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Investigation of metal built-up columns, Part I: Formulae

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ABSTRACT

Eurocodes give guidance how to design built-up columns having effective bending stiffness, smeared shear stiffness and local bow imperfection amplitude $e_0 = L/500$ under compression. The guidance is valid only for columns supported by hinges at their ends. The second order theory is presented, which allows analysis of the battened and laced built-up columns with initial imperfection under combined compression and bending with the bottom end fixed and the upper one free in the case of in-plane buckling. The application of the theory in several numerical examples is given in Part II.

KEYWORDS

built-up columns, stability, second order analysis, steel, aluminum alloy

1. INTRODUCTION

Every point on a beam-column axis undergoes displacements as a consequence of loads or restraints. The beam-column axis moves to a new, deformed position. Displacement distribution describes the shape of the function of the displacement components along the beam-column axis. Displacement distribution for the displacement components perpendicular to the beam-column axis are called deflection curves. To determine the exact shape and the internal forces, it is necessary to integrate the corresponding differential equations that link the given actions with displacement required.

The behavior of the battened and laced built-up beam-columns with sway and bow initial imperfections under combination of different external actions with various boundary conditions is investigated because the current Eurocode [1] give the rules only for limited cases. Another reasons why the built-up columns are investigated are the facts that the published numerical examples: are not complete; contain incorrect verification of out-of-plane buckling resistance of the column under combination of the axial and horizontal forces, do not solve built-up columns under combination of axial force and biaxial bending; do not solve built-up columns made from aluminum alloys.

2. DIFFERENTIAL EQUATIONS FOR STRAIGHT UNIFORM BEAM-COLUMN UNDER IN-PLANE LOADING

2.1. Non-linear differential equation of the large elastic displacements theory

Behavior of the column under axial compression force F with the large elastic displacements may be described by the following non-linear differential equation:

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$$\kappa(x) = -\frac{w''(x)}{\{1+[w'(x)]^2\}^{\frac{3}{2}}},\ \ M(x) = EI\kappa(x),\ \ EI\frac{w''(x)}{\{1+[w'(x)]^2\}^{\frac{3}{2}}} = -M(x), \tag{1}$$

where EI is the bending stiffness; M is the bending moment; w is the deflection and κ is the curvature. For the column with the length L, the bottom end ($x = 0$) fixed and the upper end ($x = L$) free the bending moment is

$$M(x) = -F[w_L - w(x)]. \tag{2}$$

The solution of the non-linear Eq. (1) leads to the solution of Eq. (3) enabling the calculation of the displacement $w(x)$ also for the axial force F greater than the critical force F_{cr}. The following equation contains complete elliptic integral of the first kind. It is called complete if the upper bound equals $\pi/2$. The quantity φ is called amplitude of elliptic integral and $\sin(\vartheta)$ is called module of elliptic integral

$$\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\phi}{\sqrt{1-[\sin(\phi)]^2[\sin(\vartheta)]^2}} - \frac{\pi}{2}\sqrt{\frac{F}{F_{cr}}} = 0. \tag{3}$$

The solution of Eq. (3) may be performed with Computer Algebra System (CAS). In the past it was necessary to use special tables of elliptic integrals. For $F/F_{cr} = 1.01$ the solution of Eq. (3) gives the value $\vartheta = 8.074$. The relative horizontal displacement of the column top w_L/L may be then calculated as follows:

$$\frac{w_L}{L} = 2\sin(\vartheta)\frac{2}{\pi}\sqrt{\frac{F_{cr}}{F}} = 0.178. \tag{4}$$

The curvature κ in Eq. (1) may be for building and civil engineering structures approximated by Eq. (5) first part because $1+(w')^2 \approx 1{,}000$. Designers in the practice therefore work with the differential Eq. (5) second part describing behavior of the beam with the small elastic displacement

$$\kappa(x) \approx -w''(x),\ EIw''(x) = -M(x). \tag{5}$$

The procedure using Eq. (1) is sometimes called the theory of the third order, what is not very convenient. In this theory non-linearity of static expressions and geometrical equations are utilized and the functions $\cos(\varphi)$, $\sin(\varphi)$ and $\tan(\varphi)$ are not approximated by the values 1, φ respectively, as it is supposed in the theory of the second order. The theory of large displacements (theory of the third order) may be necessary to perform when analyzing very flexible systems, e.g. cable-net structures.

2.2. Differential equation of the small elastic displacements theory with effect of shear forces

In the second order theory it is supposed: an equilibrium is fulfilled on the due loading deformed structure; the geometrical expressions are linearized (6); the elasticity expressions, e.g., (7), are linear; and the approximations (8) are accepted,

$$\begin{aligned}\kappa(x) &= -\frac{w''(x)}{\{1+[w'(x)]^2\}^{\frac{3}{2}}} \approx -w''(x),\\ \varepsilon_x &= u'(x) + \frac{1}{2}\left\{[u'(x)]^2 + [w'(x)]^2\right\} \approx u'(x),\end{aligned} \tag{6}$$

$$M(x) = EI\kappa(x), \tag{7}$$

$$\cos(\varphi) \approx 1,\ \ \sin(\varphi) \approx \varphi,\ \ \tan(\varphi) \approx \varphi. \tag{8}$$

Differential equation of the small elastic displacements theory with effect of the shear forces is:

$$w''(x) = -\kappa(x) + \gamma_v'(x),\ \ w''(x) = -\frac{M(x)}{EI} + \frac{V'(x)}{GA_V}, \tag{9}$$

where GA_V is the shear stiffness; V is the shear force and γ_v is the shear strain. Compared with Eq. (5) the quantity w'' in Eq. (9) is increased due to the effect of the shear forces. The relevant shear areas are defined by the formulae

$$A_{Vz} = \frac{I_y^2}{\int_A \left[\frac{S_y(s)}{t(s)}\right]^2 \mathrm{d}A},\ \ A_{Vy} = \frac{I_z^2}{\int_A \left[\frac{S_z(s)}{t(s)}\right]^2 \mathrm{d}A}. \tag{10}$$

2.3. General differential equation of the small elastic displacement theory

The above differential equations of the second order may be written in the form of the differential equations of the fourth order (12). The following one in the most general form is frequently used in engineering practice for a lot of various problems: beams with shear stiffness GA_V or without the effect of shear forces ($GA_V = \infty$ kN); beams with the transverse loading (e.g., q) without or with axial tension force (N is positive); beams with the transverse loading (e.g., q) with or without axial compression force (N is negative); eventually beams on the elastic foundation ($k_f \neq 0$ kNm^{-2}). Thanks to the analogies of different differential equations of the various problems this type of differential equation (11) may be used also for the special problems of mixed, warping or uniform (St. Venant) torsion, distortion, pipelines under earth pressure or cylindrical shells under rotational symmetrical loading,

$$\begin{aligned} w^{IV}(x,t) &- \frac{\frac{N}{EI} + \frac{EI}{GA_v}\frac{k_f}{EI}}{1+\frac{N}{EI}\frac{EI}{GA_V}} w''(x,t)\\ &+ \frac{\frac{k_f}{EI}}{1+\frac{N}{EI}\frac{EI}{GA_V}} w(x,t) = \frac{\frac{q(x)}{EI}}{1+\frac{N}{EI}\frac{EI}{GA_V}},\end{aligned} \tag{11}$$

where k_f [kNm^{-2}] is the modulus of the foundation; t [s] is the time. With dimensionless quantities:

$$\begin{aligned}\xi &= \frac{x}{L},\ \ \varepsilon^2 = \frac{N}{EI}L^2,\ \ \overline{\kappa} = \frac{EI}{GA_V}\frac{1}{L^2},\ \ \rho^2 = \frac{k_f}{EI}L^4,\\ \varsigma &= \frac{1}{2}\frac{\varepsilon^2 + \overline{\kappa}\rho^2}{1+\overline{\kappa}\rho^2},\ \ \eta = \sqrt{\left|\frac{\rho^2}{1+\overline{\kappa}\rho^2}\right|}.\end{aligned} \tag{12}$$

Equation (11) may be rewritten in the following forms (13) or (14):

$$\begin{aligned} v^{IV}(\xi,t) &- \frac{\varepsilon^2 + \overline{\kappa}\rho^2}{1+\overline{\kappa}\varepsilon^2} v''(\xi,t) + \frac{\rho^2}{1+\overline{\kappa}\varepsilon^2} v(\xi,t)\\ &= \frac{1}{1+\overline{\kappa}\varepsilon^2}\frac{q(\xi)}{EI}L^4,\end{aligned} \tag{13}$$

$$v^{IV}(\xi,\, t) - 2\varsigma v^{''}(\xi,\, t) + \text{sign}\left(\rho^2\right)\eta^2 v(\xi,\, t) = \frac{1}{1+\bar{\kappa}\varepsilon^2}\frac{q(\xi)}{EI}L^4. \tag{14}$$

2.4. Differential equation for the bending moment

In the case of stability problems and similar issues, it is no longer possible to consider all displacement components as small, and the equilibrium conditions must be formulated for the deformed structural system (the second-order theory). The superposition law no longer applies to these problems. Similarly, the superposition law does not apply to problems with material non-linearity (e.g. the elastic-plastic and rigid-plastic systems).

The differential equation of the beam with local bow initial imperfection under transverse loadings (e.g. q) and compression axial force N has for the bending moment M the form [2]:

$$M^{''}(\xi) + \varepsilon_v^2 M(\xi) = -\gamma\left[q(\xi)L^2 + 8Ne_0 + NL^2\kappa_e\right], \tag{15}$$

where e_0 is the amplitude of the local bow initial imperfection in the form of the quadratic parable (very similar to sinus) and κ_e is the given curvature, e.g. from the change of the temperature. The new dimensionless parameters are the beam shear factor γ and the beam factor ε_V, which are used in the second-order analysis. In the case of the first-order theory $\varepsilon = 0$. If the effect of the shear forces is negligible $\gamma = 1$ because the shear stiffness $S_v = GA_V \to \infty$ kN:

$$\gamma = \frac{1}{1-\frac{N}{S_v}},\quad \varepsilon_v = L\sqrt{\frac{\gamma N}{EI}} = \sqrt{\gamma}\varepsilon. \tag{16}$$

It is supposed that the normal force N and the shear stiffness $S_v = GA_V$ are constants.

3. THE METHODS FOR CALCULATION OF INTERNAL FORCES AND DISPLACEMENTS

There are many computer programs, which may be used for calculation of internal forces and displacements. Nevertheless, for explanations of the mutual relations between quantities, for creating and applying differential equations, for understanding and education of load-bearing behavior and for verification of computer results, the following methods and procedures may be convenient and helpful. They are based on: virtual work; iterative calculation; equivalent horizontal forces; amplification factors; solution of differential equations.

The principle of virtual work and iterative calculation methods will not be discussed in this paper.

3.1. Equivalent horizontal forces

Appropriate allowances should be incorporated in the structural analysis to cover the effects of imperfections, including residual stresses and geometrical imperfections, which can be a lack of verticality, a lack of straightness, a lack of flatness and a lack of fit eccentricities present in joints of the unloaded structure. According to [1, 2] equivalent geometric imperfections should be used with values, which reflect the possible effects of all type of imperfections unless these effects are included in the resistance formulae for column design. The following imperfections should be taken into account: global imperfections for frames and bracing systems; local imperfections for individual columns.

The global initial sway imperfections and the local initial bow imperfection amplitude may be determined according to the clause 5.3.2, *Fig.* 5.4 in [1].

3.2. Amplification factors, critical force and shear stiffness

The internal forces and moments may generally be determined using either: first-order analysis, using the initial geometry of the structure or second-order analysis, taking into account the influence of the displacement of the structure. The effects of the deformed geometry (second-order effects) should be considered if they increase the action effects significantly or modify significantly the structural behavior. First order analysis may be used for the structure, if the increase of the relevant internal forces or moments or any other change of structural behavior caused displacements can be neglected. This condition may be assumed to be fulfilled, if the following criterion is satisfied:

$$\begin{aligned}\alpha_{cr} &= \frac{F_{cr}}{F_{Ed}} \geq 10, \quad \text{for elastic analysis},\\ \alpha_{cr} &= \frac{F_{cr}}{F_{Ed}} \geq 15, \quad \text{for plastic analysis},\end{aligned} \tag{17}$$

where α_{cr} is the factor by which the design loading would have to be increased to cause elastic instability in a global mode; F_{Ed} is the design loading on the structure and F_{cr} is the elastic critical buckling load for global instability mode based on initial elastic stiffness.

The second order effects may be calculated by using an analysis appropriate to the structure (including step-by-step or other iterative procedures). For frames where the first sway buckling mode is predominant first order elastic analysis should be carried out with subsequent amplification of relevant action effects (bending moments, shear forces, displacements) by appropriate amplification factor,

$$k_{II} = \frac{1}{1-\frac{1}{\alpha_{cr}}} = \frac{\alpha_{cr}}{\alpha_{cr}-1}. \tag{18}$$

The quantities of the second order theory may by calculated by amplifying quantities of the first order theory by amplifying factor k_{II},

$$M_{II} = k_{II}M_I,\quad V_{II} = k_{II}V_I,\quad w_{II} = k_{II}w_I. \tag{19}$$

The formulae (19) give: exact values of M_{II}, V_{II}, w_{II} if the shape of displacement due to loading obtained in the first order analysis is affine to the shape of the buckling mode; approximate values of M_{II}, V_{II}, w_{II}, if the similarity of the shape of displacement due to loading obtained in the first order analysis with the shape of the buckling mode is at least quantitative; otherwise the formulae must not be used.

The exactness of the approximate values of M_{II}, V_{II}, w_{II} may be increased by the well-known Dischinger's factors δ and δ_w

$$M_{II} \approx \frac{1+\delta}{1-\frac{1}{\alpha_{cr}}} M_I, \quad V_{II} \approx \frac{1+\delta}{1-\frac{1}{\alpha_{cr}}} V_I, \quad w_{II} \approx \frac{1+\delta_w}{1-\frac{1}{\alpha_{cr}}} w_I. \qquad (20)$$

The critical force $N_{cr,v}$ should be calculated according to Eq. (22), if the effect of the shear forces is taken into account:

$$k_{II,v} = \frac{1}{1-\frac{1}{\alpha_{cr,v}}} = \frac{\alpha_{cr,v}}{\alpha_{cr,v}-1}, \quad \alpha_{cr,v} = \frac{F_{cr,v}}{F_{Ed}}, \qquad (21)$$

$$\frac{1}{N_{cr,v}} = \frac{1}{N_{cr}} + \frac{1}{S_v}, \quad N_{cr,v} = \frac{N_{cr}}{1+\frac{N_{cr}}{S_v}}. \qquad (22)$$

The shear stiffness for the individual single column is $S_v = GA_V$. The relevant shear areas A_V are defined in Eq. (10). The shear stiffness GA_V of the individual single columns is relatively large and therefore the influence of shear forces is usually neglected. But its influence is necessary to take into account in the cases with extremely large tension axial forces. This is the case of warping torsion of thin-walled closed profiles where the large torsional stiffness GI_t plays in analogy an extremely large fictitious axial tension force.

The following shear stiffness for the battened and laced built-up members is defined in Eurocode [1]. The shear stiffness of battened built-up members is

$$S_v = \frac{24EI_{ch,z}}{a^2\left[1+\frac{2EI_{ch,z}}{n_b I_b}\frac{h_0}{a}\right]}, \quad S_{v,max}\frac{2\pi^2 EI_{ch,z}}{a^2}, \qquad (23)$$

where $EI_{ch,z}$ is the bending stiffness of the chord; n_b and I_b are the number of battens in a panel and the second moment of the batten, respectively; h_0 is the distance of the centers of gravities of the chords and a is the distance of the battens. The shear stiffness of lacings of built-up columns is given in *Fig.* 6.9 [1].

The shear stiffness of the laced built-up columns is usually greater than the shear stiffness of the battened built-up columns due to larger value of the distance of the centers of gravities of the chords.

The critical force of the flexural buckling may be calculated from the basic formula

$$N_{cr} = \frac{\pi^2 EI}{L_{cr}^2}, \quad L_{cr} = kL, \qquad (24)$$

where L_{cr} is the buckling length. The value of k, the buckling length factor for columns, should be assessed from knowledge of the end conditions. Unless a more accurate analysis is carried out, the value of k from Table 1 may be used.

Buckling length factors k^* are increased for columns with fixed end(s) in comparison with theoretical values k. The comparisons of k^* values with theoretical ones k are presented in Table 1.

In the most European countries are used the theoretical k values in the design practice but for example in UK and Australia are used the increased k^* values. Eurocode [1] uses the theoretical k values.

The critical force $N_{cr,v}$ should be calculated according to Eq. (22) and the following is valid:

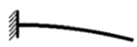
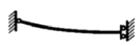
$$N_{cr,v} < N_{cr}, \quad L_{cr,v} > L_{cr}. \qquad (25)$$

3.3. Solutions of differential equation by Rubin

Rubin published in ref. [2] the formulae for calculation of beam bending moments $M(x)$, shear forces $V(x)$, displacement $w(x)$ and rotation $\varphi(x)$. There are formulae there for 12 various actions, including effect of global and local initial imperfections, non-uniform temperature and unit discontinuities and displacements. These formulae are applicable for 4 different boundary conditions:

Case 1: rotation and translation are fixed at both ends (Item 1 in Table 1 is called Euler's case 4);
Case 2: rotation and translation are fixed at one end, rotation is free and translation is fixed at other end (Item 2 in Table 1 is called Euler's case 3);
Case 3: rotation is free and translation is fixed at both ends (Item 3 in Table 1 is called Euler's case 2);

Table 1. Buckling length factor k^* compared with theoretical k values

End conditions: a) rotation; b) displacement	Scheme	k^*	k
1. a): clamped-clamped; b): fixed-fixed		0.7	0.5
2. a): clamped-hinge; b): fixed-fixed		0.85	0.7
3. a): hinge-hinge; b): fixed-fixed		1.0	1.0
4. a): clamped-clamped; b): fixed-free		1.25	1.0
5. a): clamped-partially restrained; b): fixed-free		1.5	?[1)]
6. a): clamped-free; b): fixed-fixed (cantilever)		2.1	2.0
7. a): hinge-elastically clamped; b): fixed-free		≥ 2.1[1)]	2.0

[1)] Depends on stiffness at the clamped end.

Case 4: rotation and translation are fixed at one end, both are free at other end (Item 6 in Table 1 is called Euler's case 1).

Rubin's formulae give exact values for both theories: theory of the first or the second order and they take or do not take into account the influence of shear forces. These formulae are very convenient for performing large parametrical studies.

If the column with global sway initial imperfection Φ has the boundary conditions defined by the case 6 given in Table 1 and it is loaded by the combination of external axial compression force N_{Ed} located in the point $x_N = L$ m ($\xi_N = x_N/L = 1.0$); horizontal force H_{Ed} located in interval 0 m $\le x_H \le L$ m ($0 \le \xi_H = x_H/L \le 1.0$) and bending moment $M_{Ed,e}$ located in 0 m $\le x_M \le L$ m ($0 \le \xi_M = x_M/L \le 1.0$), the following formulae may be used for the calculation of the bending moment and shear force.

The bending moment due to horizontal force $H_{Ed,tot}$ for $\xi \le \xi_H$ is:

$$M_{Ed}^{H}(\xi) = \left\{ \begin{array}{c} \dfrac{\gamma \sin[(1-\xi_H)\varepsilon]\sin(\xi\varepsilon)}{\varepsilon \sin(\varepsilon)} - \\ -\dfrac{\gamma \sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} \dfrac{\sin(\varepsilon) - \sin[(1-\xi_H)\varepsilon]}{\varepsilon \cos(\varepsilon)} \end{array} \right\} L H_{Ed,tot}. \tag{26}$$

The bending moment due to horizontal force $H_{Ed,tot}$ for $\xi > \xi_H$ is:

$$M_{Ed}^{H}(\xi) = \left\{ \begin{array}{c} \dfrac{\gamma \sin(\xi_H\varepsilon)\sin[(1-\xi)\varepsilon]}{\varepsilon \sin(\varepsilon)} - \\ -\dfrac{\gamma \sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} \dfrac{\sin(\varepsilon) - \sin[(1-\xi_H)\varepsilon]}{\varepsilon \cos(\varepsilon)} \end{array} \right\} L H_{Ed,tot}, \tag{27}$$

$$H_{Ed,tot} = H_{Ed} + N_{Ed}\Phi. \tag{28}$$

The bending moment due to external bending moment $M_{Ed,e}$ for $\xi \le \xi_M$ is:

$$M_{Ed}^{M}(\xi) = \left\{ \begin{array}{c} -\dfrac{\cos[(1-\xi_M)\varepsilon]\sin(\xi\varepsilon)}{\sin(\varepsilon)} - \\ -\dfrac{\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} \dfrac{\cos[(1-\xi_M)\varepsilon]}{\cos(\varepsilon)} \end{array} \right\} M_{Ed,e}. \tag{29}$$

The bending moment due to external bending moment $M_{Ed,e}$ for $\xi > \xi_M$ is:

$$M_{Ed}^{M}(\xi) = \left\{ \begin{array}{c} \dfrac{\cos(\xi_M\varepsilon)\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} - \\ -\dfrac{\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} \dfrac{\cos[(1-\xi_M)\varepsilon]}{\cos(\varepsilon)} \end{array} \right\} M_{Ed,e}, \tag{30}$$

$$M_{Ed}(\xi) = M_{Ed}^{H}(\xi) + M_{Ed}^{M}(\xi). \tag{31}$$

If the column with local bow initial imperfection has the boundary conditions defined by the case 3 given in Table 1 and it is loaded by the combination of external axial compression force N_{Ed} located in the point $x_N = L$ m ($\xi_N = x_N/L = 1.0$); horizontal force H_{Ed} located in interval 0 m $\le x_H \le L$ m ($0 \le \xi_H = x_H/L \le 1.0$); bending moment $M_{Ed,e,1}$ located in interval 0 m $\le x_{M1} \le L$ m ($0 \le \xi_{M1} = x_{M1}/L \le 1.0$) and bending moment $M_{Ed,e,2}$ located in the interval 0 m $\le x_{M2} \le L$ m ($0 \le \xi_{M2} = x_{M2}/L \le 1.0$), the following formulae may be used for the calculation of the bending moment and shear force distributions.

The bending moment due to horizontal force H_{Ed} for $\xi \le \xi_H$ is:

$$M_{Ed}^{H}(\xi) = \left\{ \frac{\gamma \sin[(1-\xi_H)\varepsilon]\sin(\xi\varepsilon)}{\varepsilon \sin(\varepsilon)} \right\} L H_{Ed}. \tag{32}$$

The bending moment due to horizontal force H_{Ed} for $\xi > \xi_H$ is:

$$M_{Ed}^{H}(\xi) = \left\{ \frac{\gamma \sin(\xi_H\varepsilon)\sin[(1-\xi)\varepsilon]}{\varepsilon \sin(\varepsilon)} \right\} L H_{Ed}. \tag{33}$$

The bending moment due to external bending moment $M_{Ed,e,1}$ for $\xi \le \xi_{M1}$ is:

$$M_{Ed}^{M1}(\xi) = \left\{ \begin{array}{c} -\dfrac{\cos[(1-\xi_{M1})\varepsilon]\sin(\xi\varepsilon)}{\sin(\varepsilon)} - \\ -\dfrac{\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} \dfrac{\cos[(1-\xi_{M1})\varepsilon]}{\cos(\varepsilon)} \end{array} \right\} M_{Ed,e,1}. \tag{34}$$

The bending moment due to external bending moment $M_{Ed,e,1}$ for $\xi > \xi_{M1}$ is:

$$M_{Ed}^{M1}(\xi) = \left\{ \begin{array}{c} \dfrac{\cos[(\xi_{M1}\varepsilon)]\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} - \\ -\dfrac{\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} \dfrac{\cos[(1-\xi_{M1})\varepsilon]}{\cos(\varepsilon)} \end{array} \right\} M_{Ed,e,1}. \tag{35}$$

The bending moment due to external bending moment $M_{Ed,e,2}$ for $\xi < \xi_{M2}$ is:

$$M_{Ed}^{M2}(\xi) = \left\{ \begin{array}{c} -\dfrac{\cos[(1-\xi_{M2})\varepsilon]\sin(\xi\varepsilon)}{\sin(\varepsilon)} - \\ -\dfrac{\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} \dfrac{\cos[(1-\xi_{M2})\varepsilon]}{\cos(\varepsilon)} \end{array} \right\} M_{Ed,e,2}. \tag{36}$$

The bending moment due to external bending moment $M_{Ed,e,2}$ at the bottom end for $\xi \ge \xi_{M2}$ is:

$$M_{Ed}^{M2}(\xi) = \left\{ \begin{array}{c} \dfrac{\cos[(\xi_{M2}\varepsilon)]\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} - \\ -\dfrac{\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)} \dfrac{\cos[(1-\xi_{M2})\varepsilon]}{\cos(\varepsilon)} \end{array} \right\} M_{Ed,e,2}. \tag{37}$$

The bending moment due to the local bow initial imperfection with the amplitude e_0 is:

$$M_{Ed}^{e0}(\xi) = \frac{\gamma}{\varepsilon^2} \left\{ \frac{\cos[(0.5-\xi)\varepsilon]}{\cos(0.5\varepsilon)} - 1 \right\} (-8 N_{Ed} e_0). \tag{38}$$

The shape of the local bow initial imperfection which well approximates the shape of buckling mode may be sinus (see (39) first row) or parable of the second order (see (39) second row),

$$w_0(\xi) = e_0 \sin(\pi\xi), \quad w_0(\xi) = e_0 4(\xi - \xi^2), \qquad 0 \le \xi = \frac{x}{L} \le 1, \tag{39}$$

Equation (38) supposes that initial imperfection has the shape of parable of the second order.

4. CONCLUSIONS

The formulae for calculation of internal forces of the column with initial imperfection take into account the shear stiffness S_v of battened or laced built-up columns by using parameter γ. They allow performing parametrical study with the first or second order theory. The applications of formulae (26)–(31) in numerical examples, which are part of results of the large parametrical study, may be found in Part II of the paper.

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