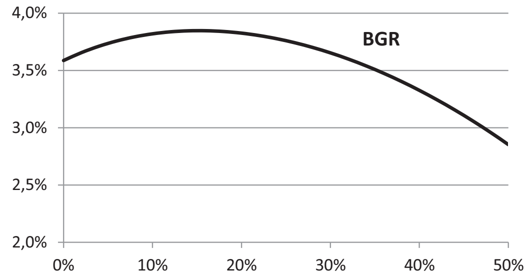

Fig. 7. BGR as a function of ψ

education is of crucial importance for growth, using the baseline scenario as a benchmark, we have calculated the BGR for any value of ψ ranging from 0 to 80%. (Higher values imply negative BGR, because investment in productive capital is so low that it does not compensate depreciation.) Fig. 7 presents the results. Note that the BGR reaches maximum equal to 3.85% at $\psi = 15\%$. Therefore, households in Poland clearly spend too little on education. However, as we argued earlier, official statistics regarding private education expenditures may well underestimate the true numbers, as large part of the private spending related to education is classified as consumption. Thus, it's hard to provide precise conclusions on that matter.

10. SUMMARY

We have proved that in the long-run our model converges towards the balanced growth path which is unique and globally asymptotically stable. The balanced growth rate (BGR) is a function of all parameters, however it can only be calculated numerically, as it requires solving a complex system of 2 non-linear equations. Despite this nuisance, some important qualitative conclusions can be drawn. For example, all tax rates are neutral, whereas the BGR is an increasing function of the rate of private savings, and a decreasing function of public consumption (as percent of GDP). Somewhat annoyingly, the relationship between the BGR and the remaining parameters of fiscal policy is ambiguous: it can be positive, negative or neutral, depending on the values of other parameters. To that end, we have provided some examples of detailed analyses supported by an intuitive explanation.

Empirical conclusions can be summarised as follows. In the period of 2000–2016, economic growth in Poland (on average 3.6% annually) was driven primarily by fast increase in human capital (growing at a rate of 5.4% per annum), and only secondarily by the accumulation of private capital (2.7% annually). The baseline scenario suggests that Poland will converge to the balanced growth path with GDP growing at the BGR equal to 3.7%. However, this rate depends on the long-run values of some instruments of fiscal policy. In particular, reducing budget deficit from the baseline value of 4.3% of GDP down to 2.3% would add approximately 0.2 percentage points to the BGR, which after 30 years would entail a cumulative effect of a 4.6% increase in GDP relative to the baseline scenario. Such a reduction in public deficit would also very significantly reduce the level of public debt.

The optimal financing structure of public debt depends on the relationship between the real rate of return on productive capital and the real rate of return on government bonds.

Straightforward extrapolation of the baseline scenario suggests that the Polish government should borrow from abroad rather than domestically. It may, however, easily change in the future.

Investing in human capital (education) is essential to economic growth. An increase in education expenditures by 1 percentage point of GDP would increase the GDP growth rate by approximately 0.34 percentage points, which after 30 years would entail a cumulative effect of a 10% increase in GDP relative to the baseline scenario. Whether the additional funding for education comes from a reduction in public or private consumption is almost irrelevant.

At present, in Poland only 3.1% of private savings is devoted to education. We show that in order to maximize the BGR it should be as much as 15%. Therefore, the current structure of private investment in Poland is far from the optimum. However, private spending on education is probably underestimated in official statistics – a substantial share of it is classified as consumption. Thus, it's hard to provide precise conclusions on that matter.

Despite methodological simplicity, our analysis provides qualitative and quantitative insights into positive effects of investing in education on economic growth in Poland as well as the negative consequences of excessive public deficit and debt. Our model captures certain 'stylized facts', especially the fast accumulation of human capital over the past 2 decades. Nonetheless, the model neglects certain phenomena which have been influencing the Polish economy. For example, on the one hand, Poland experienced large capital inflows – mainly in the form of FDI, portfolio investment, and EU convergence funds. On the other hand, there was a large migration from Poland to other EU countries. These two important facts are not included in our model, but they probably largely offset one another out.

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APPENDIX

Proof of Proposition 1

By definition, $d_F = D_F/Y$, hence $\dot{d}_F = \frac{\dot{D}_F Y - D_{Ff}\dot{Y}}{Y^2} = \frac{\dot{D}_F}{Y} - \frac{D_F}{Y}\widehat{Y}$. Recall that $\dot{D}_F = \omega\xi Y$, thus

$$\dot{d}_F = \omega\xi - \widehat{Y} d_F. \tag{A1}$$

For any given (positive) values of ω, ξ and $\widehat{Y}$, Eq. (A1) constitutes a linear differential equation of the form $\dot{d}_F = f(d_F)$. Fig. A1 presents the phase diagram of this equation. Obviously, regardless of the initial value of $d_F(t=0)>0$, over time $d_F \to \omega\xi/\widehat{Y}$.

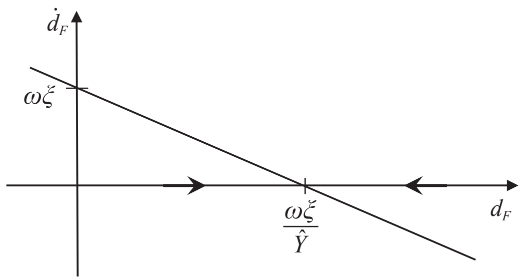

Fig. A1. The phase diagram of Eq. (A1)

Proof of Proposition 2

Let us first assume that $d_F = const.>0$. This assumption allows to proceed along the lines of proof of Propositions 1 and 2 in Konopczyński (2014). Recall that the rates of growth of private capital and human capital are given by Eqs. (34) and (35). Note that for a given (constant) value of d_F, we can treat $\widehat{K}$ and $\widehat{H}$ as functions of a single variable K/H. Provided that $E_1>0$, these functions have similar properties as their counterparts in Konopczyński (2014). In particular, the function $\widehat{K}(K/H)$ is decreasing and strictly convex. Moreover, $\widehat{K} \to_{K/H\to 0^+} +\infty$, and $\widehat{K} \to_{K/H\to+\infty} -\delta_K$. The function $\widehat{H}(K/H)$ is increasing, strictly concave, $\widehat{H}(K/H=0) = -\delta_H$, and $\widehat{H} \to_{K/H\to+\infty} +\infty$. The graphs of these functions are illustrated in Fig. A2. Due to the properties of these functions, there is exactly one point of intersection, i.e. there exists exactly one ratio $\overline{K}/\overline{H}$ for which $\widehat{K} = \widehat{H}$. The values of both functions at this point determine the balanced growth rate (the BGR). The balanced growth state is globally asymptotically stable, which is obvious from Fig. A2. In equilibrium $\widehat{K} = \widehat{H}$, which together with (2), implies that $\widehat{Y} = \widehat{K} = \widehat{H}$.

To summarize, on the one hand, we just proved that *for any given (constant) value of* d_F the economy converges to the balanced growth state where $\widehat{Y} = \widehat{K} = \widehat{H}$, which is unique and globally asymptotically stable. On the other hand, Proposition 1 implies that, along the BGR

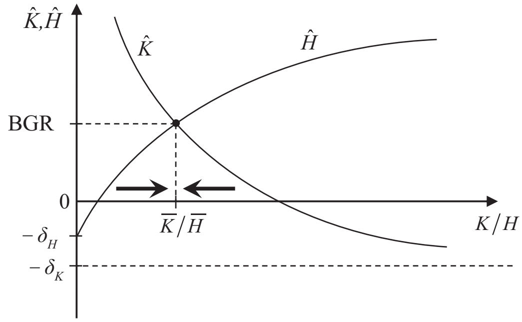

Fig. A2. Graphs of the functions $\widehat{K}(K/H)$ and $\widehat{H}(K/H)$

path, $d_F \to \omega\xi/\widehat{Y} = const$. These two facts implicate that there exists a unique balanced growth equilibrium, and it is globally asymptotically stable. In the steady state (hereafter denoted by overbar):

$$\overline{d}_F = \omega\xi/\widehat{Y} = const.>0 \tag{A2}$$