COMPARISON OF AN ITERATIVE HEURISTIC AND JOINT OPTIMIZATION IN THE OPTIMIZATION OF BONUS-MALUS SYSTEMS

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Abstract: Bonus-malus system (BMS) is a risk managing method primarily used in liability insurances. In a BMS there are finitely many classes, each having a different premium. At the start of the contract each policyholder is assigned to the 'initial class'. In each period, the policyholders are reclassified based on the number of claims. In Ágoston and Gyetvai (2020) we introduced a MILP model for the joint optimization of premiums and transition rules. The computational time of the model can be very long with realistic parameters. However, we may approximate the optimal solution by iteratively optimizing the premiums and then the transition rules. In this paper, we compare the computation of the MILP model and the iterative heuristic.

Keywords: Adverse Selection, Bonus-Malus system, Integer programming

1 INTRODUCTION

Bonus-malus systems (BMS) are used in Motor-third party liability insurances. The BMS is applied to distinguish the risky drivers from the less risky ones and incentivize drivers to be more careful.

In a BMS there are finitely many classes. For each period of the insurance, the policyholders are classified into one of these classes. Each class has a premium, so the policyholder's payment depends on the class where he/she is classified. At the beginning of the contract, each policyholder is classified into the so-called initial class. Then the classification depends on the previous period's classification and the number of claims reported in the present period. Suppose the policyholder has a claim in a period. In that case, he/she moves to a worse class with a higher premium. On the other hand, if he/she does not have a claim in the period, then he/she moves to a better class. Hence his/her premium will be lower. Therefore, the riskier policyholders will be in a worse class after several periods, and they pay more in general. The less risky policyholders will pay less in total.

Designing a BMS requires choosing the transition rules, the number of classes, the scale of premiums, and the initial class. The objective is to form a BMS that sorts policyholders as best as possible. There are many papers about the optimization of BMSs (e.g., Cooper and Hayes (1987); Lemaire (1995); Denuit et al. (2007); Heras et al. (2004); Brouhns et al. (2003); Denuit and Dhaene (2001); Mert and Saykan (2005); Najafabadi and Sakizadeh (2017)). The premium scale is the variable in these studies, and the number of classes, the initial class, and transition rules are parameters. The most relevant article for our studies is Heras et al. (2004). In this study, the authors introduce an LP model for the optimization of premiums. In Gyetvai and Ágoston (2018), we introduced a MILP model to find the optimal transition rules with a fixed premium scale. We presented a modification of this model for the joint optimization of the premiums and transition rules in Ágoston and Gyetvai (2020).

2 OPTIMIZATION MODELS

Let us assume that we can distinguish I risk-groups (types) among the policyholders. Each type has a different risk that does not change over time. In practice, transition rules are based only on claim numbers, without the consideration of the claim amount.

Let $M > 0$ be the highest number of possible claims in a period and let λ_m^i be the probability of m claims in a period for the policyholders of type i ($i = 1, \dots, I$, $\sum_{m=0}^{M} \lambda_m^i = 1$). We denote the risk-parameters (expected claim amount) for risk-group i with λ^i, ($\lambda^i = \sum_{m=0}^{M} m\lambda_m^i$). ϕ^i denotes the proportion of the type i policyholders among all of the policyholders ($\sum_{i=1}^{I} \phi^i = 1$). The BMS has $K+1$ classes indexed from 0 to K.

The classification of a policyholder only depends on the previous period classification and the number of claims of the current period. This is called the Markov property. Hence the classification of the policyholders is a regular Markov chain. Therefore exists a unique stationary probability distribution (Kemeny and Snell (1976)) that we use in the optimization. Let c_k^i denote the stationary probability of the type i policyholders is classified into class k. For the optimization of the premiums, we use an LP model (the idea appeared in Heras et al. (2004)) to minimize the difference between the expected payment and the expected claims. We denote the premium of class k by π_k. Also, let g_k^i denote the absolute deviation between the expected payment and claims for a type i policyholder in class k. Then the model is written as follows:

$$\min \sum_{i=1}^{I} \sum_{k=0}^{K} \phi^i g_k^i \tag{1}$$

Subject to

$$\pi_k c_k^i + g_k^i \geq \lambda^i c_k^i \qquad \forall i, k \tag{2}$$

$$\pi_k c_k^i - g_k^i \leq \lambda^i c_k^i \qquad \forall i, k \tag{3}$$

$$\pi_{k-1} \geq \pi_k \qquad k = 1, \dots, K \tag{4}$$

$$\pi_k \geq 0 \qquad \forall k$$
$$g_k^i \geq 0 \qquad \forall k, i$$

In Gyetvai and Ágoston (2018) we introduced a MILP model, where the transition rules are in the scope of the optimization and the premiums are considered as parameters.

Here we only give a brief description of the model. For a more detailed description, see Gyetvai and Ágoston (2018), and Ágoston and Gyetvai (2020).

Let $T_{j,m}$ denote binary variables for each possible step (j) and claim ($m = 0, 1, \ldots, M$). Hence if $T_{j,m} = 1$, then the policyholders with m claims move j classes upward (downward if $j < 0$) in the following period. Index j can be 0 as well, which means that they stay in the same class in the subsequent period. The domain of j is by $[-K : K]$. In this model the c_k^i stationary probabilities are variables that are determined by the optimal $T_{j,m}$ transition rules.

$$\min \sum_{i=1}^{I} \sum_{k=0}^{K} \phi^i g_k^i \tag{5}$$

Subject to

$$\sum_{j=\underline{J}}^{\overline{J}} T_{j,m} = 1 \qquad \forall m \tag{6}$$

$$\sum_{j=1}^{\overline{J}} T_{j,0} = 1 \tag{7}$$

$$\sum_{j=\underline{J}}^{-1} T_{j,M} = 1 \tag{8}$$

$$\sum_{l=j}^{\overline{J}} T_{l,m} \geq T_{j,m+1} \qquad \forall j, m = 0, \ldots M-1 \tag{9}$$

$$\sum_{k=0}^{K} c_k^i = 1 \qquad \forall i \tag{10}$$

$$d_{k,j,m}^i \geq \lambda_m^i c_k^i - (1 - T_{j,m}) \qquad \forall i, k, j, m \tag{11}$$

$$c_k^i = \sum_{j=\max(\underline{J}, -(K-k))}^{\min(\overline{J}, k)} \sum_{m=0}^{M} d_{k-j,j,m}^i \qquad k = 1, \ldots, K-1, \forall i \tag{12}$$

$$c_k^i = \sum_{j=0}^{\overline{J}} \sum_{\ell=0}^{j} \sum_{m=0}^{M} d_{k-\ell,j,m}^i \qquad k = K, \forall i \tag{13}$$

$$c_k^i = \sum_{j=\underline{J}}^{0} \sum_{\ell=j}^{0} \sum_{m=0}^{M} d_{k-\ell,j,m}^i \qquad k = 0, \forall i \tag{14}$$

$$\pi_k c_k^i + g_k^i \geq \lambda^i c_k^i \quad \forall i, k \tag{15}$$

$$\pi_k c_k^i - g_k^i \leq \lambda^i c_k^i \quad \forall i, k \tag{16}$$

Constraints (6)-(9) are needed for defining reasonable transition rules. Constraints (10)-(14) are for the calculation of the stationary probabilities c_k^i. And constraints (15)-(16) are for the absolute deviation of the objective function, similarily to the the premium optimization model.

In Ágoston and Gyetvai (2020) we presented another MILP model, where both premiums and transition rules can be optimized jointly. The presentation of this model would exceed the scope of this article. Hence we only describe the basic idea of the model. In this model, we start with default premiums for each class. We introduce binary variables O_k^ℓ for each class k and layer ℓ to change the default premium. Hence if $O_k^\ell = 1$, then the default premium of class k is increased with a value that the ℓth layer represented. We may consider several layers in the model, but it may increase the computational time significantly.

When realistic parameters are considered in this model, there are a considerable number of binary variables. Hence the running time can be extremely long. Separately the optimization models for the premiums and the transition rules can be calculated much faster than the joint optimization model. Hence we may use an iterative method to approximate the optimal solution. First, we calculate the optimal transition rules with a fixed premium. Then we find the optimal premiums to these transition rules, which we now consider as parameters. Then we use the optimal premiums of this model as parameters and re-optimize the transition rules. We continue it until we cannot improve the objective function further. The solution of this heuristic greatly depends on the initial model. In the initial model of transition rules optimization the premiums are outer parameters. We may also start with the premium optimization and then proceed with the optimization of the transition rules. In this case, the transition rules are outer parameters in the first model. In the comparison, we considered four types of initial premiums:

- Proportional (prop): We introduce own-classes for each risk-groups, which means the premium of these classes equals the risk-groups' expected claim. The risk-groups have that many own-classes that are proportional to their percentage of all policyholders.

- Linear (lin): We take the lowest and highest risk-groups' expected claim for the lowest and highest premium and set the classes' premium linearly.

- Minimal (min): The premium equals the highest expected claim in the worst class. In all other classes, it equals the lowest one.

- Maximal (max): In this case, only one class premium equals the minimal expected claim and each other to the highest one.

We also considered two types of initial transition rules:

- TRK: In case of any claim, the policyholders move into the worst class. Without a claim, the policyholders move one class upward.

- TR1: In case of any claim, the policyholders move one class downward. Without a claim, the policyholders move one class upward.

3 NUMERICAL RESULTS

We considered BMSs with 15 classes in the comparison of the joint optimization model to the iterative heuristic. We randomly chose their risk-groups' claim probability to test the heuristic in as many setups as possible. Hence we considered 100 randomized setups. In each setup we considered five equally sized risk-groups.

Two cases were examined: a realistic one, in which the risk parameters are generated from a 0.01 to 0.1 interval. And a not-so-realistic higher-risk setup, in which the risks are chosen from the [0.1 : 0.3] interval. In each model, the maximal number of claims per period can be up most 2. For the claim probabilities we considered Poisson distribution. We compared the iterative heuristic with six different initial solutions to the joint optimization model. We highlight the difference between the optimal solution and the running time. Figure 1 presents the relative deviation from the MILP model in both cases.

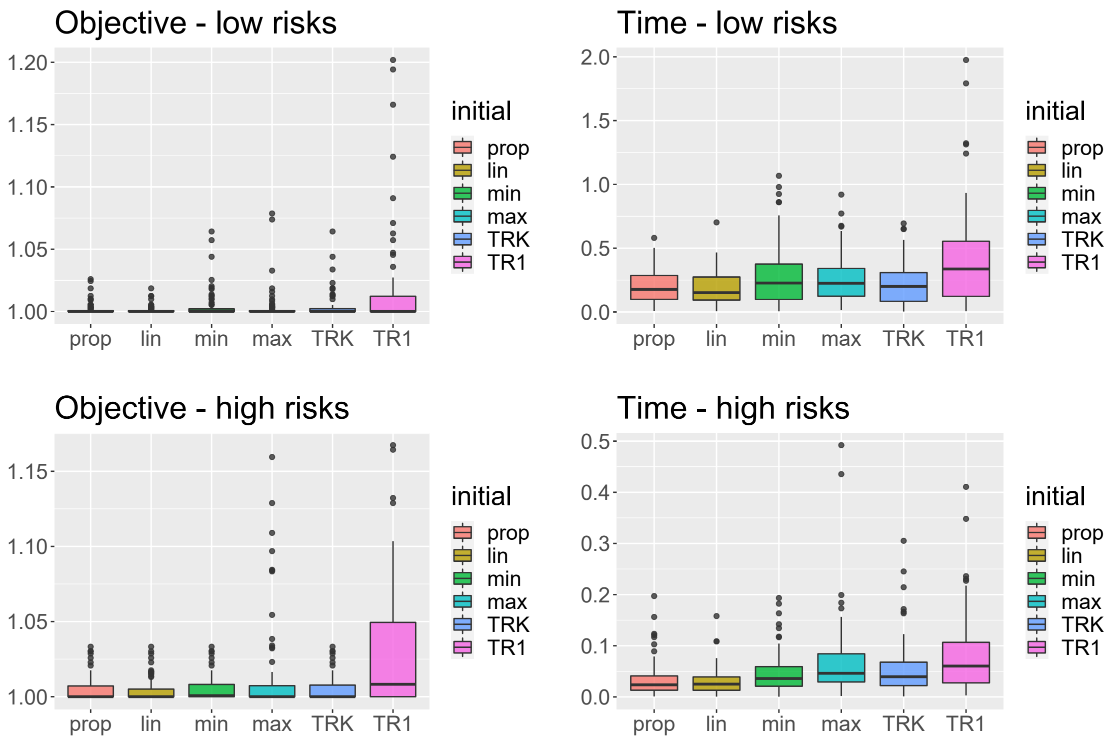

Figure 1: Objective and time change to the joint optimization model

On the left side of the figure, the objective increase to the joint model is presented. The top row shows the low-risk case and the bottom presents the high-risk case. When the risks are low, the results are similar to the objective of the joint model. The *TR1* resulted in the highest difference in average. However, even in this case, the average increase was only 1.4%. When the risks are higher, the difference between the joint model and the heuristic is higher as well. The *TR1* resulted in the highest increase in average in this case as well. The average increase was 2.9% for the *TR1* and the next in the line was the *max* with 1.1%

The running time of the iterative heuristic, in general, was much faster than the joint optimization model with each initial solution. Again, the *TR1* seems to be the worst. But even in this case, the average running time is less than half of the joint model in both setups (40% in the low and 8% in the high).

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