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PARAMETRIC UNCERTAINTY OF CENTRAL HEATING SYSTEM MODEL

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Abstract

During mathematical modeling of real technical system such as a central heating system the modeler can meet any type and rate model uncertainty. The paper shows the sensitivity analysis and methodology of sensitivity analysis based uncertainty investigation by a similar heating system exergy model case.

Rezumat

În timpul modelării matematice a sistemelor tehnice reale, cum ar fi un sistem de încălzire centrală, modelatorul poate întâlni un model de orice tip și rată de incertitudine. Lucrarea prezintă analiza de sensibilitate și metodologia analizei de sensibilitate bazată pe incertitudinea de investigare a unui sistem similar de încălzire exergetic.

1. Introduction

During mathematical modeling of real technical system we can meet any type and rate model uncertainty [5]. Its reasons can be incognizance of modelers or data inaccuracy. So, classification of uncertainties, with respect to its sources, distinguishes between aleatory and epistemic ones. The aleatory uncertainty is an inherent data variation associated with the investigated system or the environment. Epistemic one is an uncertainty that is due to a lack of knowledge of quantities or processes of the system or the environment. Aleatory uncertainty is primarily associated with objectivity, but epistemic uncertainty may be comprised of substantial amounts of both objectivity and subjectivity [3].

Ferson and Tucker written that uncertainty analysis is a systematic study in which a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified [1]

Mahdavi studied the various sources of uncertainty in building performance simulation [3]. In the eyes of Mahdavi, the potential errors due to i) inaccurate building descriptions, ii) uncertain micro-climatic assumptions, and iii) deficient building users' information are discussed, using original data and analyses.

Kalmár's paper proposed to analyze central beating systems from exergy point of view [2]. The obtained results proved that, when the supply (outgoing) temperature is lower, the exergy losses and exergy need decrease.

The aim of this paper is to show the sensitivity analysis and methodology of sensitivity analysis based uncertainty investigation by a similar heating system exergy model case.

The outline of the paper is as follows: Section 2 shows the sensitivity analysis. Section 3 presents a simply case study shortly words by central heating system exergy model. Section 4 summaries the work of author.

2. The Sensitivity Analysis

The essence of the sensitivity analysis is that the anomalies and variations of the given functional unit or aggregate are simulated by changing of its independent variables. On the basis the mathematical model of the investigated system or process, it can be determined how sensitive the dependent system or process variables will be to the simulated changes. If only one of the independent variables is changed, the investigation will be called one-parameter sensitivity test. If the number of the changed independent it variables is more than one, the several-parameter sensitivity test is used.

It is important to mention that the changes of independent variables cannot be more than about 1 or 5 % depending on the intensity of the original model's nonlinearity. Depending on the non-linearity of the

original model, the results of the sensitivity analysis can have difference from real influences of the simulated changes. But these results show the direction and order of magnitude of the real simulated changes.

To determine the $K_{y;x_i}$ sensitivity coefficient as a first step, the total differential of both sides of the initial equation

$$y = f(x_1, x_2, ..., x_n) \quad (1)$$

should be formed:

$$dy = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_1} dx_1 + \ldots + \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_n} dx_n \quad (2)$$

Then both sides of the last equation should be multiplied by same sides of the general equation and all elements should be multiplied by $\frac{x_i}{x_i}$:

$$\frac{dy}{y} = \frac{\partial f(x_1; x_2; .. x_n)}{\partial x_1} \frac{x_1}{f(x_1; x_2; .. x_n) x_1} dx_1 + \ldots + \frac{\partial f(x_1; x_2; .. x_n)}{\partial x_n} \frac{x_n}{f(x_1; x_2; .. x_n) x_n} dx_n \quad (3)$$

Introducing the sensitivity coefficient coefficients:

$$K_{y;x_i} = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_i} \frac{x_i}{f(x_1; x_2; \ldots x_n)} = \frac{\partial y}{\partial x_i} \frac{x_i}{y}$$

(4)

and

$$\frac{d\eta}{\eta} \approx \frac{\Delta \eta}{\eta} = \delta \eta$$

equation, the following linear system can be achieved:

$$\delta y = K_{y;x_1} \delta x_{y;x_2} + \ldots + K_{y;x_n} \delta x_n \quad (5)$$

Using equation mentioned above how sensitive the dependent system or process output parameters will be to the uncertainties of input ones. For example, these uncertainties can be occurred by measurement inaccuracies.

3. Case Study of an Exergy Model

Nowadays exergy analysis of energy consumption is used increasingly. Kalmár in reference [2] studied the central heating systems from exergy point of view. The exergy of a system is the maximum work possible during a process that brings the system into equilibrium with a heat reservoir. To determine heating energy and energy flows in buildings Kalmar spitted up the central heating system into following parts: room, appliances and their control, heat distributor, heat production. The exergy requirement of all part can be determined by their demand for energy.

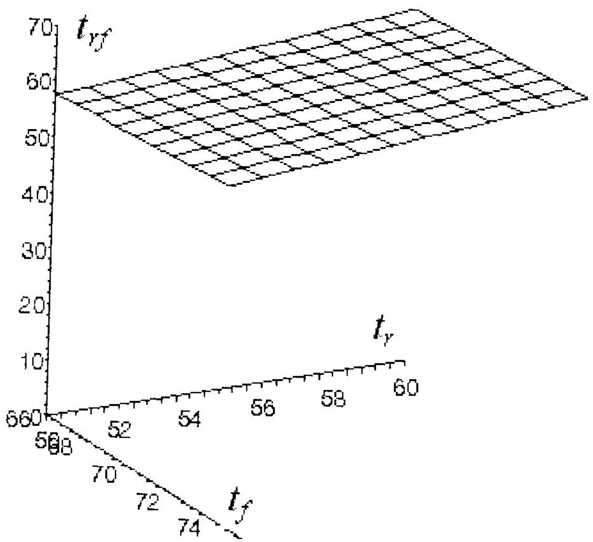

Fig. 1. The function $t_{rf}(t_f,t_r)$ in case of $t_i=21\ ^oC$

In this paper only the heating body — as an aggregate of the central heating system — namely uncertainty of its equation (that is mathematical model) will be investigated.

The surface temperature of heating body

$$t_{rf} = \frac{t_f - t_r}{\ln\left(\frac{t_f - t_i}{t_r - t_i}\right)} + t_i \qquad (6)$$

where:

t_f — outgoing temperature. [°C];
t_r — return temperature, [°C];
t_i — room air temperature. [°C].

The Figure 1. show the function (6) in case of $t_i=21\ ^oC$ room air temperature. From equations (4) and (6) the sensitivity coefficients of heating body:

$$K_{trf;tf} = \frac{\partial t_{rf}}{\partial t_f}\frac{t_f}{t_{rf}} = \left(\frac{1}{\ln\left(\frac{t_f - t_i}{t_r - t_i}\right)} - \frac{t_f - t_r}{\ln\left(\frac{t_f - t_i}{t_r - t_i}\right)^2 (t_f - t_i)} \right)\frac{t_f}{t_{rf}}$$

(7)

$$K_{trf;tr} = \frac{\partial t_{rf}}{\partial t_r}\frac{t_r}{t_{rf}} = \left(\frac{t_f - t_r}{\ln\left(\frac{t_f - t_i}{t_r - t_i}\right)(t_r - t_i)} - \frac{1}{\ln\left(\frac{t_f - t_i}{t_r - t_i}\right)} \right)\frac{t_r}{t_{rf}}$$

(8)

$$K_{trf;ti} = \frac{\partial t_{rf}}{\partial t_i}\frac{t_i}{t_{rf}} = \left(1 - \frac{(t_f - t_r)\left(\frac{t_f - t_i}{(t_r - t_i)^2} - \frac{1}{t_r - t_i}\right)(t_r - t_i)}{\ln\left(\frac{t_f - t_i}{t_r - t_i}\right)^2 (t_f - t_i)} \right)\frac{t_i}{t_{rf}} \qquad (9)$$

In case of present used 70/55 temperature-gradient system (t_f = 70 °C ; t_r = 55 °C) and t_i = 21 °C and room air temperature the values of sensitivity coefficients:

$$K_{t_{rf};t_f} = 0{,}50124$$
$$K_{t_{rf};t_r} = 0{,}50254 \qquad (10)$$
$$K_{t_{rf};t_i} = -3{,}78405 \cdot 10^{-3}$$

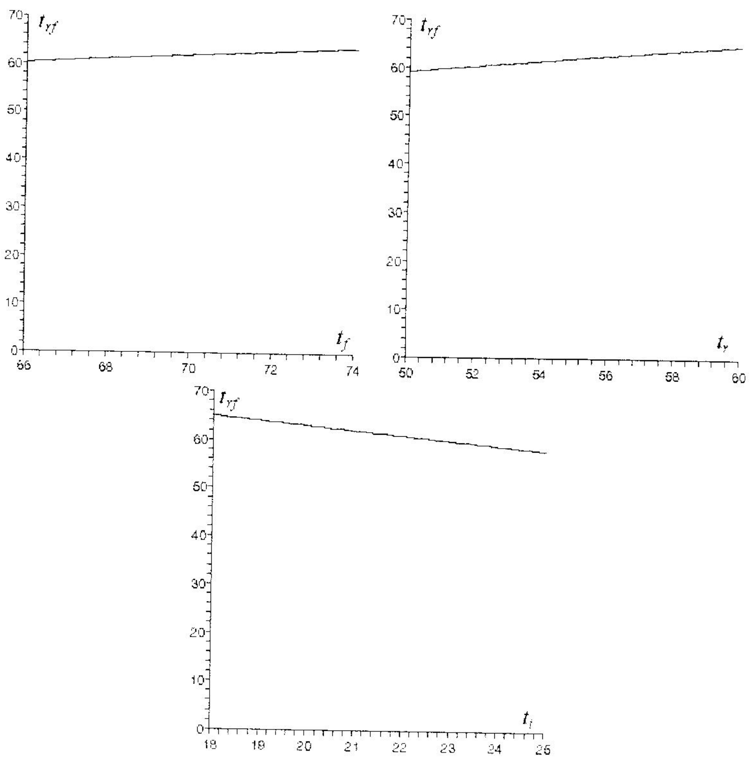

Figure 2. Singulary functions' graphs of equation (6)

In Figure 2. singulary functions determined to work point mentioned above can be seen, which well illustrate the sensitivity coefficients determined above. On the supposition that input — t_f ; t_r and t_i

— parameters have 1 °C uncertainties, the output — t_{rf} — parameter has uncertainties shown by Table I.

[%]	δt_{rf} [%]	Δt_{rf} [°C]
δt_f = 1,42857	0,71606	0,44427
δt_r = 1,81818	0,91372	0,56691
δt_i =4,76190	-1,80193 10^{-2}	-1,11799 10^{-2}

Table I. Output parameter uncertainties in cases of 1 °C differences of input ones

How conclusions can be deduced from similar analysis mentioned above? It can be established that surface temperature of heating body is changed by 0,44427 °C, if temperature of outgoing line changes by 1 °C. The industrial thermometers have similar errors in measurement that is parameter uncertainties. So, it is not improbably that the surface temperature of heating body is deviated from designed data. It can be not to be tolerated for example in an industrial chemistry situation. It is established that in everyday using the sensitivity of heating body temperature is good, it has small quantity. In practical point of view, the temperature of heating body is insensitive to t_i room air temperature.

5. SUMMARIZE

The writer of this paper would like to arouse readers' interest in importance and possibilities of use of mathematical model uncertainty analysis. The paper has shown the sensitivity analysis. Then the methodology of sensitivity test based uncertainty analysis has been shown by a short and similar case study of exergy investigation of central heating system. It is important to mention that this aleatory uncertainty analysis is only one part of complete central heating system exergy investigation. The parametric uncertainty investigation of entire system has longer and more complex work.

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