

ON THE COMPLEX OPTICAL POTENTIAL IN THE LANE-MODEL

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By comparing the total widths of the IAR's in ^{209}Bi calculated in a microscopic model to those obtained from the Lane model, it is shown that the decay of the $^{208}\text{Bi}(0^+)$ core of ^{209}Bi in a few lowest-lying IAR's can be taken into account by assuming complex optical potential in the (closed) $T_{>}$ channel.

KOMPLEX OPTIKAI POTENCIÁL A LANE-MODELLBEN.

A ^{209}Bi izobár analóg rezonanciáinak teljes szélességeit számoltuk egy mikroszkópikus modell és a Lane-modell segítségével. A kétféle módon számított szélességek összehasonlítása révén megmutatjuk, hogy a $^{208}\text{Bi}(0^+)$ törzs bomlásának hatása a (zárt) $T_{>}$ csatornában felvett komplex optikai potenciál használatával figyelembe vehető.

КОМПЛЕКСНЫЙ ОПТИЧЕСКИЙ ПОТЕНЦИАЛ В МОДЕЛИ ЛЕЙНА.

Полные ширины изобар-аналоговых резонансов ^{209}Bi были рассчитаны по микроскопической модели и модели Лейна. Сравнение двумя методами ширин показывает, что эффект распада остова $^{208}\text{Bi}(0^+)$ может быть учтен комплексным оптическим потенциалом, взятым в (закрытом) канале $T_{>}$.

Introduction

It is well known that the description of the proton elastic scattering in the neighbourhood of an isobaric analogue resonance (IAR) is quite successful in terms of the isospin dependent phenomenological optical potential proposed by A.M. Lane. The isovector part of the potential couples the proton and neutron channels resulting in the so called Lane equations [1]. Both the real and imaginary parts of the optical potential are supposed

to have the same dependence on isospin

$$U(r) = U_0(r) + U_1(r) \cdot (\vec{t} \cdot \vec{T}_0), \quad (1)$$

where \vec{T}_0 and \vec{t} are the isospins of the target and the nucleon, respectively, $U_1(r)$ is called the symmetry potential. For the target + proton system the potential terms diagonal in proton neutron representation are

$$\begin{aligned} U_{pp} &= U_0 - U_1 \frac{T_0}{2} \\ U_{nn} &= U_0 + U_1 \frac{T_0 - 1}{2}, \end{aligned} \quad (2)$$

while in the representation characterized by the isospin of the nucleon + core system (Lane-Robson equations [2])

$$\begin{aligned} U_{<} &= U_0 - U_1 \frac{T_0 + 1}{2} \\ U_{>} &= U_0 + U_1 \frac{T_0}{2}. \end{aligned} \quad (3)$$

In a ground-state target + neutron system the proton-neutron and the total-isospin formalisms coincide and the diagonal potential is equal to $U_{>}$. So for the bound system, at least when the low-lying states are considered, $U_{>}$ must be real. Still it has been observed, that the total widths of the resonances in ^{209}Bi analogous to the low-lying states of ^{209}Pb are seriously underestimated if $\text{Im } U_{>} = 0$ is assumed in the description of proton scattering on ^{208}Pb with the Lane equations. One of the reasons of this contradiction is that the underlying phenomenological picture does not take into account the fact that the core state itself, to which the neutron is bound (analogue of ^{208}Pb ground), is unstable against proton emission due to the high Coulomb energy shift.

The aim of this note is to answer the question whether or not the effect of the core decay can be incorporated in the phenomenological description, by using a properly chosen complex potential in the $T_{>} = T_0 + 1/2$ channel. To ensure that only the pure effect of the core decay be regarded, the results of the Lane equation are compared to those of a microscopic model calculation that accounts for this very effect.

The microscopic model

For the microscopic calculation of the total width the "reduced width method" [4], which was applied to IAR first in [5], has

been used.

In this method the total width of a $|\lambda\rangle$ state $\Gamma_{\text{tot}}^\lambda$ appears as the sum of the partial widths $\Gamma_{\lambda c}$ to channels c

$$\Gamma_{\text{tot}}^\lambda = \sum_c \Gamma_{\lambda c} \quad (4)$$

and

$$\Gamma_{\lambda c} = 2P_c \gamma_{\lambda c}^2, \quad (5)$$

where P_c is the barrier penetration factor, and $\gamma_{\lambda c}^2$ the reduced width, is connected to the $\langle r, c | \lambda \rangle$ coefficients of the expansion of $|\lambda\rangle$ in terms of channel function times single particle function, and a good estimate for the reduced width (correct in a square-well model) is the following

$$\gamma_{\lambda c}^2 = \left[\sum_r (-1)^r \langle r, c | \lambda \rangle \right]^2 \frac{\hbar^2}{mR^2}, \quad (6)$$

where r runs over the radial quantum numbers of the single-particle states, m is the reduced mass, R is the well radius. The expansion coefficients are to be determined by the diagonalization of the Hamiltonian on the basis consisting of the states $|r, c\rangle$.

The pure IAR is defined as the sum of the target + proton and analogue (of the target) + neutron system. So, for a doubly magic target it is reasonable to choose the elements of the basis as

$$\begin{aligned} & |N_1, p j \rangle, \quad |[(N, n j')^{-1} (N, p j')]^{0^+} (N_1, n j) \rangle, \\ & |N_1+1, p j \rangle, \quad |[(N, n j')^{-1} (N+1, p j')]^{0^+} (N_1, n j) \rangle, \\ & |N_1+2, p j \rangle, \quad |[(N, n j')^{-1} (N+2, p j')]^{0^+} (N_1, n j) \rangle, \dots \end{aligned} \quad (7)$$

where N and j' denote the radial quantum number and the angular momentum of a single-particle state outside the $N=Z$ core in the target, j' runs through the states filled with neutrons, N_1 and j stand for the radial quantum number and angular momentum of a single-particle state of the lowest shell not occupied by neutrons in the target, p and n denote proton and neutron, respectively, the target ground state is regarded as the vacuum, $[]^{0^+}$ means angular momentum coupling to $J^\pi = 0^+$. Because of

the Coulomb interaction the proton single-particle states corresponding to bound neutron single-particle states may lie in the continuum. Unbound proton states labelled by j give rise to an elastic proton width (Γ_p), while those labelled by j' produce non-vanishing inelastic proton widths. The latter ones describe the decay of the core state (analogous to the target ground state), which is sometimes referred to as the "phonon component" since it contains (neutron) holes and (proton) particles coupled to 0^+ .

In this paper oscillator single particle wave functions have been used. The single-particle energies for neutron have been taken as the experimental binding energies in the ^{209}Pb [6], for protons the energy shift has been calculated using the value of the $2g_{9/2}$ single-particle energy from [7]. For the energy difference between the shells characterized by $r=N(N_1)$ and $r=N+1(N_1+1)$ the value 13 MeV has been accepted. The total and proton partial widths for several resonances have been calculated on two different bases consisting of the first line and of the first + second lines of (7), respectively. A residual interaction of the form

$$V^R = \sum_{i>j} g(1-\alpha + \alpha(\vec{\sigma}_i \cdot \vec{\sigma}_j)) \delta(\vec{r}_i - \vec{r}_j)$$

has been used with parameters $g=881.79 \text{ MeV}\cdot\text{fm}^3$ and $\alpha=0.135$. The results are presented in Table 1.

The phenomenological calculation of the total width

In the course of this calculation the hypothesis was accepted that the decay of the ^{209}Bi "phonon component" of the analogue resonances in ^{209}Bi can be simulated by absorption. By varying the imaginary strength in the proton channel between reasonable limits no appreciable change was experienced in the value of the total width if the usual condition for the closed $T_>$ channel, $\text{Im } U_> = 0$, was kept. This fact is not much surprising since the decay takes place in the neutron channel. So we kept all the optical parameters at usual values and varied the imaginary strength in the $T_>$ channel so as to give the minimum mean square deviation of the total widths from the values calculated microscopically. The optical potential in the Lane equations was the following:

$$U_0 = (V_0 + iW_0)f(r) + V_{S0} \frac{2}{r} \frac{df(r)}{dr} (\vec{l} \cdot \vec{\sigma})$$

and

$$U_1 = (V_1 + iW_1)f(r)$$

where

$$f(r) = - \left[1 + \exp \left(\frac{r - r_0 A^{1/3}}{a} \right) \right]^{-1}$$

with $r_0 = 1.19$ fm $a = 0.75$ fm $V_0 = 56.4$ MeV $V_{SO} = 5.8$ MeV
 $V_1 = 0.5$ MeV and $W_{pp} = W_0 - W_1 T_0 / 2 = 4.5$ MeV.

The parameter values are the same as in [8], W_{pp} came from [9] by extrapolation. The scattering function $S(E)$ as a function of the energy resulted from the coupled Lane equations by numerical integration and was parametrized with the

$$S^1(E) = e^{2i\delta(E)} - i \frac{e^{2i\xi} \Gamma_p}{E - E_0 + \frac{1}{2} i \Gamma_{tot}}$$

one level formula.

Here E_0 , Γ_{tot} and Γ_p are the position, the total width and the elastic proton width of the IAR. By assuming a linear energy dependence for the complex non-resonating background phase shift, the best-fit values of the parameters have been determined. Γ_{tot} and Γ_p are presented in Table 1.

Table 1

Values of the total and partial elastic proton width of IAR's in the ^{209}Bi calculated microscopically and phenomenologically.

J^π	7x7		$W_s = +0.11$		14x14		$W_s = +0.19$		$W_s = 0.0$	
	Γ_{tot}	Γ_p	Γ_{tot}	Γ_p	Γ_{tot}	Γ_p	Γ_{tot}	Γ_p	Γ_{tot}	Γ_p
$9/2^+$	182.8	20.8	202.9	39.6	246.0	47.0	304.0	39.7	52.0	39.2
$5/2^+$	233.5	71.5	239.5	85.8	351.2	152.2	325.4	85.0	114.5	85.5
$1/2^+$	247.1	85.1	222.0	84.2	282.4	83.4	296.4	83.8	116.0	84.6
$7/2^+$	208.0	46.0	222.9	63.4	303.2	104.2	324.7	63.1	78.9	63.5
$3/2^+$	252.8	90.8	225.8	83.9	392.3	193.3	306.1	82.7	110.7	83.4

Results, conclusion

In Table 1, the numerical results are summarized. Those of the microscopic calculations are presented in the columns denoted by 7x7 and 14x14. The notation refers to the dimension of the bases used. It can be seen that in both cases it was possible to find a $W_>$ value which reasonably reproduced the total widths. The partial widths are not much influenced by the change of $W_>$, so the increase of the Γ_p values from the 7x7 basis to the 14x14 one is not reproduced by the phenomenological model. (Though we did not aim at making comparison with experiment, it is worth noting that it is the microscopic model which seriously overshoots the partial widths.)

In the last column the widths calculated with the usual (Bondorf) condition are shown. Changing the value W_{pp} from +4.5 MeV to +9.0 MeV caused at most 20 % increase in the total widths and not more than 30 % in the partial ones.

Taking into account that no many-parameter fitting was attempted and only $W_>$ was varied, from the results presented one may conclude that this simple way of incorporating the core-decay in the Lane-model is successful.

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