

Filtered Multiplicative Bases of Restricted Enveloping Algebras

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Abstract We study the problem of the existence of filtered multiplicative bases of a restricted enveloping algebra $u(L)$, where L is a finite-dimensional and p -nilpotent restricted Lie algebra over a field of positive characteristic p .

Keywords Filtered multiplicative basis · Restricted enveloping algebra

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1 Introduction and Results

Let A be a finite-dimensional associative algebra over a field F . Denote by $\text{rad}(A)$ the Jacobson radical of A and let \mathfrak{B} be an F -basis of A . Then \mathfrak{B} is called a *filtered multiplicative basis* (f.m. basis) of A if the following properties hold:

- (i) for every $b_1, b_2 \in \mathfrak{B}$ either $b_1 b_2 = 0$ or $b_1 b_2 \in \mathfrak{B}$;

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(ii) $\mathfrak{B} \cap \text{rad}(A)$ is an F -basis of $\text{rad}(A)$.

Filtered multiplicative bases arise in the theory of representation of associative algebras and were introduced by H. Kupisch in [10]. In their celebrated paper [4] R. Bautista, P. Gabriel, A. Roiter and L. Salmeron proved that if A has finite representation type (that is, there are only finitely many isomorphism classes of finite-dimensional indecomposable A -modules) over an algebraically closed field F , then A has an f.m. basis.

In [9] an analogous statement was proposed for finitely spaced modules over an aggregate. (Such modules give rise to a matrix problem in which the allowed column transformations are determined by the module structure, the row transformations are arbitrary, and the number of canonical matrices is finite). This statement was subsequently proved in [14].

The problem of existence of an f.m. basis in a group algebra was posed in [4] and has been considered by several authors: see e.g. [2, 3, 5–7, 11]. In particular, it is still an open problem whether a group algebra KG has an f.m. basis in the case when F is a field of odd characteristic p and G is a nonabelian p -group.

Apparently, not much is known about the same problem in the setting of restricted enveloping algebras. The present paper represents a contribution in this direction. In particular, because of the analogy with the theory of finite p -groups, we confine our attention to the class \mathfrak{F}_p of finite-dimensional and p -nilpotent restricted Lie algebras over a field of positive characteristic p . Note that under this assumption, the aforementioned result in [4] can be applied only in very special cases. Indeed, for $L \in \mathfrak{F}_p$, from [8] it follows that $u(L)$ has finite representation type if and only if L is cyclic, that is, there exists an element which generates L as a restricted subalgebra.

Our main results are the following three theorems.

Theorem 1 *Let $L \in \mathfrak{F}_p$ be an abelian restricted Lie algebra over a field F . Then $u(L)$ has a filtered multiplicative basis if and only if L decomposes as a direct sum of cyclic restricted subalgebras. In particular, if F is a perfect field, then $u(L)$ has a filtered multiplicative basis.*

A restricted Lie algebra $\mathfrak{L} \in \mathfrak{F}_p$ is called *powerful* (see e.g. [15]) if $p = 2$ and $\mathfrak{L}' \subseteq \mathfrak{L}^{[p]^2}$ or $p > 2$ and $\mathfrak{L}' \subseteq \mathfrak{L}^{[p]}$. Here $\mathfrak{L}^{[p]^i}$ denotes the restricted subalgebra generated by the elements $x^{[p]^i}$, $x \in L$.

Theorem 2 *Let $L \in \mathfrak{F}_p$ be a nonabelian restricted Lie algebra over a field F . If L is powerful then $u(L)$ does not have a filtered multiplicative basis.*

Theorem 3 *Let $L \in \mathfrak{F}_p$ be a restricted Lie algebra over a field F . If L has nilpotency class 2 and $p > 2$ then $u(L)$ does not have a filtered multiplicative basis.*

An example showing that Theorem 3 fails in characteristic 2 is also provided. Finally, we remark that for odd p no example of noncommutative restricted enveloping algebra having an f.m. basis seems to be known.

2 Preliminaries

Let A be a finite-dimensional associative algebra over a field F . If \mathfrak{B} is an f.m. basis of A then the following simple properties hold (see [5]):

- (F-I) $\mathfrak{B} \cap \text{rad}(A)^n$ is an F -basis of $\text{rad}(A)^n$ for every $n \geq 1$;
- (F-II) if $u, v \in \mathfrak{B} \setminus \text{rad}(A)^k$ and $u \equiv v \pmod{\text{rad}(A)^k}$ then $u = v$.

Let L be a restricted Lie algebra over a field F of characteristic $p > 0$ with a p -map $[p]$. We denote by $\omega(L)$ the *augmentation ideal* of $u(L)$, that is, the associative ideal generated by L in $u(L)$. In [13], the *dimension subalgebras* of L were defined as the restricted ideals of L given by

$$\mathfrak{D}_m(L) = L \cap \omega(L)^m \quad (m \geq 1).$$

These subalgebras can be explicitly described as $\mathfrak{D}_m(L) = \sum_{i p^j \geq m} \gamma_i(L)^{[p]^j}$, where $\gamma_i(L)^{[p]^j}$ is the restricted subalgebra of L generated by the set of p^j -th powers of the i -th term of the lower central series of L . Note that $\mathfrak{D}_2(L)$ coincides with the Frattini restricted subalgebra $\Phi(L)$ of L (cf. [12]).

It is well-known that if L is finite-dimensional and p -nilpotent then $\omega(L)$ is nilpotent. Clearly, in this case $\omega(L)$ coincides with $\text{rad}(u(L))$ and $u(L) = F \cdot 1 \oplus \omega(L)$. Consequently, if $u(L)$ has an f.m. basis \mathfrak{B} , then we can assume without loss of generality that $1 \in \mathfrak{B}$. For each $x \in L$, the largest subscript m such that $x \in \mathfrak{D}_m(L)$ is called the *height* of x and is denoted by $v(x)$. The combination of Theorems 2.1 and 2.3 from [13] gives the following.

Lemma 1 *Let $L \in \mathfrak{F}_p$ be a restricted Lie algebra over a field F , and let $\{x_i\}_{i \in I}$ be an ordered basis of L chosen such that*

$$\mathfrak{D}_m(L) = \text{span}_F \{x_i \mid v(x_i) \geq m\} \quad (m \geq 1).$$

Then for each positive integer n the following statements hold:

- (i) $\omega(L)^n = \text{span}_F \{x \mid v(x) \geq n\}$, where $x = x_{i_1}^{\alpha_1} \cdots x_{i_l}^{\alpha_l}$,
 $v(x) = \sum_{j=1}^l \alpha_j v(x_{i_j})$, $i_1 < \cdots < i_l$ and $0 \leq \alpha_j \leq p-1$.
- (ii) *The set $\{y \mid v(y) = n\}$ is an F -basis of $\omega(L)^n$ modulo $\omega(L)^{n+1}$.*

For a subset S of L we shall denote by $\langle S \rangle_p$ the restricted subalgebra generated by S . Moreover, if $z \in L$ is p -nilpotent, the minimal positive integer n such that $z^{[p]^n} = 0$ is called the *exponent* of z and denoted by $e(z)$.

3 Proofs

Proof of the Theorem 1 Assume first that $L = \bigoplus_{i=1}^n \langle x_i \rangle_p$. Then, by the PBW Theorem for restricted Lie algebras (see [16], Chapter 2, Theorem 5.1), we see that $u(L)$ is isomorphic to the truncated polynomial algebra

$$F[X_1, \dots, X_n] / (X_1^{p^{e(x_1)}}, \dots, X_n^{p^{e(x_n)}}).$$

Consequently the algebra $u(L)$ has an f.m. basis.

Conversely, suppose that $u(L)$ has an f.m. basis \mathfrak{B}_1 with $1 \in \mathfrak{B}_1$ and put $\mathfrak{B} = \mathfrak{B}_1 \setminus \{1\}$. Let $n = \dim_F L/L^{[p]}$ and $\mathfrak{B} \setminus \omega(L)^2 = \{b_1, \dots, b_n\}$. By Lemma 1, one has $b_i = x_i + h_i$, where $x_i \in L \setminus L^{[p]}$ and $h_i \in \omega(L)^2$ for every $i = 1, \dots, n$. From [12] it follows at once that $\{x_1, \dots, x_n\}$ is a minimal set of generators of L as a restricted subalgebra. We shall prove by induction on $e = \max\{i \mid L^{[p]^i} \neq 0\}$ that L has a cyclic decomposition. If $e = 1$, then $L = \bigoplus_{i=1}^n \langle x_i \rangle_p$.

Now let $e > 1$ and suppose that L does not decompose as a direct sum of restricted subalgebras. Since $L = \sum_{i=1}^n \langle x_i \rangle_p$ and the p -map is p -semilinear, note that $L^{[p]^e}$ is just the vector subspace generated by the p^e -th powers of the generators x_i having exponent $e + 1$. Therefore, without loss of generality we can assume that

$$e + 1 = e(x_1) = \dots = e(x_m) \geq e(x_s), \quad (m + 1 \leq s \leq n)$$

and $\{x_1^{[p]^e}, \dots, x_m^{[p]^e}\}$ is an F -linearly independent set with

$$\text{span}_F \{x_1^{[p]^e}, \dots, x_m^{[p]^e}\} = L^{[p]^e}.$$

In turn, we can reindex the elements x_{m+1}, \dots, x_n so that there exists a maximal $m \leq k < n$ such that

$$H = \langle x_1, \dots, x_m, \dots, x_k \rangle_p = \bigoplus_{i=1}^k \langle y_i \rangle_p$$

for suitable y_1, \dots, y_k in L with $y_i = x_i$ for $i = 1, \dots, m$. Consequently, for every $s > k$ there exists a minimal number f_s such that

$$x_s^{p^{f_s}} = \sum_{i=1}^k \sum_{j=0}^{e(y_i)-1} \mu_{i,j}^{(s)} y_i^{[p]^j} \quad (\mu_{i,j}^{(s)} \in F). \quad (1)$$

Denote by J the associative ideal of $u(L)$ generated by the elements $b_1^{p^e}, \dots, b_m^{p^e}$. Clearly $J \subseteq \omega(L)^{p^e} \subseteq L^{[p]^e} u(L)$. Suppose by contradiction that $J \neq L^{[p]^e} u(L)$. If r is the maximal positive integer such that

$$(L^{[p]^e} u(L) \cap \omega(L)^r) \setminus J \neq \emptyset,$$

then there exists $v = x_i^{p^e} x_1^{a_1} \dots x_n^{a_n} \in \omega(L)^r \setminus J$ such that

$$v \equiv b_i^{p^e} b_1^{a_1} \dots b_n^{a_n} \pmod{\omega(L)^{r+1}}.$$

Consequently

$$v - b_i^{p^e} b_1^{a_1} \dots b_n^{a_n} \in (L^{[p]^e} u(L) \cap \omega(L)^{r+1}) \setminus J,$$

contradicting the definition of r . Therefore $J = L^{[p]^e} u(L)$, which implies that $u(L)/J \cong u(L/L^{[p]^e})$. Moreover, it is easily seen that $\mathfrak{B}_1 \cap J$ is an F -basis of J , hence the elements $b_i + J$ with $b_i \notin J$ form an f.m. basis of $u(L)/J$. Consequently, by induction we have that $\mathcal{L} = L/L^{[p]^e}$ is a direct sum of restricted subalgebras.

As the images of y_1, \dots, y_k are F -linearly independent in $\mathcal{L}/\Phi(\mathcal{L})$, from [12] it follows that there exists a restricted subalgebra P of L with $L^{[p]^e} \subseteq P$ such that

$$\mathcal{L} = H/L^{[p]^e} \oplus P/L^{[p]^e}.$$

As a consequence, for every $s > k$ we have $x_s \equiv v_s + w_s \pmod{L^{[p]^e}}$ with $v_s \in H$ and $w_s \in P$ and, moreover, it follows from Eq. 1 that $w_s^{[p]^{fs}} \in L^{[p]^e}$. One has

$$v_s = \sum_{i=1}^k \sum_{j=0}^{e(y_i)-1} k_{i,j}^{(s)} y_i^{[p]^j}, \quad (k_{i,j}^{(s)} \in F). \quad (2)$$

Since $x_s^{[p]^{fs}} \equiv v_s^{[p]^{fs}} \pmod{L^{[p]^e}}$, we conclude that $\mu_{i,j}^{(s)} \in F^{p^{fs}}$ provided $j < e$. We claim that for every $1 \leq i \leq k$ the coefficient $\mu_{i,e}^{(s)}$ is also in $F^{p^{fs}}$. Indeed, write

$$w_s = \sum_{b \in \mathfrak{B}} \lambda_b b \quad (\text{for suitable } \lambda_b \in F).$$

Then, as \mathfrak{B} is a filtered multiplicative basis of $u(L)$, it follows that

$$w_s^{p^{fs}} = \sum_{b \in \mathfrak{C}} \mu_b^{p^{fs}} b^{p^{fs}} \quad (3)$$

where \mathfrak{C} is a subset of \mathfrak{B} and the μ_b 's are nonzero elements of F . Moreover, since $w_s^{[p]^{fs}} \in L^{[p]^e}$ we have

$$w_s^{p^{fs}} = \sum_{i=1}^m \alpha_i b_i^{p^e}. \quad (4)$$

As \mathfrak{B} is a filtered F -basis of $u(L)$, by comparing Eqs. 3 and 4 we conclude that for every $i = 1, 2, \dots, m$ there exists $\beta_i \in F$ such that $\alpha_i = \beta_i^{p^{fs}}$. At this stage, since $b_i^{p^e} = x_i^{[p]^e}$ for every $i = 1, 2, \dots, m$, the relations (1) and (2) allow to conclude that for every $1 \leq i \leq k$ one has

$$\mu_{i,e}^{(s)} = \left(k_{i,e}^{(s)}\right)^{p^{fs}} + \beta_i^{p^{fs}} \in F^{p^{fs}},$$

as desired (here $\beta_{m+1}, \dots, \beta_k = 0$).

Now, by Eq. 1 and the above discussion we have $x_s^{p^{fs}} = z^{p^{fs}}$ for some $z \in H$. Therefore $(x_s - z)^{p^{fs}} = 0$ and then the minimality of f_s forces $\langle x_s - z \rangle_p \cap H = 0$. This contradicts the definition of k , yielding the claim.

Finally, if F is perfect, then L decomposes as a direct sum of cyclic restricted subalgebras (see e.g. [1], Chapter 4, Theorem in Section 3.1). The proof is done. \square

Unlike group algebras, a commutative restricted enveloping algebra need not have an f.m. basis. Indeed, we have the following

Example Let F be a field of positive characteristic p containing an element α which is not a p -th root in F . Consider the abelian restricted Lie algebra

$$L_\alpha = Fx + Fy + Fz$$

with $x^{[p]} = \alpha z$, $y^{[p]} = z$, and $z^{[p]} = 0$. Suppose that $u(L_\alpha)$ has an f.m. basis. By Theorem 1, L_α is a direct sum of cyclic restricted subalgebras. Since $L_\alpha^{[p]} \neq 0$ and $L_\alpha^{[p]^2} = 0$, we have $L_\alpha = \langle a \rangle_p \oplus \langle b \rangle_p$ with $e(a) = 2$ and $e(b) = 1$.

Let $b = k_1 x + k_2 y + k_3 z$, $k_i \in F$. Since $\alpha \notin F^p$, we get $0 = b^{[p]} = (k_1^p \alpha + k_2^p) z$, so $k_1 = k_2 = 0$ and $0 \neq a^{[p]} \in Fz = \langle b \rangle_p$, a contradiction.

Lemma 2 Let A be a finite-dimensional nilpotent noncommutative associative algebra over a field F . Suppose that A has a minimal set of generators $\{u_1, \dots, u_n\}$ such that:

- (i) $[u_i, u_j] \in A^3$ for every $i, j = 1, \dots, n$;
- (ii) $u_i u_j \notin A^3$ for every $i, j = 1, \dots, n$;
- (iii) $\text{span}_F\{u_i u_j \mid 1 \leq i < j \leq n\} \cap \text{span}_F\{u_i^2 \mid i = 1, \dots, n\} \subseteq A^3$.

Then A has no f.m. basis.

Proof By contradiction, assume that there exists an f.m. basis \mathfrak{B} of A . Clearly, we have $\dim_F A/A^2 = n$ and, by property (F-I), $\mathfrak{B} \setminus \omega(L)^2$ is a minimal set of generators of A as an associative algebra. Write $\mathfrak{B} \setminus A^2 = \{b_1, \dots, b_n\}$. Obviously

$$b_k \equiv \sum_{i=1}^n \alpha_{ki} u_i \pmod{A^2}, \quad (\alpha_{ki} \in F)$$

and the determinant of the matrix $M = (a_{ki})$ is not zero. Now

$$\begin{aligned} b_r b_s &\equiv \sum_{i=1}^n \alpha_{ri} \alpha_{si} u_i^2 + \sum_{\substack{i,j=1 \\ i < j}}^n (\alpha_{ri} \alpha_{sj} + \alpha_{rj} \alpha_{si}) u_i u_j \\ &\quad - \sum_{\substack{i,j=1 \\ i < j}}^n \alpha_{rj} \alpha_{si} [u_i, u_j] \pmod{A^3}. \end{aligned}$$

By assumption (i) of the statement we have that $[u_i, u_j] \equiv 0 \pmod{A^3}$, so

$$\begin{aligned} b_r b_s &\equiv \sum_{i=1}^n \alpha_{ri} \alpha_{si} u_i^2 \\ &\quad + \sum_{\substack{i,j=1 \\ i < j}}^n (\alpha_{ri} \alpha_{sj} + \alpha_{rj} \alpha_{si}) u_i u_j \equiv b_s b_r \pmod{A^3}. \end{aligned} \quad (5)$$

Suppose $b_r b_s \in A^3$ for some r, s . Because of Eq. 5 and the assumptions (ii) and (iii) of the statement we have $\alpha_{ri} \alpha_{si} = 0$ and $\alpha_{ri} \alpha_{sj} + \alpha_{si} \alpha_{rj} = 0$ for every i, j . It follows that $\alpha_{ri} \alpha_{sj} - \alpha_{si} \alpha_{rj} = 0$. Consequently, all of the order two minors formed by the k -th and s -th lines of the matrix M are zero, which is impossible because $\det M \neq 0$. Hence $b_r b_s, b_s b_r \notin \omega(L)^3$ and $b_r b_s \equiv b_s b_r \pmod{\omega(L)^3}$ for every r, s . By property (F-II) of the f.m. bases we conclude that $b_r b_s = b_s b_r$. Thus A is a commutative algebra, a contradiction. \square

Proof of the Theorem 2 Let S be a minimal set of generators of L as a restricted Lie algebra. Then, as L is powerful, by Lemma 1 we conclude that S is a minimal set of the nilpotent associative algebra $\omega(L)$ satisfying the hypotheses of Lemma 2, and the claim follows. \square

Proof of the Theorem 3 Suppose, by contradiction, that $u(L)$ has an f.m. basis \mathfrak{B}_1 with $1 \in \mathfrak{B}_1$, so that $\mathfrak{B} = \mathfrak{B}_1 \setminus \{1\}$ is an f.m. basis of $\omega(L) = \text{rad}(u(L))$. Put $n = \dim_F \mathfrak{D}_1(L)/\mathfrak{D}_2(L)$ and write $\mathfrak{B} \setminus \omega(L)^2 = \{b_1, \dots, b_n\}$. Consider an F -basis B of

L as in the statement of Lemma 1 and let u_1, \dots, u_n be the elements of B having height 1. Thus, by Lemma 1(ii), the set $\{u_j + \omega(L)^2 \mid j = 1, \dots, n\}$ forms an F -basis of $\omega(L)/\omega(L)^2$. Then, for every $k = 1, \dots, n$ there exist $\alpha_{k1}, \dots, \alpha_{kn} \in F$ such that

$$b_k \equiv \sum_{i=1}^n \alpha_{ki} u_i \pmod{\omega(L)^2}, \quad (k = 1, \dots, n).$$

Set $\bar{u}_k = \sum_{i=1}^n \alpha_{ki} u_i$. Clearly, $\{\bar{u}_1, \dots, \bar{u}_n\}$ is an F -linearly independent set which generates L as a restricted subalgebra.

Now, if L is powerful then, by Theorem 2, $u(L)$ cannot have any f.m. basis, a contradiction. Therefore $L' \not\subseteq L^{[p]}$ and so there exist $1 \leq r < s \leq n$ such that the element $c_{rs} = [\bar{u}_r, \bar{u}_s]$ is not in $L^{[p]}$. Since L is nilpotent of class 2, we have that

$$\mathfrak{D}_2(L) = L' + L^{[p]} \supset L^{[p]} = \mathfrak{D}_3(L),$$

hence c_{rs} has height two. Furthermore, one has

$$b_s^2 \equiv \bar{u}_s^2 \pmod{\omega(L)^3}; \quad b_s b_r \equiv \bar{u}_r \bar{u}_s - c_{rs} \pmod{\omega(L)^3}.$$

Since L is nilpotent of class 2, it follows that

$$\begin{aligned} b_r b_s^2 &\equiv \bar{u}_r \bar{u}_s^2 \pmod{\omega(L)^4}; \\ b_s u_r b_s &\equiv \bar{u}_r \bar{u}_s^2 - \bar{u}_s c_{rs} \pmod{\omega(L)^4}; \\ b_s^2 b_r &\equiv \bar{u}_r \bar{u}_s^2 - [\bar{u}_r, \bar{u}_s^2] = \bar{u}_r \bar{u}_s^2 - 2\bar{u}_s c_{rs} \pmod{\omega(L)^4}. \end{aligned}$$

Therefore the elements

$$v_1 = b_r b_s^2, \quad v_2 = b_s^2 b_r \quad \text{and} \quad v_3 = b_s b_r b_s$$

are F -linearly dependent modulo $\omega(L)^4$. In view of property (F-I),

$$(\mathfrak{B} \cap \omega(L)^3) \setminus \omega(L)^4$$

is an F -basis for $\omega(L)^3$ modulo $\omega(L)^4$. Consequently, it follows that either $v_i \in \omega(L)^4$ for some $i \in \{1, 2, 3\}$ or $v_j \equiv v_k \pmod{\omega(L)^4}$ for some $j, k \in \{1, 2, 3\}$. In each case we have a contradiction to Lemma 1, and the proof is complete. \square

We remark that the previous result fails without the assumption on the characteristic of the ground field. Indeed, let F be a field of characteristic 2 and consider the restricted Lie algebra $L = Fa + Fb + Fc$ with $[a, b] = c$, $[a, c] = [b, c] = 0$, and $a^{[2]} = b^{[2]} = c^{[2]} = 0$. Then it is straightforward to show that

$$\{1, a, b, ab, ab + c, ac, bc, abc\}$$

is an f.m. basis of $u(L)$.

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