

AKADÉMIAI KIADÓ

Pollack Periodica •
An International Journal
for Engineering and
Information Sciences

16 (2021) 1, 71–76

DOI:


10.1556/606.2020.00272

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ORIGINAL RESEARCH
PAPER



Pressure distribution on rolling-slide contact problem

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Received: September 23, 2020 • Revised manuscript received: November 5, 2020 • Accepted: November 9, 2020

Published online: March 2, 2021

ABSTRACT

In this paper the generalized model of contact of two plan bodies that roll and slide or just roll is presented. The analytical solution of the correct distribution of pressure in the contact is realized in the presence of the normal compressive force, which realizes the contact, as well as of the force pair or of a tangential force. This model can be used on sprocket teeth due to sliding rolling or rolling itself on the gear pole. The results obtained have been experimentally confirmed by many researchers as well as by the damage that the wheels actually suffer from the teeth.

KEYWORDS

Hertz solution, sprocket teeth, rolling, contact trace, friction, elastic-plastic properties, rigid body

1. INTRODUCTION

Most models for analyzing contact problems in all cases of flipping and sliding rely on the classic Hertz solution [1–7]. In this context where the Hertz solution belongs only to the case with normal force, a generalized model for sliding roller contact problems is presented, taking into account the normal force and the pair of forces or tangential forces. In the case when there are also a couple of forces the problem is no longer symmetrical, and not moved away from the presence of the elastic body, only that its treatment is as a non-linear elastic problem, and the flip is only related to the elastic properties of the bodies in contact as it can be seen in Fig. 1.

In different research work has been determined the connection experimentally, which justified it with the existence of micro-dispersions in the rigid body [1–4]. Furthermore it has been seen the immersion of the cylinder in the plane of motion as a property of material and geometry, which does not depend on load. According to the classical contact mechanics it has been found that the elastic deformation and the coefficient of friction in the roll δ is depended from the load.

2. PROBLEM OF CONTACT PLAN IN TRIGONOMETRIC SERIES

The basic equation of contact with static conditions in the contour is given by the relations [3],

$$\int_a^b p(t) \cdot \ln \left| \frac{x-t}{b-t} \right| \cdot dt = \frac{f(x)}{K}; p(a) = p(b) = 0; P = \int_a^b p(x) \cdot dx, \quad (1)$$

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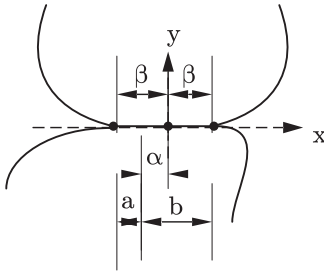


Fig. 1. Geometric parameters plan contact

where $K = 2/\pi \cdot (1/E_1 + 1/E_2)$, E_1 , E_2 are the Young's module, x is the point coordinate, t is the point coordinate where pressure is, $f(x)$ is the function of vertical distance of contact surfaces at the point with coordinates x .

To facilitate the solution, two new variables φ , θ [2] have been used instead of x , t :

$$x = \alpha + \beta \cdot \cos \varphi; t = \alpha + \beta \cdot \cos \theta, \quad (2)$$

where $\alpha = a + b/2$, is the middle of the contact trace and $\beta = a - b/2$ is half the contact trace, see Fig. 1.

The solution of the equation in the trigonometric series is seen in [2] in the form

$$p(\theta) = \sum_{n=1}^{\infty} C_n \cdot \sin(n \cdot \theta), f(\varphi) = \sum_{n=0}^{\infty} B_n \cdot \cos(n \cdot \varphi), \quad (3)$$

because it immediately meets the static conditions. Constants C_n has been determined from the basic contact equation, which are as follows:

$$\beta \cdot \int_0^{\pi} \sum_{n=1}^{\infty} C_n \cdot \sin(n \cdot \theta) \cdot \ln \left| \frac{\cos \varphi - \cos \theta}{1 - \cos \varphi} \right| \cdot \beta \cdot \sin \theta \cdot d\theta. \quad (4)$$

By comparing the two sides of the equation we obtain the constants

$$C_n = -\frac{2}{\pi \cdot \beta \cdot K} \cdot \sum_{i=0}^{\infty} (n + 2 \cdot i - 1) \cdot B_{n+2 \cdot i-1}, \quad (5)$$

and the static integral condition is transformed:

$$-F = \beta \cdot \int_0^{\pi} \sum_{n=1}^{\infty} C_n \cdot \sin(n \cdot \theta) \cdot \sin \theta d\theta = \frac{\pi \cdot \beta \cdot C_1}{2}. \quad (6)$$

3. ROLLING CONTACT PROBLEM WITH COUPLE FORCE

The function of surfaces $f(x)$ in the case when the configuration is symmetrical with the normal axis of contact can be decomposed into polynomial series in even force, and up to the second order we have the solution of Hertz, see Fig. 2.

Moving to the other angular variables, φ , θ ,

$$f(\varphi) = A(\alpha + \beta \cos \varphi)^2, \quad (7)$$

and the constants are:

$$C_1 = -\frac{2}{\pi \cdot \beta \cdot K} \cdot 2 \cdot B_2 = -\frac{2}{\pi \cdot \beta} \cdot F; C_n = 0 \text{ for } n \geq 2. \quad (8)$$

Eventually:

$$p(\theta) = C_1 \cdot \sin \theta. \quad (9)$$

To calculate the moment from the pressure that serves to balance the externally active pair M , the following equation will be used:

$$M_O = \int_a^b p(t) t dt = - \int_0^{\pi} C_1 \sin \theta (\alpha + \beta \cos \theta) \beta \sin \theta d\theta \\ = \frac{C_1 \pi \beta \alpha}{2} = 0. \quad (10)$$

For symmetric pressure distributions, resulting from the symmetrical configuration of surfaces, the Hertz solution cannot be used because $\alpha = 0$, excrete $M_o = 0$. During the application of forces and the force pair due to the deformation of the bodies a marked change of surfaces in the contact area occurs. Only by accepting this fact we can solve this problem and it is clear that the pressure must be anti-symmetric at the moment and symmetrical to the compressive force [3]. It is clear that from the series decomposition of the surfaces will get cubic order approach [1].

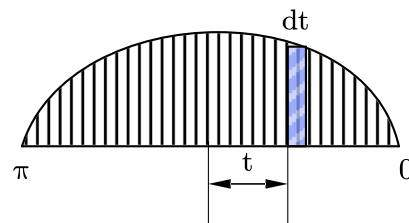
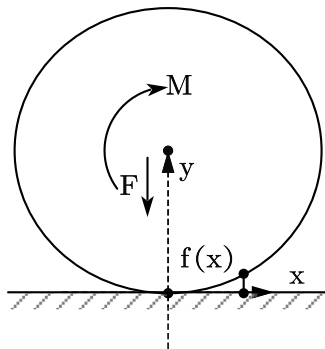


Fig. 2. The symmetric pressure from the Hertz solution does not balance the active moment

$$f(x) = A \cdot x^2 + B \cdot x^3. \quad (11)$$

Moving on to the other angular variables, θ ,

$$f(\varphi) = A(\alpha + \beta \cos \varphi)^2 + B(\alpha + \beta \cos \varphi)^3, \quad (12)$$

where the coefficients are:

$$\begin{aligned} B_0 &= A \left(\alpha^2 + \frac{\beta^2}{2} \right) + B \left(\alpha^3 + \frac{3\alpha\beta^2}{2} \right), \\ B_1 &= 2A\alpha\beta + B \left(3\alpha^2\beta + \frac{3\beta^3}{4} \right), \\ B_2 &= \frac{A \cdot \beta^2}{2} + \frac{3 \cdot B \cdot \alpha \cdot \beta^2}{2}, \quad B_3 = \frac{B \cdot \beta^3}{4}. \end{aligned} \quad (13)$$

From static conditions:

$$F(\alpha) = \alpha^3 + \frac{A}{B}\alpha^2 + \frac{2}{9} \left(\frac{A}{B} \right)^2 \alpha + \frac{PK}{6B} = 0 \quad (14)$$

and the pressure constants will be:

$$\begin{aligned} C_1 &= -\frac{2}{\pi \cdot \beta \cdot K} \cdot 2 \cdot B_2, \quad C_2 = -\frac{2}{\pi \cdot \beta \cdot K} \cdot 3 \cdot B_3. \\ C_n &= 0 \text{ for } n \geq 3 \end{aligned} \quad (15)$$

and the shape pressure distribution, see Fig. 3 [1, 2],

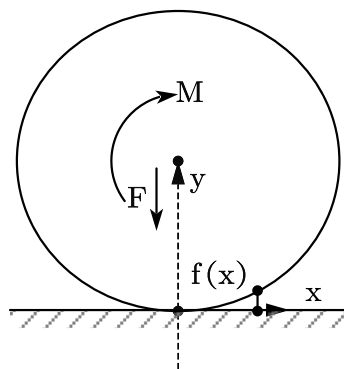
$$p(\theta) = C_1 \cdot \sin \theta + C_2 \cdot \sin(2 \cdot \theta) = p_1(\theta) + p_2(\theta). \quad (16)$$

4. THEORETICALLY DEFINITION OF ROLLING FRICTION FACTOR

The roller contact pressure $p(\theta)$ is reduced at the origin of the axle system, placed at the initial contact of the surfaces:

$$\begin{aligned} -F &= \int_a^b p(x) dx = \frac{\pi \beta C_1}{2}, \quad M = \int_a^b p(x) x dx \\ &= \frac{\pi \beta \left(C_1 \alpha + \frac{C_2 \beta}{2} \right)}{2}. \end{aligned} \quad (17)$$

In this way it becomes quite clear that the pressure constants are related to the external load:



$$C_1 = -\frac{2 \cdot P}{\pi \cdot \beta} \text{ dhe, } C_2 = -\frac{4 \cdot (\mp M + F \cdot \alpha)}{\pi \cdot \beta^2}. \quad (18)$$

The constant C_1 is related only to the compressive force F , while the constant C_2 is related to both the force and the moment. The moment has the + sign during the anti-clockwise rotation and the force F is always positive, because if it was not in pressure there is no contact. Recognizing that the configuration in the contact area changes considerably and therefore it does not remain symmetrical, which gives the opportunity to solve the contact problem with torque and compressive force. Since β and α are not known in advance, the iterative method is used to solve this nonlinear elastic problem. First, the compressive force F is set and β is determined (Hertz contact problem).

From Eq. (15):

$$\begin{aligned} 4A\alpha + 6B\alpha^2 + 3B\beta^2 &= 0, \\ \rightarrow 4 \frac{FK + \alpha 2\pi K C_2}{\beta^2} \alpha - \frac{4\pi K C_2}{\beta^2} \alpha^2 - 2\pi K C_2 &= 0. \end{aligned} \quad (19)$$

By denoting with the symbol the ratio of pressure constants are:

$$\lambda = \frac{C_2}{C_1}, \rightarrow C_2 = \lambda \cdot C_1 = -\lambda \cdot \frac{2 \cdot F}{\pi \cdot \beta}. \quad (20)$$

Substitute and take:

$$\alpha^2 + \frac{\alpha \cdot F}{\pi \cdot \left(-\lambda \cdot \frac{2 \cdot F}{\pi \cdot \beta} \right)} - \frac{\beta^2}{2} = 0, \rightarrow \alpha^2 + \frac{\alpha \cdot \beta}{2 \cdot \lambda} - \frac{\beta^2}{2} = 0. \quad (21)$$

The roots of this quadratic equation are:

$$\alpha_{1,2} = \frac{1}{2} \cdot \left(\frac{\beta}{2 \cdot \lambda} \pm \sqrt{\left(\frac{\beta}{2 \cdot \lambda} \right)^2 + 2 \cdot \beta^2} \right) = \frac{\beta}{4 \cdot \lambda} \cdot \left(1 \pm \sqrt{1 + 8 \cdot \lambda^2} \right). \quad (22)$$

The positive root is accepted because it does not satisfy the physics of contact, i.e., when $\lambda = 0$ (there is not a pair forces), and must get $\alpha = 0$ (Hertz problem).

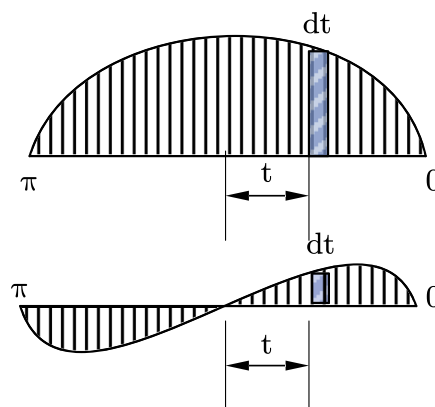


Fig. 3. Symmetric and anti-metric pressure in contact with compressive force and moment

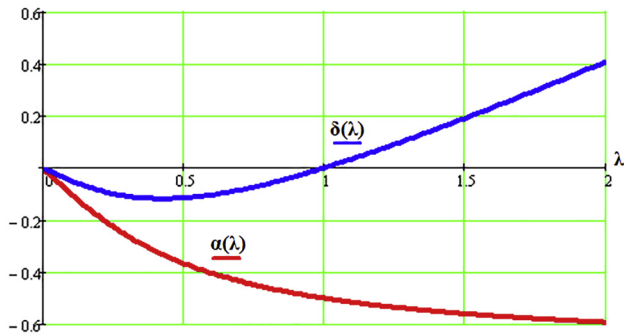


Fig. 4. The coefficient of friction in the roll δ and the displacement of the middle of the trace α depending on the ratio of the pressure constants λ in the case when $\beta = 1$

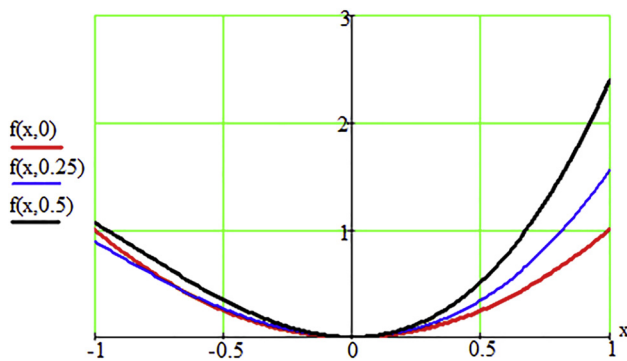


Fig. 5. Configuration of surfaces after deformation depending on the ratio of pressure constants $\lambda = 0, 0.25, 0.5$ for the case when $F \cdot K / \beta^2 = 1$ and $f(x) = Ax^2 + Bx^3$

The coefficient of friction in the roll represents the displacement of the normal contact force in the direction of motion from the starting point of contact, see Figs 4 and 5:

$$\delta = \frac{M}{F} = \frac{F \cdot \left(\alpha + \frac{\lambda \cdot \beta}{2} \right)}{P} = \alpha + \frac{\lambda \cdot \beta}{2}. \quad (23)$$

5. ROLLING CONTACT BEARING CAPACITY

5.1. Criteria I. Condition according to Greenwood analogous to that of Zommerfeld

At the back of the contact trace, where the constituent terms of pressure act with the opposite sign, the resultant of the pressures is thought to be equal to zero, for the cylinder to begin to roll. This is also the condition given by Greenwood [8],

$$C_1 \cdot \frac{\pi \cdot \beta}{4} = C_2 \cdot \frac{\beta}{3}, \rightarrow \lambda = \frac{C_2}{C_1} = \frac{3 \cdot \pi}{8} = 1.178, \quad (24)$$

whereas the middle of the contact trace will be, see Fig. 6:

$$\alpha = \frac{\beta}{4 \cdot \lambda} \cdot \left(1 - \sqrt{1 + 8 \cdot \lambda^2} \right) = -0.526 \cdot \beta. \quad (25)$$

Figure 7 presents α and δ at flip, which is after the value $\lambda = 1$, where the flip coefficient changes sign. The moment

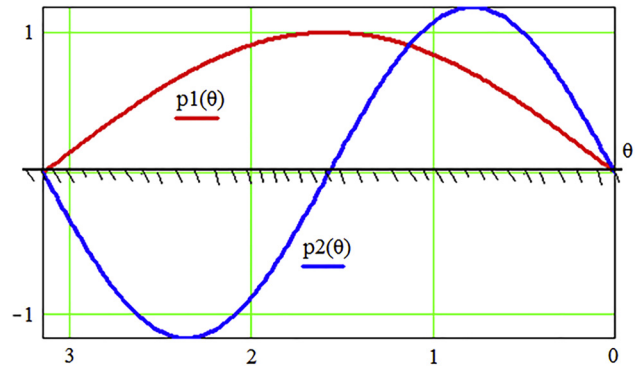


Fig. 6. Terms of contact pressure for the case when $\lambda = 3 \cdot \pi / 8$ and $C_1 = 1$

changing from zero value to its limit value of the roll M , reveals the values λ in the interval $1 - 1.178$, for $M = 0$ there is $\lambda = 1$ and $C_1 = C_2$. As it is clear in this case, a small stimulus causes the loss of the stability of contact, since there is pressure in the contact. This solution should be applicable to very flexible materials, which lose durability for small irritations like as tires.

Analytically the coefficient of friction in the roll and the limit rolling moment for this case are, see Fig. 7,

$$\delta = \alpha + \frac{\lambda \cdot \beta}{2} = -0.526 \cdot \beta + \frac{1.178 \cdot \beta}{2} = 0.0628 \cdot \beta, \quad (26)$$

$$M = -0.0628 \cdot \beta \cdot F.$$

5.2. Criterion II. Condition analogous to that of Reynold's

For contact body materials like steel, copper, etc., the theory of hysteresis does not provide an answer, so Johnson [9] is forced to accept plastic deformations in contact, which change the hysteresis and reinforce the material. Considering that these materials are more solid and don't lose their durability so quickly will be seen that solutions beyond the value should be $\lambda > 1$. In this case will be accepted conditions where the pressure cannot be negative,

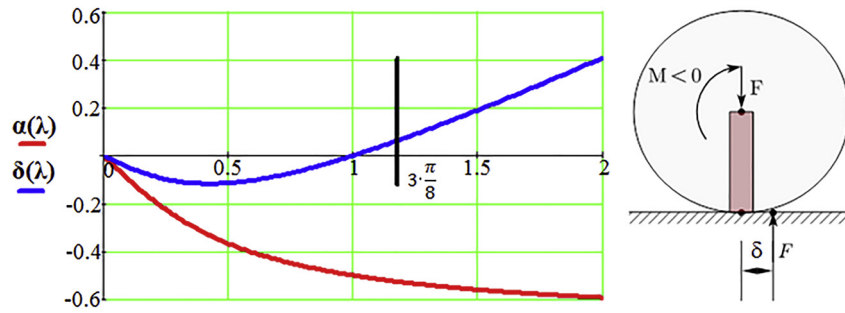
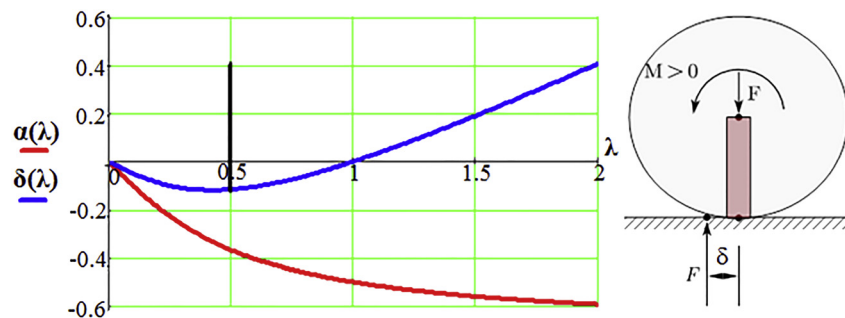
$$p(\theta_s) = \frac{dp(\theta_s)}{d\theta_s} = 0. \quad (27)$$

In this way all the parameters of the rolling boundary condition according to this criterion will be, see Fig. 8:

$$\alpha = \frac{\beta}{4 \cdot \lambda} \left(1 - \sqrt{1 + 8 \cdot \lambda^2} \right) = -0.366 \beta, \quad \delta = \alpha + \frac{\lambda \beta}{2} = -0.116 \beta, \quad M = 0.116 \beta F. \quad (28)$$

5.3. Criterion III. Contact saturation condition

In the friction of the grip, the contact saturation occurs, i.e. the limit of the sliding state is reached, and then the sliding begins, where the friction force does not change anymore, if the normal force and the coefficient of friction do not change. This idea is applied to the roller contact, so the

Fig. 7. Graph of α and δ in contact for the case when $\lambda = 3\pi/8$ and the approximate stability loss schemeFig. 8. Graph of α and δ in contact for the case when $\lambda = 0.5$

maximum value of the roller coefficient can be found and from the mathematical analysis the first derivative must be zero [1],

$$\delta = \frac{\beta}{4\lambda} (1 - \sqrt{1 + 8\lambda^2}) + \frac{\lambda\beta}{2}, \frac{d\delta}{d\lambda} = \left(\frac{\sqrt{1 + 8\lambda^2} - 1}{4\lambda^2} - \frac{2}{\sqrt{1 + 8\lambda^2}} + \frac{1}{2} \right) \beta = 0, \lambda = 0.424. \quad (29)$$

This way all the parameters of the rolling boundary state will be, see Fig. 9:

$$\alpha = \frac{\beta}{4\lambda} (1 - \sqrt{1 + 8\lambda^2}) = -0.331\beta, \delta = \alpha + \frac{\lambda\beta}{2} = -0.119\beta, M = 0.119\beta F. \quad (30)$$

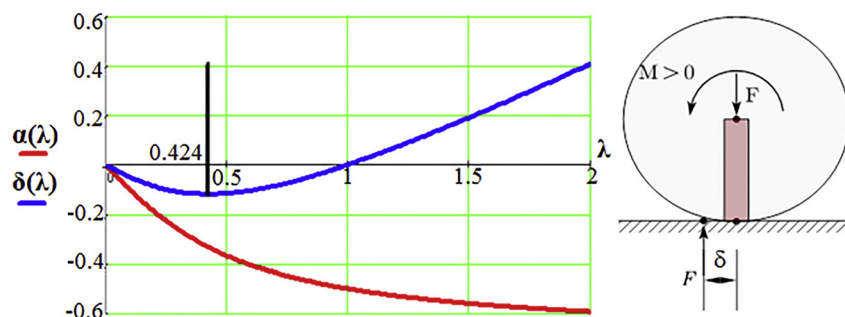
6. PRESSURE DISTRIBUTION IN CONTACT

In this case, which happens to have a pair of forces, the pressure solution has two terms, see Fig. 10,

$$p(\theta) = C_1 \sin \theta + C_2 \sin(2\theta), \lambda = \frac{C_2}{C_1} = 0.424; C_1 = -\frac{2F}{\pi\beta}, \quad (31)$$

as well as the known parameters for this condition:

$$\delta = -0.119\beta, M = 0.119\beta P, \alpha = -0.331\beta, A = 1.56 \frac{PK}{\beta^2}, B = 0.565 \frac{FK}{\beta^3}. \quad (32)$$

Fig. 9. Graph of α and δ in contact for the case when $\lambda = 0.424$

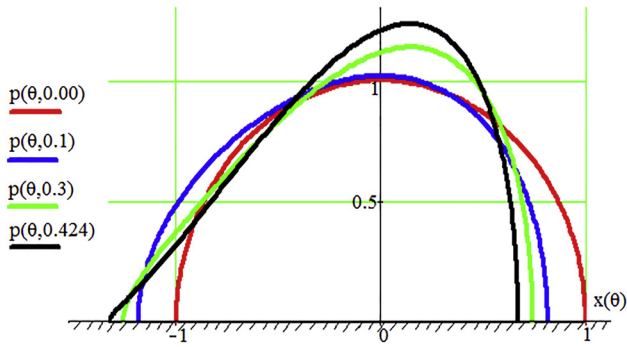


Fig. 10. Development of pressure in the rolling contact and movement of the contact trace during the setting of the rolling moment until the moment of saturation

The maximum pressure will be:

$$p_{\max} = 1.234 \cdot p_{\text{Hercit}}. \quad (33)$$

Firstly, the compressive force P are set and then gradually the moment to the saturation limit by playing with the parameter λ from 0 to 0.424 and seeing how the transition from the beginning to the end of the rolling contact pressure occurs and how it moves contact during flip.

7. CONCLUSION

Based on the generalized model in the contacts of two plan bodies that roll and slide or just roll and the analytical research study following conclusions were drawn as follows:

- Hertz's solution cannot be used for real bodies sliding or rolling;
- The coefficient of friction in the roll δ is displayed because the bodies are deformable and have elastic-plastic properties, which makes difficult the mechanical use of the Hertz solution;
- Sliding roller contact is also a stability problem, which also depends on the properties of the material;
- During rolling, the contact configuration changes strongly in the contact area, and only by taking this fact into

account, can determine the pressure distribution in the contact;

- Rolling occurs when the contact as in the case of sliding reaches the saturation phase;
- Contact pressure from series decomposition consists of two terms, symmetric and anti-metric;
- The coefficient of friction in the roll can be analytically determined.

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