

# THE THEORY OF VARIATION IN STUDY SYSTEMS

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## **Abstract**

*Introducing new definitions for system and its main systemic properties, new evolution preservation principles, composition laws for systems, speeds, impulses, inertia, pressure, etc in n-dimensional space are stated. Also, new numerical methods in non-linear equations and new relaxing solutions for specific differential equations are presented, together with critical points, expansion of the studies in other applications in physics, automatics, robotics, quantum mechanics, biology, astronomy, ecology, etc. The composition laws for speeds, impulses, inertias, relative variation in n-dimensional space, can be considered relativity's theory develop, easy applicable in the researchers current activity, engineers, etc., having a strong integrator character of different scientific branches, promoting the idea of a future common methodology. The principle of total relative variation conservation is similar to the Hamilton's principle in the classic physics, and having a universal n-dimension expression is easily applicable in all applications, including engineering. The paper opens new horizons in scientific frontier research, sustaining the numerical methods development and the tendency of information digitization, of the standardizing methodological research.*

*Keywords:* system model; nonlinear; deterministic; dynamical; superposition; discrete.

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## 1. OBJECTIVES, IMPORTANCE, IMPACT

The fundamentation on new basis of the systems study, with principles, laws, methods and new knowledges. The new models system can be useful for researchers, projectors, students in the study of the current problems, or in some of the limits of the science problems. Considering that the fundamental request of any modelling, is to be able to reproduce better the observed dates, in the conditions of a more simply model.

## 2. MODELS, DIEFINITIONS, PRINCIPLES, PROPERTIES OF THE SYSTEMS IN THE N DIMENSIONAL DOMAIN

From G.Galilei to nowadays, the specialists susstain that the fundamental states of the matter is the moving and not the rest. Generally a system can be described in proportion with its components variation independently of the nature of these variations (space, energy, information, mass, speed, etc.). Depending of the organization level of the systems, of the interactions type, the variations can be regular or irregular. How a system in the most general case can cross different organizational level in the both directions from the presystemic level (for example sub quantum level characterized from the independent/equiprobable behavior of the components), passing from different levels of interdependence/action, and culminating in superior organizations such as symmetry, harmony, synchronism, etc., the variations of the components become more or less regular.

It has been ascertained that the superior levels of organization are characterized also through the equiprobability of the components actions. Practically has been ascertained that the construction process of the structure ( see the defining of the structure notion gave farther on) is of long standing, the decomposition can be sometimes very fastly (right explosive).

Generally we can imagine just a repetitive cycle of these levels of organization.

The systems can be rigorously defined if apart from relations of evolution are known included the in-out laws [5], [6], [7], [8].

In practice the finite physical systems often are modelled through equations, equations systems, identities.

For describing the system we will prefer one of the following descriptions:

a. In a metrical space

$$S(x_1, \dots, x_n) \subseteq \mathfrak{R}^n$$

With a system of nonlinear functions in the form

$$F_j = F_j(x_1, \dots, x_i, \dots, x_n) \quad (j = 1, \dots, m) \quad (1. 1.)$$

b.  $x$  state vector  $n$ -dimensional with components  $x_1, x_2, \dots, x_n$ ;

$f(x, t)$  field vector with the components  $f_1, f_2, \dots, f_n$  under the form:

$$\dot{x} = f(x, t)$$

Observation.[4]: a differential equation of superior order, under the form:

$$x^{(n)} + f(x, x', x'', \dots, x^{(n-1)}, t) = 0$$

Can be brought in the form b, if there are introduced the dependent variables  $x_1, x_2, \dots, x_n$  defined in this way:

$$x'_1 = x_2$$

$$x'_{n-1} = x_n$$

$$x'_2 = x_3$$

$$x'_n = -f(x_1, x_2, x_3 \dots x_{n-1}, x_n, t)$$

The elements of the variations can appear under different forms (space, information, energy, mass, etc.).

In the natural systems in some conditions the variation (space, information, etc.) it can be transformed in energy, and the energy can be transformed in the mass. The transformations can take place in the both directions.

Starting from the notions of hypervolume  $V(x_1, \dots, x_n)$  we define the operator of the relative variation. [1],[2]:

$$\Omega(V) = dV/V = \sum_{i=1}^n a_i (dx_i / x_i) = \sum_{i=1}^n a_i \omega_i \quad \omega(V) = \sum_{i=1}^n (dx_i / x_i) = \sum_{i=1}^n (\omega_i)$$

with the properties:

$$\omega(a.b) = \omega(a) + \omega(b)$$

$$\omega(a + b) \geq \omega(a) + \omega(b)$$

The main direction is attached to the term  $\max_i \sum_j a_{ij} \cdot \omega_j$

In the case of the classical systems the hypervolume can represents for example the function of transfer T of the system.

$$\Omega(V) = \Omega(T) = dT/T = \Omega(Y) - \Omega(X)$$

in which the variables Y are outputs, the possible functions objective, respectively variables X represent the inputs, the perturbations of the system.

In the situation of the other partitions ( states, external perturbations, commands etc.), the relation aforesaid can be completed ( the positive terms have output or objective character, and the negative terms have input character, perturbations, etc.)

In some special systems, the hypervolume can be defined in this way:

$$V = \prod x_i^{p_i}, \text{ the exponents } p_i \text{ can be weights or dimensional coefficients, concentrations, etc.}$$

The hypervolume can be introduced as a generating function of the system (1.1) defined in this way:

$$V = \prod F_j \quad \text{with the definition functions of the system.}$$

In the literature in accordance of  $\Omega(V)$  the systems can be classified as conservative (hamiltonians)

when  $\Omega(V) = 0$ , respectively dissipative with  $\Omega(V) < 0$  [9].

The specific punctual values  $a_i$  for the variables of the type in for a concrete system 1.1 will take the form

$$a_i = x_i \sum_j (1/F_j) \cdot (\partial F_j / \partial x_i)$$

If the laws of the interior interaction are completed with the relations of interactions with the exterior environment, we can define a **model of complete system**.

If the system 1.1 corresponds to the conditions

$$F_j(x_1, \dots, x_i, \dots, x_n) = c_j \quad (j = 1, \dots, m)$$

And  $\Omega(V) = 0$  (elements  $dx_i/x_i$  unique)

A complete system is formed.

This system – apparently isolated – [9] similar to the systems orientated to objects from informatics includes even the effects of the exterior world.

The complete systems have specific properties, for example are not sensible at perturbations, etc., they have oscillating properties, increasing the causal effects, the functional laws usually are valid including in the asymptotic limit domains etc. We can obtain a form of complete system, for example if we express the relations of the system under the form of equalities, identities.

For example, from the relation  $\lambda = \Omega T$  known in physics, we can obtain the forms  $E = (\Omega T / \lambda) \equiv 1$  and  $d\Omega / \Omega + dT / T - d\lambda / \lambda \equiv 0$  where  $\lambda$  represents wavelenght,  $\Omega$  angular speed, T period.

**The relaxing state.** The distant states of relaxation of the systems are the asymptotic states, where because of the high entropy, at too low energies or too high, becomes dominant the interdependence/probability equal with the components and it must be checked the laws validity of initial working. It could be demonstrated that the relaxing states for a 1.1 system (to which tend asymptotic the free evolution, even if it will not reach this state especially), it can be obtained with the system of relations:

$$\Omega(V) \longrightarrow \omega(V), \quad a_i = x_i \sum_j (1/F_j) \cdot (\partial F_j / \partial x_i) = \pm 1.$$

These solutions can supply the final states of relaxing, the evolution direction, respective, the necessary time until the relaxation.

It can be observed that for the independent components with  $V = k \cdot \prod x$ , results  $\Omega(V) = \omega(V)$ .

It can be noticed that the differential equations can be brought to the form  $\Omega(V)$  through elementary transformations, then compared with the form  $\omega(V)$  can be obtained directly the expressions  $a_i$ . In this way, for the individual differential equations can be obtained the expressions  $a_i$  more simply.

For example, in the case of classical volumic dilation, the classic law of dilation is valid until the volume reaches  $2V_0$  proper to the state of relaxation. In the case of some sistem with fluid structure, it is possible that the extreme point of relaxing is a point of working of the system.

Analogous, in the annex, are illustrated, starting from the equation Srödinger for a system with slow interaction, new relations for the energetic levels of relaxing (asymptotics), asymptotic solutions of new type for the differential equation Ricatti, used in automatics and physics, asymptotic solutions for the general harmonious motion, asymptotic expression for the first fundamental square form of the surface in the differential geometry.

### Critical points ( of equilibrium)

Analogous the conception of proper values from the theory of the liniar systems, we introduce for the nonlinear systems the critical points, like those points in which the relative variation reaches values with integer numbers:

$$(dx_i / x_i) = p_i \approx (\pm 1)^* k, k = 0, 1, 2, 3, \dots$$

For discret values we can take approximately:

$$0 \text{ if } (dx_i / x_i) < 0,5$$

$$P_i = 1 \text{ if } (dx_i / x_i) \in [0,5-1,5)$$

$$2 \text{ if } (dx_i / x_i) \in [1,5-2,5) \dots\dots$$

These points help us at the determination of the attraction points (attractors) of the system. So, a system can have more critical points. In the current actual practice it is used the proper point of the value  $k=0$ . Some authors are calling the critical points as points of local equilibrium.

The systems, in the evolution course, can reach more critical points. Between critical points, become characteristic the undulating properties with high speeds, until when around the critical points the speeds decrease, it becomes a priority the material (inertial) properties.

Because of the undulating conduct which occur at high speeds ( at smaller temporal intervals ), the evolution has a leap character between the critical points. In the case of interaction of two or more systems, the evolution tends to critical (new) points, which can be points of accumulation of type cycle limit, with entropy and proper potential energy:

$$\left\{ \frac{\Delta\lambda_i}{\lambda_i} \right\} \Rightarrow \frac{\Delta\Lambda}{\Lambda} = 0,1,2,\dots$$

For example, for two systems: 
$$\frac{\lambda_2 - \lambda_1}{\lambda_2} - \frac{\lambda_2 - \lambda_1}{\lambda_1} = -\frac{\lambda_2 - \lambda_1}{\lambda_2} \times \frac{\lambda_2 - \lambda_1}{\lambda_1} \Rightarrow (-k), k \in \mathbb{Z}$$

Between the critical points, the evolution is possible only at the reaching of some energetic thresholds  $\Delta E = h \cdot \nu$ .

The complex systems, without the dinamical components, contain both inertial components and functional components. Due to the variation of the wavelength ( $\Delta\lambda_i$ ) are obtained results with changes of speed. It is possible that through a successful superior position of the luminous sources, can be realised luminous motors?

It is extremely important the correct defining of the equation of evolution. The results in the domain of chemical kinetics underline that, to obtain oscillator evolutions is necessary at least two relations and a supplementary condition, some relations being nonhomogeneous. The presence of the attraction zones, of the limit cycles, characteristic for the systems with proper organization, imposes the necessity of the existence of the nonlinear character, of a spatial and temporal order. If it is kept the place of the components,  $\omega_i$  in their growing succesion, the system have not a chaotic conduct. [9]. The converse is not true, because symmetrical pairs of some oscillations can change the place in temporary succesion. By replacing the pair with only a variation, we can still obtain a growing succesion. If the inferior term of the succesion changes the place, appear temporal salts. Condition of uncertainty type can also appear because of the insufficient number of relations. We are underlining that the theory of catastrophes through the study of the ramifications approaches incomplete systems (fragments of system) with a insufficient number of relations, without equilibrium states (example poles in the case of liniar systems). In the case when the number of the relations is large, comparable with the number of variables, the importance of the interactions is growing, the group and the phase effects are with approximation identically, the speeds do not pass the speed of the light in vacuum. At the reduction of the relative variations at the level of the values threshold of the critical points, the afferent energetic levels can disappear (the phenomenon of contraction). At the systems with large number of variables, with less relations, the case of weak interactions, to which the volume increases, the inertia diminishes, the speeds of phase can pass the speed of light in vacuum. It is expected that at the systems with weak interactions, the critical points would have monochromatic characteristics. The system can be presented simultaneous through diverse essential proper critical points (example the independent systems). The existence of the positive or negative reaction can be appreciated according the positive or negative sign of the product of relative variations  $\prod \omega_i$ , respective of the coefficients  $\prod a_{ij}$ .

If the product is negative and also exist positive factors, the evolution is chaotic (the motivation is similar to the utilization of the Liapunov exponents [9]).

### Conditions of periodicity

It could be illustrated that similar with the Barchausens condition from the study of the electric oscillations, if it is added a negative reaction  $\beta$ , the necessary condition of periodicity (symmetry) becomes  $\sum (dx_i / x_i) + (d\beta / \beta) = 0$ , or  $\forall \beta = \pm 1$ ,  $\forall$  representing the hypervolume, respective the function of the transfer of the system. In practice it can be built new experiences with pairs of synchronizing symmetries, which are giving rise to rotations.

At a series of systems well definite (for example at the complete systems) the components have different variations but equal speeds (for example the complex chemical systems, the speed of the colors components of the white light, the speed of the human parameters which is satisfying the Weber law of the equality of thresholds of human sensibility).

### Conditions of causality, controllability

We are telling that  $x_1$  depends causal of  $x_2$  if  $\omega(X_1) \sim \omega(X_2)$

The system is controllable if it  $\sum \omega(y_i)$  can be brought at zero from any point through  $\omega(x_i)$ .

We have to observe that in the conditions above it is not necessary to know the relations form of interdependence.

### The synthesis of the evolution relations

Starting from a classic series of powers

$$y = y_0 + a_1 \cdot h + a_2 h^2 + a_3 h^3 + \dots + a_n h^n = y_0 ( 1 + (a_1 / y_0)h + (a_2 / y_0)h^2 + \dots + (a_n / y_0)h^n )$$

...

It can be illustrated a relation of type error of form

$$(y - y_0) / y_0 = (a_1 / y_0)h + (a_2 / y_0)h^2 + \dots + (a_n / y_0)h^n = b_1 h + b_2 h^2 + \dots + b_n h^n )$$

On the other side, a relation of error can be also obtained through the expression of a relation of type Tailor of form:

$$y = y_0 (1 + c_1 \cdot h (1 + c_2 h (1 + c_3 h \dots)))$$

$$(y - y_0) / y_0 = c_1 \cdot h + c_1 c_2 h^2 + c_1 c_2 c_3 h^3 + \dots + c_1 \dots c_n h^n \quad c_n = (1/n) \cdot (f^n / f^{n-1}) \Big|_{x_0}$$

$$c_1 = a_1 / y_0 \quad c_1 c_2 = a_2 / y_0 \quad \dots$$

Having recurrence relations:

$$c_n = a_n / a_{n-1}$$

We are recommending the last development, which has a high convergence speed, it is easily to be used inclusive in the discrete domain.

Expressing one by one error relation of the above type for the essential variables of the system, replacing the observed/measured values for these variables in these relations, we can achieve the superposition of these expressions in one general expression for example through the next method:

a.  $\varepsilon = [(Y - Y_0) / Y_0] = 1 - \prod (1 - \varepsilon_i)$  – after the composition law of the errors of grade 1.

$$\begin{aligned} \varepsilon = \sum \varepsilon_i - (\varepsilon_1 \cdot \varepsilon_2 + \varepsilon_1 \cdot \varepsilon_3 + \dots + \varepsilon_1 \cdot \varepsilon_n + \\ + \varepsilon_2 \cdot \varepsilon_3 + \varepsilon_2 \cdot \varepsilon_4 + \dots + \varepsilon_2 \cdot \varepsilon_n + \\ \dots \\ + \varepsilon_{n-1} \cdot \varepsilon_n) + \\ (\varepsilon_1 \cdot \varepsilon_2 \cdot \varepsilon_3 + \varepsilon_1 \cdot \varepsilon_2 \cdot \varepsilon_4 + \dots + \varepsilon_{n-2} \cdot \varepsilon_{n-1} \cdot \varepsilon_n) + \end{aligned}$$

$$\dots\dots\dots$$

$$(-1)^{n+1} \cdot (\varepsilon_1 \cdot \varepsilon_2 \dots \varepsilon_n).$$

Example for  $n=2$ ,  $\varepsilon = (\varepsilon_1 + \varepsilon_2) - \varepsilon_1 \cdot \varepsilon_2$

**For a total error of maximum 5%, in the case for example  $n=4$ , it is necessary that the error of the components has not to pass 1%.**

**The correspondence relation**

We can name as a correspondence relation between the classic models of system and of those proposed (including the relativist models) for example the HUBLE law from astronomy:

$$v = c \cdot \frac{d\lambda_i}{\lambda_i} \quad \text{and generalising}$$

$$x_i = n \cdot \lambda_i \quad \text{if} \quad \lambda_i < x_i \quad \text{and,}$$

$$\lambda_i = k \cdot x_i \quad \text{if} \quad x_i < \lambda_i \quad i = 1, n$$

Finally we obtain

$$v = c \cdot [\Omega(V) \pm \Omega(r)] = c \cdot [\omega(V) \pm \omega(r)], a = \sum b_i v_i$$

where  $v$  is the speed,  $c$  the light speed in vacuum, respective  $\lambda_i$  wavelenght.

The above expression expresses the composition relation of the speeds in the  $n$ -dimensional – respective at a correct defining of the functions,  $[1,1]$  including the effects of the extern average, when the system on the whole is not moving, so  $v = 0$  – it is speaking about the preseving of the total speed. If the total speed is not equal with 0, for example the system is not defined complete, the system or the ether can move. The values  $\omega(r)$ , are  $\Omega(r)$  reference system, (in the upper example are expressions of the values  $n$  and  $k$ ). The sign  $+$  have the terms with  $x_i < \lambda_i$ , respective the sign  $-$  have the terms with  $x_i > \lambda_i$ . The condition  $\omega(r) = 0$  correspond to the case  $x_i = \lambda_i$ .

The expression  $a_i$  was defined before. The upper relation was determinated for the validity domain of the Huble relation.

According to the above defined relation, the speed has components of every variable element of the hypervolume, irrespective of their physique significance. Another correspondence can be expressed through the transfer function concordant to the upper relation.

**The interaction**

If  $dx_i / x_i \ll p_i$ , and  $(dx_i / x_i)$  is the movement measure, the expression  $p_i - (dx_i / x_i)$  can be the inertia measure around the critical point  $p_i$ . The interaction measure is the product of the movement and inertia:

$$S = \sum_{i=1}^n \sum_{j=1}^n a_i (dx_i / x_i) \cdot (p_j - a_j (dx_j / x_j))$$

$$\text{or, } S = [P \cdot \Omega(V) - (\Omega(V))^2]$$

The condition of realizability involves an finite interaction for every temporary interval:

$$S < \text{const.}$$

It can be observed that in the case  $S = 0$  we have two solutions:  $\Omega(V) = 0$  and  $\Omega(V) = P$

Where the  $p_j$  values correspond to the critical points value of the variables  $j$ .

The expression of the relative impulse:

$$H = v * [P - (\Omega(V) \pm \Omega(r))] = c * \left[ [\Omega(V) \pm \Omega(r)] * P - [\Omega(V) \pm \Omega(r)]^2 \right]$$

In the same way there can be expressed new relations for the pressure, density, thermic conductivity, etc.

**The superposition of the inertias** is also very important:

$$\Omega(V) = dV/V = \sum_{i=1}^n a_i (dx_i / x_i) = \sum_{i=1}^n a_i * \omega_i \quad \omega(V) = \sum_{i=1}^n (dx_i / x_i) = \sum_{i=1}^n (\omega_i)$$

$$P = \sum_{i=1}^n (p_i)$$

$$I = P - \Omega = \sum (p_i - a_i \omega_i) = 1 - \prod_{i=1}^n (1 - p_i + \Omega_i) \Rightarrow$$

$$\Omega = P - 1 + \prod_{i=1}^n (1 - p_i + \Omega_i)$$

The expression of the inertia theory is suggested by the law of the error composition of the first rank from the error theory.

In this way we expressed the superposition of the relative variations, so the subsystems superposition.

Through the observed/measured variables composition brought at the error form, we can realize the subsystems superposition in only one relation.

The Einstein's energy-mass relation following the results presented in the upper part can be expressed in this way:

$$\Delta E = \Delta m * c^2 = c^2 * (P - \Omega) = c^2 * \left( 1 - \prod_{i=1}^n (1 - p_i + \Omega_i) \right)$$

Using an estimator of the form  $\varepsilon = \min(\mathbf{p} - \boldsymbol{\omega})$  having controls of the type  $\mathbf{u} = \mathbf{p}$ , the theory of the variational calculus can be reformulated, the maximum principle, the study of the optimal filters (the **Wiener-Hopf** equation).

### **The principle of preserving the total relative variation**

In the case of the systems can be defined the principle of preserving the total relative variation, which explains the possible variations between the variables with different physical significants with the condition of the constant keeping or at minimum level of the total relative variation. ( in the annex is explained the levitation possibility)

$$\sum_0^V S = \sum_0^V \left[ P * \Omega(V) - (\Omega(V))^2 \right] = \text{MIN/CONST}$$

The principle reminds of the Hamilton principle from physics, but having only a unique form becomes easily applicable in practice, even at the discret systems. The minimization can be realised for example even in the discret case following the principles of dynamic programming.



### The adaptability

The low changing elements are called strong elements (pulley). In order that the system can be adaptable at soft elements (faster changing, more unstable), it is necessary that it contains soft elements in sufficient relative quantity and variation

$$\sum_{j=1}^p \frac{dx_j}{x_j} \geq \sum_{i=p+1}^n \frac{dx_i}{x_i}$$

Where p is the number of the terms of the soft elements.

The relative variation of the hypervolume is function of strong and soft elements:

$$\omega = \sum_{j=1}^p \frac{dx_j}{x_j} + \sum_{i=p+1}^n \frac{dx_i}{x_i}$$

We can particularize the Boltzmann relation of entropy under the form:

$$S = K_B \ln\left(\sum_{j=1}^p \frac{dx_j}{x_j} / \sum_{i=1}^n \frac{dx_i}{x_i}\right)$$

We can estimate the measure of the system adaptability and through the minimum number of interactions, the relative variation threshold  $\omega_p$  necessary, the necessary time, upon that, the soft elements produce structural variations in the system.

The adaptation is a dissipative process, which diminishes the entropy, (diminishes the relative hypervolume of the system too).

### The evolution preserving law

Generalising the inertia law of Newton from physics, we are defining the evolution preserving law even in the interaction conditions:

A system is preserving the continue evolution if it is preserving its curve form of the evolution.

We can sustain for example that the system evolution is according at the evolution curve, if the past, the present, the future of the system is on the curve.

The system evolution can change as a result of the intern working, or of the anisotropy changing of the environment. Omitting the environment changing, with regard to the evolution preserving law we can stipulate only the indispensable conditions. For the formulation of the sufficient conditions must be included in the model inclusive the environment variations. In the case of a complete system this is possible if there are known the variation laws of the environment. We can enumerate as a necessary conditions for example:

-if the system is keeping its form of the evolution equations

-if it is satisfying an invariate expression, which contains at least some three consecutive points of the trajectory, etc.

Similar to the notion of curvature of the space from physics (inclusive the relativity theory) we will be able to define expressions of evolution threshold, after that we can characterize the movement, for example on the form:

$I(k) = 1$  stable linear evolution on the initial curve (every k point of the trajectory)

$I(k) = \text{const} \neq 1$  nonlinear stable evolution of constant curvature

$I(k) \neq I(k-1)$  variable, k discontinuity point of the trajectory.

A first relation can be defined for example as:

The curvature ratio

$$I(k) = (x_k - x_{k-1})(x_k - x_{k-2}) / (x_{k+1} - x_{k-1})(x_{k-1} - x_{k-2})$$

**of 4 consecutive points** of the trajectory are taken at equal temporal intervals. It can be verified that the upper relation has the invariate value equal with 1 for the linear systems and it respects the curvature conditions upper defined. **The upper conditions can stand at the durable evolution verification base.**

We have to remember that the relativity theory sustains the preservation of the equations form of evolution for the inertial systems.

### The stability condition

If the hypervolume  $V > 0$  and  $\omega(V) \leq 0$ , the system is stable in Liapunov sense [6].

So, the conditions  $V > 0$  and  $\omega \leq 0$ , can be used for the analysis of the systems stability.

We have to remark that in this way we can analyse the stability without knowing the evolution law, only on the base of measured or observed variables.

### The symmetry

Concordant to the theory of differential equations, a system is symmetrical if it is satisfying the condition:

$$(dx_1 / x_1) = (dx_2 / x_2) = \dots = (dx_n / x_n)$$

**For the critical points of the symmetrical systems, is valid the expression:**

$\prod \frac{\Delta \lambda_i}{\lambda_i} = k$  integer number. The relation is known in the  $\{\mathbf{P}, \mathbf{V}, \mathbf{T}\}$  space from thermodynamics.

### The synchronism

The evolution is called synchronic if the different elements  $(dx_i / x_i)$  own similar importance (probability), so are satisfying the relation  $\sum \alpha_i (dx_i / x_i) = \text{const}$  in the conditions

$$\alpha_1 = \alpha_2 = \dots = \alpha_n .$$

### The structure, the observability

In universe everything varies. The variaton measure and the observers resolution are the conditions for which depends what is the structure and what is the variation. In these conditions the variable  $x_i$  is an structural element of the system if is satisfying the condition:

$$\max \left( \frac{dx_i}{x_i} \right) \leq C_0$$

where  $C_0$  is a constant of observer, a level that can still be observed as a variation.

The structure of the system is composed from that  $x_i$  elements, that are satisfying the upper condition, so are appearing in the description of the system model.

Similar to the anterior definition we are defining the **variation observability**:

The variation of the element system  $x_i$  is observable, if

$$\min \left( \frac{dx_i}{x_i} \right) \geq C_m \qquad \max \left( \frac{dx_i}{x_i} \right) \leq C_M$$

and

where  $C_m$  and  $C_M$  depend of the observer capacity.

Generally  $C_0 \leq C_M$  but the visible domain can contain structural elements with variations, respective everything that belongs to the definite interval of the two upper limit values.

The structure is created in long-standing time, and usually is breaking up suddenly. The science studies detailed the matter disintegration, but it is studied less the transformation process of the energy in matter.

### Spectral interpretations

It is obvious that in some communication systems in the case of very weak signals, it is recommended the utilization of the relative expressions, for example in the case of a  $x$  signal of order of size  $\sim 10^{-12}$  m and  $dx \sim 10^{-14}$  m even the signal identification is easily remaking the signal under the form:

$$\frac{dx_i}{x_i} \quad (\sim 10^{-2} \text{ m}).$$

With the help of the spectral variables, we can classify the systems after their state  $(i=1,n)$ :

- a)  $\text{mod}\left(\frac{dx_i}{x_i}\right) \ll 1$  processes with inertial states
- b)  $\text{mod}\left(\frac{dx_i}{x_i}\right) \approx 0,1, 2, 3, \dots$  (integer number) – processes with critical states
- c)  $\text{mod}\left(\frac{dx_i}{x_i}\right) > 1$  states of avalanche, oscillatory, explosive, catastrophies

### A solution of the system model (1.1)

As we know the probabilities of the operational research, specially the methods of improvement without restrictions, a system of nonlinear relations of the type (1.1.) can be solved easier searching the solution of a equivalent system of type

$G_j = F_j(x_1, \dots, x_i, \dots, x_n) - c_j = 0 \quad (j = 1, \dots, m)$  under the form

$$\min \left( \Phi = \sum_{j=1}^m G_j^2(x_1, \dots, x_i, \dots, x_n) \right)$$

Using a known computing method (for example the method of the n-dimensional simplex) [1],[2], [11].

The solvation of a nonlinear relations system through the minimization of a functional, appart from the method generality is advantageous even through the fact that it does not presume conditions between the number of relations and the number of variables, there can be introduced new variables and relations (for example for the replacing of a nonlinear differential relation with a set of linear relations).

If we include in the expression of the functional an objective function expected from minimum type, we can obtain even optimum solutions. The possibility of the generalization in the case of the multiobjective problems is obvious. Combining the method with the relaxation principle and with the evolution description with new dynamic series of rapid convergence similar to the Volterra type relations, we can solve problems of variational type too.

In literature [4] are known replacing methods of some nonlinear relations of type (1.1) with sets of linear relations.

The minimization of the functional combined with the  $n$ -dimensional simplex method is advisable, not existing continuity and derivatibility conditions, so in general are avoided the minimum local cases. In the annex we present a concrete example using this method.

In treating the upper problems we do not need the mass notion. A series of other problems can make the object of a future article.

### 3.APPLICATIONS

#### 3.1 THE MODEL SOLVATION OF COMPLETE SYSTEM

In the next part we expand the numerical method from the chapter [2,2] of rezolving a nonlinear system of equations:

We can consider a nonlinear system:

$$F_j(x_1, \dots, x_i, \dots, x_n) = 0 \quad (j=1, \dots, m) \quad 3.1$$

We construct the equivalent problem for the calculus of the values  $x_1, \dots, x_n$ , which provide the functional minimization:

$$\min \left( \Phi = \sum_{j=1}^m F_j^2(x_1, \dots, x_i, \dots, x_n) \right) \quad 3.2.$$

Where the functions are given under the form (3.1). We have to observe that the solution  $x_1, \dots, x_n$ , of the system (3.1) provide in the same time the functional minimum (3.2), is true the reciprocal too. In these conditions the solvation method can be replaced with the problem of minimization of a functional without supplementary restrictions. The last problem is more general and easier, using one from the known methods of improvement without restrictions, for example:

- the method of unidimensional variation
- the Box method
- the Hooke-Jeeves method (pattern search)
- the Rosenbrock method
- methods of gradient type
- the method of type Newton-Rapson
- methods of contraction, etc.

The Box method (in literature known as different denominations: the Spandley method or Hex method, or the  $n$ -dimensional simplex), does not presume the continuity, the derivation of the functions. In the  $n$ -dimensional space we construct a regular figure with  $n+1$  vertexes with equal distances between vertexes named simplex (for example in the three-dimensional a tetrahedron, etc.). We calculate the  $\Phi$  in the simplex vertexes. The afferent vertex to the highest value, repetitive we replace with a vertex of opposite sense, symmetrical, obtaining a new simplex. Repeating the method we skip the anterior selected vertexes, in case of need for precision we reduce the dimension of the simplex side, obtaining the vertex of expected value. In literature [11] are indicated the expressions of obtaining the simplex vertexes, of choosing the new vertex.

The model solving steps are:

- We read the number of variables ( $n$ ), the initial size of the  $n$ -dimensional ( $a$ ) simplex side, the error threshold ( $\epsilon$ ) for the  $\Phi$  calculus, the maximum number of iterations ( $t$ )
- We construct the vertexes  $V_1, \dots, V_i, \dots, V_{n+1}$  of the simplex, in this way:

$$V_i = V_i(x_i^1, \dots, x_i^j, \dots, x_i^n) \quad (3.3).$$

$$p = \frac{a}{n\sqrt{2}}(n-1+\sqrt{n+1})$$

$$q = \frac{a}{n\sqrt{2}}(-1+\sqrt{n+1})$$

$$x_i^j = \begin{cases} 0 & \text{if } i=1 \\ q & \text{if } j+1 \neq i, i > 1 \\ p & \text{if } j+1 = i, i > 1 \end{cases}$$

■ We calculate the  $\Phi$  value in every vertex of the simplex

■ We choose the highest value (the worst possibility) of  $\Phi$  in the simplex vertexes, vertex that was not selected before and we keep its value (  $x_i^R$  )

■ We calculate the new symmetrical vertex in the place of the selected one for elimination (this will be the image of the vertex that have to be eliminated in the mirror of the other vertexes) using the relation (3,4)

$$\blacksquare \quad x_i^N = \left[ \frac{2}{n} \left( \sum_{j=1}^{n+1} x_i^j - x_i^R \right) \right] - x_i^R \quad (i=1, \dots, n) \quad 3.4.$$

■ We repete the algorithm from the point 3 until the way out condition in one of the vertexes is satisfied,  $\Phi \leq \varepsilon$  or the iterations number has reached the t value.

■ We post the point coordinates, the  $\Phi$  value, the iterations number,  $\varepsilon$ , t, a values

If the approximation is not acceptable, we reduce the size of the simplex side, and we continue from point 3 omitting the selected vertexes choosing a less bad vertex.

There are not restrictions between the number of relations and the number of variables. If the problem has more solutions, in literature are often introduced supplementary aim functions, obtaining optimum solutions [16].

## NUMERICAL SIMULATION

In the presentation we use the research results [16] realised with the support of Cluj Sapientia University in 2004. We are foreshadowing the solvation of a nonlinear electric network open with the relations system:

$$\begin{cases} 4x^3 - y^3 - 3x^2y + 3xy^2 + y - 5 = 0 \\ 3x^3 + y^2 + x - 9 = 0 \end{cases} \quad 3.2.1.$$

We construct the functional (3.2):

$$\Phi = F_1^2(x, y) + F_2^2(x, y) =$$

$$= [4x^3 - y^3 - 3x^2y + 3xy^2 + y - 5]^2 + [3x^3 + y^2 + x - 9]^2 \quad 3.2.2.$$

For the functional minimization we have used the Box method (of the n-dimensional simplex) starting from the point (0,0) with a side equal with 2, the partial results being reproduced in the next table:

The simulation steps.

Table nr.1.

The iteration	X	Y	$\Phi$
$a = 2$	$p = 0,5174$	$q = 0,9314$	
1	0,000 0	0,000 0	106, 000
2	0,517 4	1,931 4	617, 781
3	1,931 5	0,517 4	48,8 40
4	-1,41 40	1,414 0	1488 ,000
5	1,931 4	0,517 4	617, 781
$a = 0,1$	$p = 0,0965$	$q = 0,0258$	
1	0,517 4	1,931 4	48,8 40
2	0,613 9	1,957 2	672, 117
3	0,543 2	2,127 9	48,1 40
4	0,446 7	2,002 1	60,4 60
5	0,613 9	1,957 2	37,6 80
6	0,639 8	2,053 8	36,1 37
7	0,710 5	1,983 1	26,7 74
8	0,736 4	2,079 7	24,6 99
9	0,808 1	2,009 0	16,4 75
0	0,833 9	2,105 6	14,2 77
1	0,905 6	2,034 9	7,71 1
1	0,931 5	2,131 6	5,92 4
2	1,003 2	2,060 9	1,71 2
3	1,029 1	2,157 8	1,02 3
4	1,100 8	2,087 1	0,22 7
5			

In the beginning we have used the simplex side  $a = 2$ , but because the functional value has repeated from the vertex 5, it was necessary the continuation from point 3 with a smaller side ( $a = 0,1$ ). After 15 iterations we obtained the solution  $x = 1,1008$  és  $y = 2,0871$ , which would be possible improving only with a smaller side. For verification, we obtained  $\Phi$  minimum with the packet of MATLAB programs. After 154 iterations, on the next graph can be observed that the upper point is one of the problem solutions, the calculating solution being:  $x = 1,0831$  și  $y = 2,0260$ .

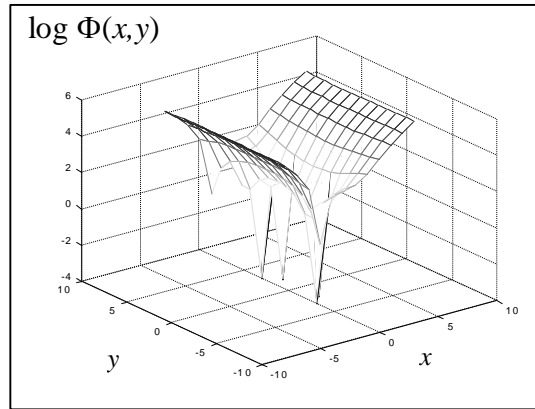


Figure 1. The graphic representation of the functional

### 3.2. Relaxing conditions, ( the tendency towards independence/equiprobability)

$$\omega_R = \frac{dV}{V} = \sum_{i=1}^n \varepsilon_i \frac{dx_i}{x_i} = const.$$

$$\varepsilon_i = \varepsilon_k$$

$$k = 1, n$$

**The gases dilation:**

$$V = V_0(1 + \gamma \cdot \Delta t)$$

$$V - V_0 / V_0 = \gamma \cdot \Delta t \rightarrow$$

$$\gamma \cdot \Delta t = 1$$

$$V_{\max} = 2 \cdot V_0$$

**The harmonious movement:**

$$J\varepsilon'' + 2D\varepsilon' + t\varepsilon = M$$

$$\Rightarrow \varepsilon' J \frac{\varepsilon''}{\varepsilon'} + t \frac{\varepsilon'}{\varepsilon} \frac{\varepsilon}{\varepsilon'} + 2D \frac{\varepsilon'}{\varepsilon} \varepsilon = M$$

$$\varepsilon' \cdot J = t \frac{\varepsilon^2}{\varepsilon'} + 2D\varepsilon$$

$$\varepsilon'^2 \cdot J = t\varepsilon^2 + 2D\varepsilon\varepsilon' \Rightarrow$$

$$\varepsilon' = [2D \cdot \varepsilon \pm 2\varepsilon(D^2 + Jt\varepsilon^2)^{0.5}] / 2J$$

### The thermodynamic system

$$dq = du + dw$$

$$\frac{dq}{q} = \frac{du}{u} + \frac{dw}{w}$$

$$\frac{u}{q} = \frac{w}{q} = 1$$

$$u = w = q$$

### Quantum power levels

The wave function  $y(x)$  satisfies the Schrodinger equation

$$y'' + \frac{2m}{h^2}(E - V(x))y = 0$$

$$\frac{y''}{y'} + \frac{2m}{h^2}(E - V(x)) \frac{y}{y'} = 0$$

$$\frac{2m}{h^2}(E - V(x)) \frac{y^2}{y'^2} = 1 \Rightarrow$$

$$y'^2 = \frac{2m}{h^2}(E - V(x)) y^2$$

The asymptotic solution of the Ricatti equation:

$$R u''(x) + R' u'(x) + [Q' - P] u(x) = 0$$

$$R \frac{u''}{u'} + R' u' + [Q' - P] \frac{u^2}{u'} = 0$$

$$R \cdot u' = R' u + [Q' - P] \frac{u^2}{u'} \Rightarrow$$



$$u'_{1,2} = [R' u \pm u(R'^2 + 4R(Q' - P))^{0.5}] / 2R$$

**Asymptotic expression for the first fundamental square form of the surface from the differential geometry.**

$$ds^2 = Edu^2 + 2Fdudv + Gdv^2$$

$$\frac{1}{u^2 v^2} \frac{ds^2}{s^2} - \frac{E}{s^2 v^2} \frac{du^2}{u^2} - \frac{2F}{s^2 u^2 v^2} \frac{du dv}{u v} - \frac{G}{s^2 u^2} \frac{dv^2}{v^2} = 0 \Rightarrow$$

$$E = -\frac{s^2}{u^2} \quad G = -\frac{s^2}{v^2} \quad F = -\frac{s^2}{2udv} \quad \text{replacing in the initial relation} \Rightarrow$$

$$\frac{ds^2}{s^2} = -\left(\frac{du^2}{u^2} + \frac{dv^2}{v^2} + \frac{du}{u}\right)$$

### 3.3. A LEVITATION, THE INERTIAS COMPOSITION

$$I = P - (dV/V) = P - \Omega = 1 - \prod_{i=1}^n (1 - p_i + a_i(d x_i/x_i)) = 1 - \prod_{i=1}^n (1 - p_i + a_i^* \omega_i) \Rightarrow$$

$$\Omega = P - 1 + \prod_{i=1}^n (1 - p_i + \Omega_i) \quad \omega = 1 - \prod_{i=1}^n (1 - \omega_i)$$

If some relative variations are increasing, the inertia can decrease.

### 3.4. CRITICAL POINTS

$$\text{mod}\left(\frac{dx_i}{x_i}\right) = k \approx 0, 1, 2, 3, \dots$$

Example: The Volterra model for the surviving of the fish species

$$\frac{dx}{x} = (A - \varepsilon) - By$$

$$\mathbf{k=0} \Rightarrow \frac{dy}{y} = -(C + \varepsilon) + Dx$$

$$y = \frac{A - \varepsilon}{B}$$

$$x = \frac{C + \varepsilon}{D}$$

### 3.5. A connection to the theory of relativity

$$T = T_0 \sqrt{1 - \beta^2} \quad \omega(T) = (T / T_0) - 1 = \sqrt{1 - \beta^2} - 1$$

$$\beta^2 = -(\omega^2 + 2\omega) \longrightarrow v^2 = -c^2(\omega^2 + 2\omega)$$

Where T is the temperature,  $\beta = v/c$ , v system speed, c the speed of light.

#### 4. BIBLIOGRAPHY

- [1] SZEL A., Principle and Models for System Evolution, edited by Hungarian Technical Society of Transylvania International Technical Review Vol 45/2009 page 35- 44, ISSN 1454-0746.
- [2] SZEL A., Principle and Models for System Evolution, edited by Hungarian Technical Society of Transylvania International Conference of Computer Science and Energetic-Electrical Engineering-Oradea 2007, pp. 174-179, ISSN 1842-4546.
- [3] HAY KIN, S., Adaptive Filter Theory, Prentice Hall 2001
- [4] RADU P. V. and ION V. S., Sisteme Dinamice , Curs Institutul Politehnic din Bucuresti 1993
- [5] KALMAN R.E. and FALB P.L. and ARBIB M.A., *Teoria sistemelor dinamice*. Editura Tehnică București, 1975
- [6] ZADEH L.A. and POLAK E. *Rendszerelmélet*. Műszaki Könyvkiadó Budapest, 1972
- [7] IONESCU V. and VARGA A., *Teoria sistemelor*. Editura ALL București, 1994
- [8] BELEA C., *Teoria sistemelor. Sisteme neliniare*. Editura Didactică și Pedagogică București, 1985
- [9] BOURCEANU G. and GROSU I. and BELDIE C., *Evoluție și autoorganizare în sisteme departe de echilibru*. Editura Tehnică București, 1989
- [10] XXX *CadetWin-99: CNN application development environment and toolkit under Windows*. Version 3.0. Budapest, Analogical and Neural Computing Laboratory, Computer and Automation Institute, Hungarian Academy of Science 1999
- [11] CURIEVICI I., *Optimizări în industria chimică*. Editura Didactică și Pedagogică București, 1980
- [12] PIERCE R. G., *Solution Methods for Nonlinear Models*. University of Surrey 1997
- [13] SZÉL A., *Perfecționarea modelelor de prognoză* București, Comisia Națională de Informatică, contract nr.257/1994 Bucuresti 1994

- [14] SZÉL A. and GACSADI A. and KAKAS S., *Általános rendszermodell fejlesztése az inercialitás függvényében és alkalmazásai Kolozsvár Sapientia Alapítvány, (K/449-2003. contract) 2004*
- [15] ANOSOV D. V. and ARNOLD V.I., *Dynamical Systems I. 1988, Springer-Verlag, Berlin, Heideberg, New York, London, Paris, Tokio.*
- [16] ARNOLD V. I., *Dynamical Systems III (Mathematical Aspects of Classical and Celstial Mechanics), 1988, Springer-Verlag, Berlin, Heideberg, New York, London, Paris, Tokyo.*
- [17] ARNOLD V. I. and NOVIKOV S.P. *Dynamical Systems IV. Symplectic Geometry and its Applications, 1990, Springer-Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo, Hong Kong.*
- [18] BELTRAMI E., *Mathematics for Dynamic Modeling, 1999, Academic Press, INC. Boston, Orlando, San Diego, New York, Austin, London, Sydney, Tokyo, Toronto.*