Report on the work done in 2007

February 26, 2008

1 Network Coding

One of the fastest developing areas of the mathematical theory of communication networks is network coding. The motivation of the theory is the observation that the performance of a network can be considerably improved if nodes are allowed to transmit combinations of the contents of different packages instead of just forwarding the packages themselves. The connectivity properties of the graph or hypergraph describing the network play an important role in the construction of the coding and transmission methods. Graph and hypergraph orientation problems arise when undirected networks are considered. This topic is closely related to several different fields, like Coding Theory and Information Theory. The wide-ranging expertise of the researchers participating in the project offers an excellent possibility for joint work on this topic.

2 High-speed optical networks and new reliable protocols

Perhaps the most important applications of graph connectivity results concern networking. Z. Király and V. Grolmusz [1, 2] constructed highly original routing schemes, capable for directing packets in high-speed optical network designs, where the main bottleneck in the present technology is the low speed of the optical-electronic-optical transitions. The construction is completely optical, and uses non-trivial combinatorial methods for routing. Our aim is to extend this method, to find other applications of the protocols and to disclose the similarities to the network coding.

Pálvölgyi and Király designed a new error-correcting protocol for multicasting and uni-directional transmission [4], the ongoing research should discover the full power and possible practical usage of this protocol.

3 Dual packing theory

Packings in graphs play an important role both in graph theory and in combinatorial optimization. The packing problems have differently looking but similar dual objects, usually called barriers or tight sets, and, in almost all cases, a canonical dual object (see eg. Gallai-Edmonds' theorem). Due to the previously started research [5], there is a possibility that in a general enough setting, using λ-submodularity, and submodularity with respect to intricately defined intersection and union, these all have the same basic underlying structure, i.e., the canonical dual is the "intersection" of all – properly defined – duals. The aim of the present research is to work out the details of this uniform theory.

4 Graph drawing

The two-dimensional representation of graphs is gaining more and more importance since computers appeared and hence made the visualization of large data structures possible. This can be crucial to observe and to better understand them. One very natural criteria is to demand the edges of the planar lay-out of the graph to be straight-line segments and then one can try to minimize the different directions of these segments. Our previous research ([3], [6]) examined this parameter as a function of the maximum degree of the graph to be represented.

5 Codes and Extremal Combinatorics

Miklos Ruszinkó would like to continue his on-going research on codes and extremal set systems. Due to their applications e.g., in molecular biology [9], these objects are getting more and more important. In [11] so far the best known bounds for superimposed codes were obtained and in [10] the multiple user tracing codes were introduced. Some results in [10] were later improved by Alon and Asodi [7], [8], but there is still an exponential gap between the known upper and lower bounds. The goal is to perform research on related extremal and coding problems.

6 Hypergraphs: finite and infinite

Peter Komjath investigated those systems which must always occur in an uncountably chromatic system of k-element set system. This is well described for k=2 but even for k=3 we are far from the complete characterization. A few, rather complicated examples are know, in particular, Andras Hajnal and Komjath [12] proved consistent that there are two triple systems which can separately be omitted but no uncountably chromatic triple system omits both.

In a different direction [13], [14], [15], it is described what configuration must occur if all k-element subsets of a set are colored in such a way that no two disjoint sets get the same color.

Gyula Katona was working on Hamiltonian chains in hypergraphs. The notion of the Hamiltonian chain was defined in his joint paper with Kierstead in 1999. The paper also raised several open questions. One of them, a Dirac-type theorem, was solved meanwhile [16]. Katona has several other papers on related questions (see his publication list enclosed with this proposal). The most interesting open problem is to establish an Ore-type theorem, another is to determine the maximum number of edges in a non-Hamiltonian uniform hypergraph.

7 Combinatorics on Protein 3D Structures

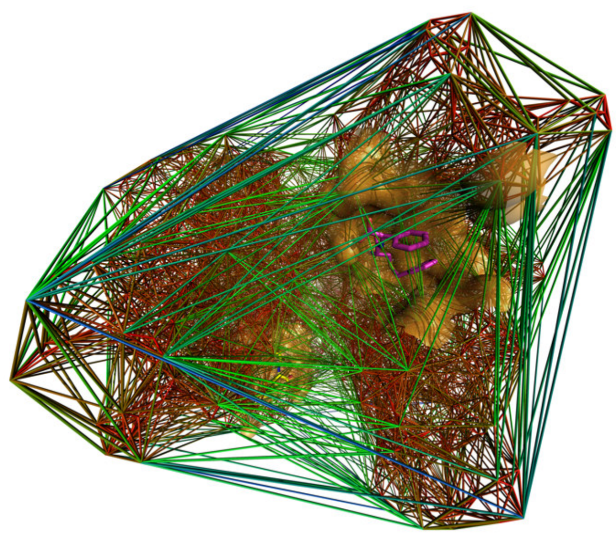

Figure 1: *The Delaunay decomposition of the PDB entry 10gs.*

The fast growing Protein Data Bank (PDB) (http://www.rcsb.org/pdb/home/home.do) contains more than 46,000 entries, each describing 3-dimensional structure of protein and nucleic-acid molecules. In an earlier work of ours, this huge database was rigorously re-structured and cleaned of errors, resulting in the Rich Structure Protein Data Bank (RS-PDB) [17], [18]. In this cleaning procedure we assigned a graph-structure to the proteins, where differently labeled edges correspond to differently bounded atom-pairs. The database contains around 18 million atoms. This cleaning-procedure makes possible to apply combinatorial methods for the otherwise unstructured data. In [19] sequential and spatial characteristics of the proteins were examined; in [20] a combinatorial structure to discretize all the known 3D protein molecules (see Fig. 1) was defined on the atoms of the proteins.

References

[1] V. Grolmusz, Z. Király: Secure routerless routing, *Proc. ACM SIGCOMM workshop on Future Directions in Network Architecture* (2004), 21–27.

[2] V. Grolmusz, Z. Király: Generalized Secure Routerless Routing, Proc. Networking—ICN 2005 Lecture Notes in Comp. Sci. **3421** (2005), 454–462.

[3] B. Keszegh, J. Pach, D. Pálvölgyi, and G. Tóth: Drawing cubic graphs with at most five slopes, in: Graph Drawing 2006, Lecture Notes in Computer Science 4372, Springer, (2007), 114-125.

[4] Z. Király, D. Pálvölgyi: Reliable Data Transmitting Without Feedback, Proc. HSN workshop, (2007), Balatonkenese

[5] Z. Király: λ-supermodular functions and dual packing theory, Proc. The 5th Hungarian-Japanese Symposium on Discrete Mathematics and Its Applications, (2007), Sendai

[6] J. Pach and D. Pálvölgyi: Bounded-degree graphs can have arbitrarily large slope numbers, *Electronic J. Combinatorics* **13**/1 (2006), N1.

[7] N. Alon, V. Asodi: Tracing a single user, *European Journal of Combinatorics*, 27(8), 2006, pp. 1227-1234.

[8] N. Alon, V. Asodi, Tracing many users with almost no rate penalty, *IEEE Transactions on Information Theory*, 53 (1), 2007, pp. 437-439.

[9] M. Csürös, A. Milosavljevic, A Pooled Genomic Indexing (PGI): Analysis and design of experiments, *Journal of Computational Biology,* 11(5), 2004, pp. 1001-1021.

[10] M. Csürös and M. Ruszinkó, Single-user tracing and disjointly superimposed codes, *IEEE Transactions on Information Theory*, 51(4), 2005, pp. 1606-1611.

[11] M. Ruszinkó, On the Upper Bound of the Size of the r-Cover-Free families, Journal of Combinatorial Theory, Series A, 66(2), 1994, pp. 302-310.

[12] A. Hajnal, P. Komjath: Obligatory subsystems of triple systems, to appear

[13] P. Komjath: An uncountably chromatic triple system, manuscript.

[14] C. Avart, P. Komjath, T. Luczak, V. Rodl: Colorful flowers, to appear.

[15] P. Komjath: Some remarks on colorful flowers, manuscript

[16] Rucinski, Rodl and Szemeredi *A Dirac-type theorem for 3-uniform hypergraphs, Combinatorics Probability & Computing* 15 (1-2): pp 229-251 (2006))

[17] Z. Szabadka, V. Grolmusz: High Throughput Processing of the Structural Information of the Protein Data Bank, *Journal of Molecular Graphics and Modeling*, 25 (2007) pp. 831-836

[18] Z. Szabadka, V. Grolmusz: Building a Structured PDB: The RS-PDB Database, *Proceedings of the 28th IEEE EMBS Annual International Conference*, New York City, Aug. 30-Sept 3, 2006., pp. 5755-5758.

[19] G. Iván: An optimized distance function for comparison of protein binding sites, *Proceedings of the Second Brazilian Symposium on Bioinformatics, Angra dos Reis, Brazil, August 2007; Springer Lecture Notes in Bioinformatics* No. 4643, pp. 93-100.

[20] R. Ördög, Z. Szabadka, V. Grolmusz: Analyzing the Simplicial Decomposition of Spatial Protein Structures, BMC Bioinformatics 2008, 9(Suppl 1):S11.