Some applications of possibilistic mean value, variance, covariance and correlation[1]

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Abstract: In 2001 we introduced the notions of possibilistic mean value and variance of fuzzy numbers. In this paper we list some works that use these notions. We shall mention some application areas as well.

1 Possibilistic mean value, variance, covariance and correlation

A *fuzzy number* A is a fuzzy set $\mathbb{R}$ with a normal, fuzzy convex and continuous membership function of bounded support. The family of fuzzy numbers is denoted by $\mathcal{F}$. Fuzzy numbers can be considered as possibility distributions [11, 15]. A fuzzy set C in $\mathbb{R}^2$ is said to be a joint possibility distribution of fuzzy numbers $A, B \in \mathcal{F}$, if it satisfies the relationships $\max\{x \mid C(x,y)\} = B(y)$ and $\max\{y \mid C(x,y)\} = A(x)$ for all $x, y \in \mathbb{R}$. Furthermore, A and B are called the marginal possibility distributions of C. Let $A \in \mathcal{F}$ be fuzzy number with a γ-level set denoted by $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$, $\gamma \in [0,1]$. A function $f\colon [0,1] \to \mathbb{R}$ is said to be a weighting function if f is non-negative, monoton increasing and satisfies the following normalization condition

$$\int_0^1 f(\gamma) d\gamma = 1.$$

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Different weighting functions can give different (case-dependent) importances to γ-levels sets of fuzzy numbers. It is motivated in part by the desire to give less importance to the lower levels of fuzzy sets [14] (it is why f should be monotone increasing).

The f-weighted *possibilistic mean value* of $A \in \mathcal{F}$, defined in [12], is defined as

$$E_f(A) = \int_0^1 E(U_\gamma) f(\gamma) d\gamma = \int_0^1 \frac{a_1(\gamma) + a_2(\gamma)}{2} f(\gamma) d\gamma,$$

where U_γ is a uniform probability distribution on $[A]^\gamma$ for all $\gamma \in [0, 1]$. If $f(\gamma) = 1$ for all $\gamma \in [0, 1]$ then we get

$$E(A) = \int_0^1 E(U_\gamma) f(\gamma) d\gamma = \int_0^1 \frac{a_1(\gamma) + a_2(\gamma)}{2} d\gamma,$$

which the possibilistic mean value of A originally introduced in [3].

The f-weighted *possibilistic variance* of $A \in \mathcal{F}$, defined in [12], can be written as

$$\mathrm{Var}_f(A) = \int_0^1 \sigma^2_{U_\gamma} f(\gamma) d\gamma = \int_0^1 \frac{(a_2(\gamma) - a_1(\gamma))^2}{12} f(\gamma) d\gamma.$$

If $f(\gamma) = 1$ for all $\gamma \in [0, 1]$ then we get

$$\mathrm{Var}(A) = \int_0^1 \sigma^2_{U_\gamma} d\gamma = \int_0^1 \frac{(a_2(\gamma) - a_1(\gamma))^2}{12} d\gamma.$$

which the possibilisticvariance of A originally introduced in [3].

The f-weighted *measure of possibilistic covariance* between $A, B \in \mathcal{F}$, (with respect to their joint distribution C), defined by [13], can be written as

$$\mathrm{Cov}_f(A, B) = \int_0^1 \mathrm{Cov}(X_\gamma, Y_\gamma) f(\gamma) d\gamma,$$

where X_γ and Y_γ are random variables whose joint distribution is uniform on $[C]^\gamma$ for all $\gamma \in [0, 1]$.

The f-weighted *possibilistic correlation* of $A, B \in \mathcal{F}$, (with respect to their joint distribution C), defined in [9], can be written as

$$\rho_f(A, B) = \frac{\int_0^1 \mathrm{cov}(X_\gamma, Y_\gamma) f(\gamma) d\gamma}{\left(\int_0^1 \sigma^2_{U_\gamma} f(\gamma) d\gamma\right)^{1/2} \left(\int_0^1 \sigma^2_{V_\gamma} f(\gamma) d\gamma\right)^{1/2}}$$

where V_γ is a uniform probability distribution on $[B]^\gamma$. Thus, the possibilistic correlation represents an average degree to which X_γ and Y_γ are linearly associated as compared to the dispersions of U_γ and V_γ.

It is clear that the standard probabilistic calculation is not used here. Really, a standard probabilistic calculation might be the following

$$\frac{\int_0^1 \operatorname{cov}(X_\gamma, Y_\gamma) f(\gamma) d\gamma}{\left(\int_0^1 \sigma^2_{X_\gamma} f(\gamma) d\gamma \right)^{1/2} \left(\int_0^1 \sigma^2_{Y_\gamma} f(\gamma) d\gamma \right)^{1/2}}.$$

That is, the standard probabilistic approach would use the marginal distributions, X_γ and Y_γ, of a uniformly distributed random variable on the level sets of $[C]^\gamma$.

Theorem 1.1 ([9]). *If $[C]^\gamma$ is convex for all $\gamma \in [0,1]$ then $-1 \leq \rho_f(A,B) \leq 1$ for any weighting function f.*

2 Applications

The possibilistic mean value, variance, covariance and correlation have been extensively used in our later works for real option valuation [4, 8], project selection [2, 5, 6, 10], capital budgeting [1] and optimal portfolio selection [7]. The concept of possibilistic mean value and variance is used in many different areas and by many different authors (see [16-63]). For example, these notions are applied by Zhang, et al [16] when they investigate possibilistic mean-variance models in portfolio selection problems; Dutta et al [17] when they investigate a continuous review inventory model in a mixed fuzzy and stochastic environment; Thiagarajah et al [20] when they introduce an option valuation model with adaptive fuzzy numbers; Dubois [23] when he discusses possibility theory and statistical reasoning; Beynon and Munday [25] when they elucidate of multipliers and their moments in fuzzy closed Leontief input-output systems; Zarandi et al [41] when they present an intelligent agent-based system for reduction of the bullwhip effect in supply chains; Lazo et al [51] when they determine of real options value by Monte Carlo simulation and fuzzy numbers. The notion of possibilistic correlation is used in [64-65]. The notions of f-weighted possibilistic mean value and variance are used in [66-77]. For example, Zhang and Li [73] use these notions to portfolio selections problems with quadratic utility function in a fuzzy environment; and Garcia [76] uses these notions to fuzzy real option valuation in a power station reengineering project.

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