THE TENSILE STRENGTH: THE MOST FUNDAMENTAL MECHANICAL CHARACTERISTICS OF CONCRETE

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Concrete is an inhomogeneous building material. It has a considerable and reliable compressive strength and a relative low tensile strength which can be even exhausted locally under unfortunate conditions. It is quite obvious that the concrete tensile strength was always reprehended as the most unreliable concrete property.

A simple relationship between tensile- and compressive strength is introduced. The mechanical background of the relation tensile- to compressive strength in case of 'normal' and high strength concretes is elucidated. Mechanical bond, too, relies completely on the tensile strength. In the design of structural concrete members the tension fields are more characteristic than the compression fields. Effective concrete strengths are not successful. Tensile strength can be applied as 'yield condition' for the lower bound solution in the theory of plasticity.

The paper intends to contribute to the acceptance of the tensile strength as the more fundamental concrete characteristics.

Keywords: tensile strength, compressive strength, bond, lower bound theorem

1. INTRODUCTION

Concrete is an inhomogeneous building material. It has a considerable and reliable compressive strength and a relative low tensile strength which can be even exhausted locally under unfortunate conditions, e.g. due to hydration heat of cement or to its plastic shrinkage. It is quite obvious that the concrete tensile strength was always reprehended as the most unreliable concrete property.

As the compressive strength of the conventional test specimens (cubes or cylinders) was rather insensitive to most of the aforementioned influences and it was convenient to be measured, it became accepted by the material science, the design office and the construction site as the fundamental mechanical characteristics of concrete.

Several other properties were deduced empirically by the help of best-fit formulas using the compressive strength as basic variable.

According to a sad terminology, students learn to 'neglect' the concrete tensile strength at dimensioning any SC member. Even Model Codes use this verb. In other standards and notebooks tensile strength will be 'ignored'. At dimensioning of watertight or prestressed concrete structures the tensile strength will be relied on with a shy consciousness of guilt.

Reinforced concrete consists of: concrete, reinforcement, discrete cracks and bond.

Loaded in axial tension concrete fails to longitudinal elongation, loaded in axial compression it fails to transversal elongation. In practice these failures are characterized with the stresses deduced from the failure load divided with the specimen's nominal cross section area perpendicular to the direction of the failure load. A special tension loading/failure type is the inclined splitting of concrete cover due to dowel action of a rebar. Here neither the acting load level nor the effected concrete surface is known thus practical detailing rules are given in order to eliminate this type of concrete failure.

Tensile strength is the most fundamental characteristics of concrete.

This paper intends to contribute to the acceptance of the tensile strength as the more fundamental concrete characteristics.

2. THE 'ROLE' OF TENSILE STRENGTH AT THE COMPRESSIVE STRENGTH

Concrete has three constituents: the aggregate, the cement matrix and the bond on the interface between them. Both, the aggregates and the matrix have their Young's Modulus and Poisson's ratio and a bond strength. In concrete classes, say, C50, $E_{ag} > E_{cem}$, Loading a specimen in uniaxial compression (deformation) the compressive trajectories 'run' from the aggregate to aggregate like in a motocross-course. The change of direction of the compressive trajectories cause tensile stresses between the aggregate and cement matrix. The specimen fails at a relative low compressive strength, the ratio compressive to tensile strength is relative low, nevertheless the practice since the beginning of reinforced concrete construction accustomed to praise the low/normal strength concretes on account of its relative high tensile strength.

At design of high strength concretes the concrete technologist knowingly or out of habit improves the Young's Modulus of the cement matrix. Loading a specimen in uniaxial compression (deformation) the compressive trajectories 'run' straight through aggregate and matrices, the bond strength between aggregate and matrix is barely loaded: the specimen fails at a relative high compressive strength. The practice reprehends the high strength concretes for their relative low tensile strength, although HC and UHC should be praised for their relative high compressive strength. Of course in case of loading which causes direct tensile stresses the 'lower' tensile strength should be taken into account.

At design of high strength lightweight concretes the concrete technologist (knowingly or out of habit) reduces the Young's Modulus of the cement matrix to the relative low E_{ag} of the lightweight aggregate, thus achieving a relative homogenous material, due to the 'straight' compressive trajectories a relative high compressive strength will be achieved.

Do we neglect the concrete tensile strength at calculation of ultimate flexural moment?

We do not neglect it at all: the flexural failure occurs in a cracked section where the tensile strength has been exceeded.

Note: The conical failure patterns (well-known from the usual compression tests) are the 'results' of the influence of the friction between the steel loading plate and the specimen, hence causes a false perception in the superficial viewer. Concrete is not a frictional material at all. The Mohr-Coulomb material law is not valid in case of concrete.

3. RATIO OF COMPRESSIVE TO TENSILE STRENGTH

In MC2010 the mean value of uniaxial tensile strength f_{ctm} in [MPa] is defined as:

$f_{ctm} = 0.3\ (f_{ck})^{2/3}$ for concrete grades ≤ C50

$f_{ctm} = 2.12\ \ln(1 + 0.1(f_{ck} + 8))$ for concrete grades > C50

Defining the ratio

$$\chi = f_{ck}/f_{ctm} \quad (1)$$

we get it as function of the characteristic compressive strength, f_{ck} (*Figure 1*).

Figure 1 reveals that the simple linear function

$$\chi = 0.13\ f_{ck} + 6 \quad (2)$$

describes quite exactly the interrelation of tensile to compressive strengths, hence $f_{ctm} = f_{ck} / \chi$.

4. BOND

Without mechanical bond the higher strength rebars could not be exploited economically, the triumphal march of reinforced concrete in the last over 70 years would not be possible.

Bond is a direct consequence of concrete tensile strength, even if *Table 1* taken form MC 2010 (2013) deftly conceals

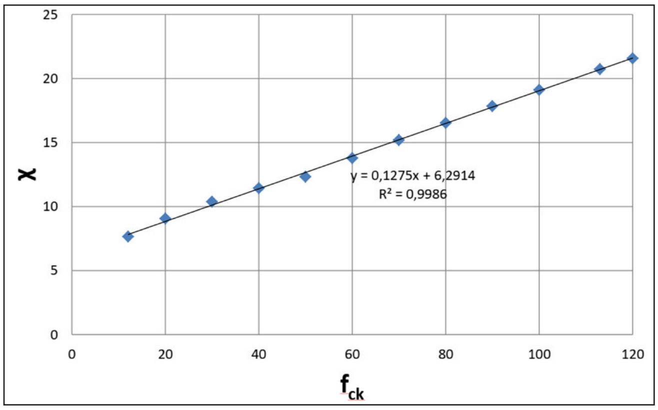

Figure 1: Ratio of compression strength to tensile strength (χ) as function of f_{ck}

this showing different powers < 1 of the mean concrete compressive strength. (Such terms always stay in formulas and equations substituting the tensile strength, which is neglected by the modern r.c. theories and models, isn't it?

Figure 2 shows the analytical bond stress-slip relationship as given in MC 2010.

The mechanical bond between rebar and the concrete around develops when relative displacement occurs between them. The rebar's ribs are supported by the concrete brackets, as shown in *Figure 3*. The slip, as shown in *Figure 2* comes from the deformation of the concrete brackets. The compressive stresses loading the brackets let develop longitudinal and circular tensile stresses, and then first internal cracks (in red in *Figure 3*) which are called Goto-cracks (after the researcher who first showed them). The curved course of the bond stress-slip relationship reveals the influence of

Figure 2: Analytical bond stress-slip relationship (monotonic loading) acc. to MC 2010 (2013)

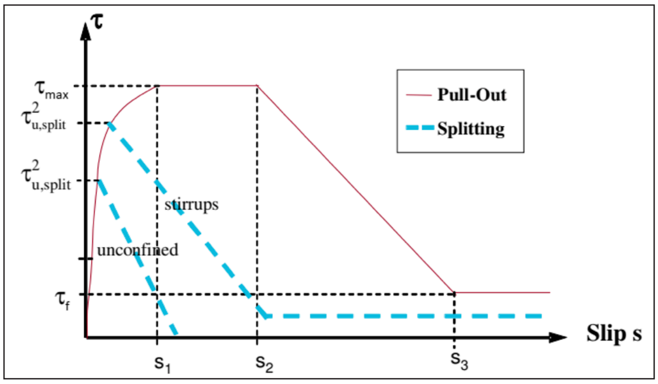

Table 1: Parameters defining the mean bond stress-slip relationship of deformed bars acc. to MC 2010 (2013)

	1	2	3	4	5	6
	Pull-Out (PO)		Splitting (SP)			
	$\varepsilon_s < \varepsilon_{s,y}$		$\varepsilon_s < \varepsilon_{s,y}$			
	Good bond cond.	All other bond cond.	Good bond cond.		All other bond cond.	
			unconfined	stirrups	unconfined	stirrups
τ_{max}	$2.5\sqrt{f_{cm}}$	$1.25\sqrt{f_{cm}}$	$7.0\cdot\left(\frac{f_{cm}}{25}\right)^{0.25}$	$8.0\cdot\left(\frac{f_{cm}}{25}\right)^{0.25}$	$5.0\cdot\left(\frac{f_{cm}}{25}\right)^{0.25}$	$5.5\cdot\left(\frac{f_{cm}}{25}\right)^{0.25}$
s_1	1.0mm	1.8mm	$s(\tau_{max})$	$s(\tau_{max})$	$s(\tau_{max})$	$s(\tau_{max})$
s_2	2.0mm	3.6mm	s_1	s_1	s_1	s_1
s_3	c_{clear} 1)	c_{clear} 1)	$1.2s_1$	$0.5c_{clear}$ 1)	$1.2s_1$	$0.5c_{clear}$ 1)
α	0.4	0.4	0.4	0.4	0.4	0.4
τ_f	$0.40\tau_{max}$	$0.40\tau_{max}$	0	$0.4\tau_{max}$	0	$0.4\tau_{max}$

1) c_{clear} is the clear distance between ribs

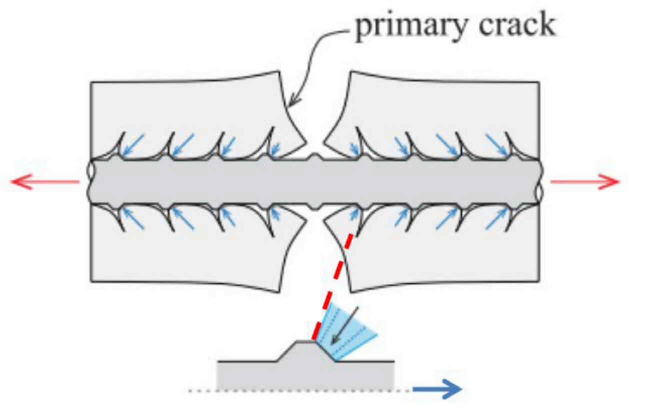

Figure 3: Primary- and Goto-cracks, compressive stresses and crack development at a rebar's rib

the Goto-cracks: the stiffness of bond decreases. In extensive pull-out tests in the Laboratory of the Institute for Reinforced Concrete Structures of TU Budapest, Hungary, Windisch (1984, with the active support of Balázs, that time student there) showed that the bond parameters - identical to s_3 - s_1 and s_2, too depend on the clear rib spacing, hence on the bar diameter. Therefore the constant values given in MC 2010 are not correct. Moreover, in the more than 300 PoT-s the slips on both, loaded and unloaded ends of the specimens were measured: the slips on the unloaded ends show a much stiffer bond characteristics than those measured on the loaded one (Windisch, 1985). Nevertheless, MCs and the producers of FEM models do not take this into account. What a pity!

Increasing the slip the circular tensile stresses around the rebars increase as well. These can cause the longitudinal splitting of the concrete cover which further decreases the bond-stiffness or even leads to total loss if bond (blue lines in *Figure 2*).

Bond completely relies on the tensile strength of concrete.

5. MODELS FOR REINFORCED CONCRETE MEMBERS: COMPRESSION FIELD OR TENSION FIELD?

Soil has, similar to concrete a low tensile strength compared to its compressive strength. Experts of soil mechanics continued to check form, position and load bearing capacity of sliding surfaces in soil structures even after introduction of the theory of plasticity.

Similar to soil structures, the condition of structural concrete members can/could be better described with tension fields than with compression fields, unless the member will become over-reinforced.

Models operating with compression fields (Strut-and-Tie, Modified Compression Field Theory and others) adjust their predicted capacities to the test results applying efficient compressive strength values. Nevertheless, during the decades and hundreds of different applications no generally valid efficient compressive strength value has been found. Each test yield different efficiency factors. The fundamental problem is that in most of the cases compression is not the fundamental variable. A characteristic example is a dapped end with usual reinforcing pattern tested by Desnerck et al. (2018): NS-REF made of C30/37 failed at 402 kN whereas LS-REF (C12/15) at 400 kN. Try to define here a common efficiency factor! For this reason STM and MCFT have - no matter how well they are marketed – no future.

6. APPLICATION OF TENSILE STRENGTH IN THE THEORY OF PLASTICITY

In the theory of plasticity for the approximation of the failure load two limit values can be determined: according to the lower limit - and the upper limit theorem, resp.

Lower bound theorem:

The structure won't collapse if only it is possible to find a statically admissible stress field corresponding with the load. In such situation the bearing capacity is at least the same as the corresponding load or even higher.

Statical admissibility of the stress field requires that:

- stress field is in equilibrium with external load,
- stress field satisfies the internal equilibrium condition,
- stress field satisfies the statical boundary conditions,
- stress do not exceed the limit value.
- a proportional increase in load is assumed. i.e. all loads remain proportional to each other. This allows the entire load system to be controlled with one load parameter. If this is not the case, the load combinations must be examined individually.

If an equilibrium distribution of stress can be found which balances the applied load and nowhere violates the yield criterion, the body (or bodies) will not fail, or will be just at the point of failure.

Upper bound theorem:

The structure will collapse, if only it is possible to find a kinematically admissible velocity field such that total work of external load is not less than total work of internal forces. The bearing capacity is at most the same as the one correspondingly with the load, but it may be lower.

Kinematic admissibility of the velocity field requires that

- velocity field satisfies the kinematic boundary conditions
- velocity field is such that displacement is continuous
- total work of external load on velocity is positive.
- a state of motion is kinematically permissible if the resistance and motion correspond to the flow condition and the flow law and the geometric boundary conditions are observed.
- A mechanism is a kinematically permissible state of motion and has one degree of freedom.

Applying the lower bound theorem in case of concrete structures then the tensile strength can be considered as yield criterion, too.

The notion: "yield condition" and the very often pronounced reference: "tensile strength of concrete will be neglected" led to erroneous perceptions concerning the tensile strength of concrete:

Lower bound failure loads in case of (the letters in italic mark the relevant 'yield criterion'):

• plain concrete members loaded in uniaxial tension

$$N_u = A_c * f_{ct}$$

• unreinforced concrete members loaded in pure flexure

$$M_u = W_c * f_{ct}.$$

Conclusion: the well-known cracking loads are lower bound values.

7. CONCLUSIONS

The tensile strength of concrete is the most fundamental mechanical characteristics of concrete. The compressive strength is a cleverly used tensile strength. The test specimens loaded in pure compression fail when discrete tensile cracks perpendicular to the direction of the compressive load occur. This is valid in case of 2D and 3D compressive loading, too.

Mechanical bond is based on the concrete tensile strength around the rebars. Codes should give direct reference to the role of tensile strength at bond problems.

As in many cases the ultimate load of reinforced concrete members are quite insensitive to variation of the compressive strength, hence models where the basic variable is the effective compressive strength face with serious problems.

The tensile strength of concrete is a fully valid yield criterion for determination of failure loads according to the lower limit theorem.

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NOTATIONS

A_c ideal concrete area

E_{ag}, E_{cem} Young's Modulus of elasticity of aggregates and cement matrix, resp.

N_u, M_u Ultimate cracking tensile force and bending moment of plain concrete members

W_c Cross section modulus of uncracked concrete cross section

c_{clear} clear distance between ribs

f_{ck} characteristic value of concrete compressive strength, MPa

f_{ctm} mean value of uniaxial concrete tensile strength, MPa

s_1, s_2, s_3 characteristic slip values of the analytical bond stress-slip relationships

χ ratio of characteristic value of concrete compressive strength to uniaxial concrete tensile strength

τ bond stress values

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