

Determination of the conditions of laminar/turbulent flow transition using pressure compensation method in the case of Ga75In25 alloy stirred by RMF

Arnold Rónaföldi, András Roósz, Zsolt Veres *

MTA-ME Materials Science Research Group, ELKH, HungaryUniversity of Miskolc, Hungary

ARTICLE INFO

Communicated by Lijun Liu

Keywords:

A1 Fluid flows
A1 Magnetic fieldsA1
Stirring
A1 Magnetic Taylor numberA1
Reynold number
B1 Ga75In25 alloy

ABSTRACT

The effect of melt-flow on the microstructure has been investigated by performing several unidirectional solidification experiments where the melt was flown in a rotating magnetic field. It is a well-known fact that the angular frequency of the melt cylinder always differs from the angular frequency of the magnetic field. However, it proves to be very difficult to determine during the experiments. In our present study, the magnetic Taylor number and the Reynolds number were determined as a function of the radius of melt cylinder, the magnetic induction, and the angular frequency of magnetic field using the real angular frequency defined by the so-called pressure-compensation method developed earlier in the case of Ga75In25 alloy. The simulations developed for performing similar experiments can be checked as well as the different experiments can be compared correctly by using the obtained measurement results. By knowing the critical Reynolds number belonging to the laminar/ turbulent flow transition, the value of critical magnetic induction belonging to the transition was determined as a function of the radius of the melt cylinder.

1. Introduction

The developing microstructure (primary and secondary dendrite arm spacing, micro- and macrosegregation) and by this the mechanical properties of solid material are significantly influenced by the melt flow taking place in the mushy zone during the solidification of metallic melts. Several research teams deal with the simulation of melt flow and with the calculation of microstructure (and mechanical properties) developing under the influence of melt flow. In the course of the industrial solidification processes (continuous and semi-continuous steel- Al- casting, etc. different methods of mould casting, die casting methods), the velocity of melt flow can be merely stated by estimation in the mushy zone so the simulations cannot be validated exactly by comparing them.

During the majority of experiments used for validation, the melt was solidified by unidirectional heat extraction and was flown by rotating magnetic field (RMF) [1–9]. The experiments are mainly characterized by the magnetic Taylor number (Ta_m) [4,5].

$$Ta_m = \frac{1}{2} \frac{\sigma B^2 R^4 \omega_0}{\rho \nu^2} \quad (1)$$

where

σ (S/m) - the specific electrical conductivity of melt
 B (T) - magnetic induction
 ω_0 (rad/s) - angular frequency of magnetic field
 R (m) - radius of sample
 ρ (kg/m³) - melt density
 ν (m²/s) - kinematic viscosity of melt

A significant part of flow simulations supposes that the flow is laminar (owing to the simplification). It can be decided by the Reynolds number if a flow is laminar or turbulent. The Reynolds number characterizing the flow in case of cylinder symmetric melt flow is as follows:

$$Re = \frac{\omega_0 R^2}{\nu} \quad (2)$$

“If the Reynolds number lies between 0 and 2320, the flow is considered streamlined or laminar. Reynolds number between 2320 and 4000, indicates an unstable flow condition ranging from streamlined to

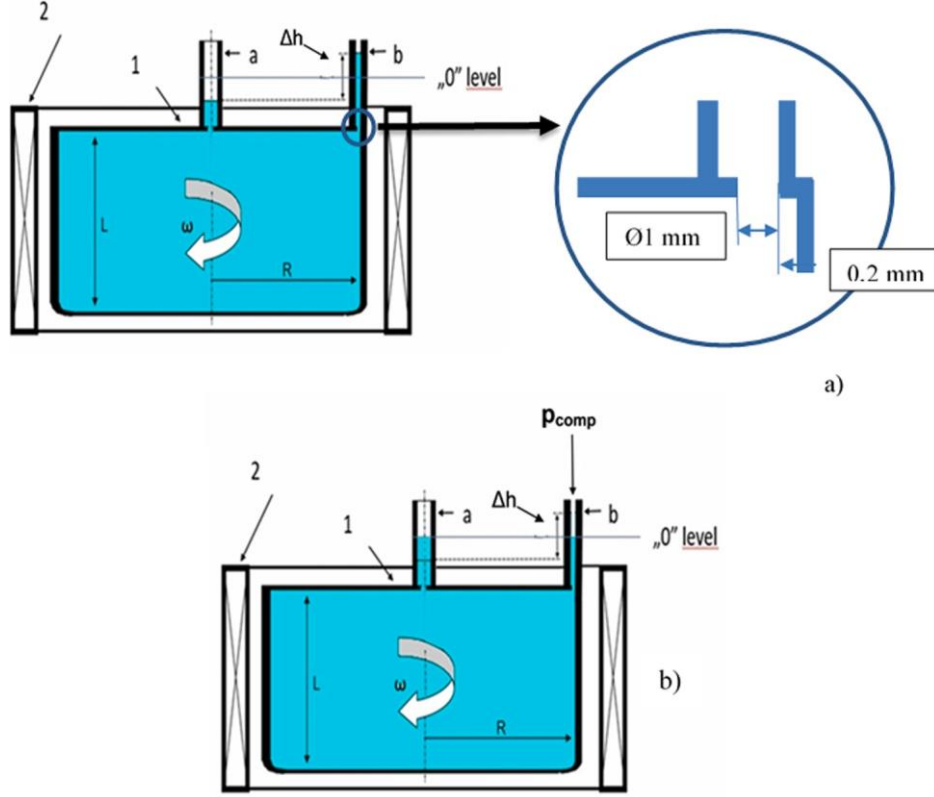


Fig. 1. (a) The melt level in the gauges when the magnetic induction is switched on (1), closed tank, (2) RMF inductor, “a” and “b” the gauges. (b) The melt level in the gauges at the pressure compensation.

turbulent. Reynolds number >4000 indicates a turbulent flow meaning the flow velocity is critical” [10].

“The flow became fully turbulent at 4200. The accepted design value for pipe flow transition is now taken to be 2300. This is accurate for commercial pipes” [11].

“Using Reynolds number, it becomes easy to determine whether the fluid flow is laminar or turbulent. Following are the boundary values of a circular pipe that can be used to determine the flow pattern [12]:

- If $Re > 2300$, then the flow is said to be laminar.
- If $2300 < Re < 4000$, then the flow is said to be transient.
- If $Re > 4000$, then the flow is said to be turbulent”.

An incorrect value is obtained both concerning the Taylor number and Reynolds number in case if the ω value calculated from the synchronous revolution number of the magnetic field is substituted in Equations (1) and (2) as the melt does not rotate by this revolution number. The cause of it is the internal friction and the viscosity of the melt. According to our experiences, the revolution number of melt approximates the synchronous revolution number of magnetic field merely at an extremely high B value but it does not reach this number. This fact shall be taken into consideration when determining the magnetic Taylor number or Reynolds number serving for the characterization of experiments as well as the laminar/turbulent transition.

So, the magnetic Taylor number characterizing correctly the experiment is as follows (see in the Appendix):

$$Ta_m^* = \frac{1}{2} \frac{\sigma B^2 R^4 (\omega_0 - \omega)}{\rho \nu^2} \quad (3)$$

and the Reynolds number is as follows:

$$Re^* = \frac{\omega R^2}{\nu} \quad (4)$$

where ω is the real angular frequency of the rotating melt.

It can be concluded from the aforementioned facts that it is extremely important to know the value of the real angular frequency (ω) developing in the melt at a given B and ω_0 value so that we can evaluate the solidification experiments. There are several different calculations- and modelling methods for determining this value but it is necessary to determine the value of real angular frequency using experiments to validate these methods.

In the case of metals and alloys having a low melt point, the flow rate can be determined by determining the revolution number of turbines of low inertia dipping into the melt [13]. Another possible method that can be principally used in the case of any melting points is to determine the shape of free surface developing under the influence of rotation and to calculate the peripheral speed from it. In the first case, the melt flow is reduced by the moment caused by the inertia and friction of the turbine moreover it does not reach the revolution number of melt. In the second case, it is extremely difficult to determine exactly the shape of the free surface especially if we would like to determine it at a relatively high temperature (e.g. in a temperature range of 550–670 °C in case of Al- alloys).

The aim of our present work was to determine the experimental parameters (B , R) at which the melt flow of Ga75In25 alloy changes from laminar to turbulent. The real angular frequency developing under

the influence of stirring by using rotating magnetic field was determined as a function of the magnetic induction by a so-called pressure compensation method (PCM) developed earlier by applying different sample-diameters [14]. The aforementioned failures and difficulties are eliminated by the equipment operating based on the PCM method and



Fig. 2. Photo of the free surface of the rotated Ga [13].

therefore it is suitable for validating the results obtained by calculations and for determining the magnetic induction belonging to the laminar/ turbulent transition.

2. Experiments

2.1. The principle of determining the real angular frequency

It is a well-known fact that the pressure changes along the radius in the rotating liquid column as the liquid parts move by different velocity at the places having different radius. By the above fact, a rotating paraboloid shape of liquid surface develops in the case of a free surface.

The pressure change can be measured along the radius in case if the liquid is rotated without free surface i.e. in a closed tank. A higher value of pressure belongs to a higher radius. This phenomenon can be used for determining the average revolution number of rotating liquid e.g. for determining the revolution number of metallic melt stirred by the rotating magnetic field (RMF) as well.

The “ Δp ” pressure-difference related to the pressure prevailing in the axis of rotation can be calculated by knowing the velocity-differences of melt-parts that are present at any point of the closed probe and at any place having “ r ” radius. The peripheral speed is zero in the axis of rotation so:

$$\Delta p = \frac{\rho[v(r)]^2}{2} = \frac{\rho\omega^2 r^2}{2} \quad (5)$$

where: $v(r)$ = peripheral speed developing at radius r .

The ω angular frequency can be calculated from the measured pressure:

$$\omega = \frac{1}{r} \sqrt{\frac{2\Delta p}{\rho}} \quad (6)$$

2.2. Principle construction of measuring unit

It is very difficult to measure pressure developing in the melt in the closed probe without disturbing the melt flow therefore the pressure is not measured directly in the closed probe. In case if we want to perform the pressure measurement at the $R = R$ mm place, it is necessary to complete the closed tank with 2 gauge connections, one at the axis ($r = 0$) and the other at the periphery (R) of the tank indicated with “a” and “b” in Fig. 1.a. So, the two gauges connections and the tank create a “communication vessel”. The melt-level is the same in the gauge

Table 1

Physical parameters of Ga75wt%In25wt% alloy.

Melting point, °C	15.7
Density Ga, kg/m ³ (at m.p)	6350
Density In, kg/m ³ (at m.p)	7020
Density Ga75In25*, kg/m ³ (at m.p)	6517.5
Kinematical viscosity, m ² /s	$3.41 \cdot 10^{-7}$
Specific electrical conductivity, MS/m	3.58
Relative magnetic permeability	1
Penetration distance**, at 50/100/150/200 Hz, mm	36/26/21/18

$$\rho_{GaIn} = 0.75\rho_{Ga} + 0.25\rho_{In}.$$

$$** \delta = \frac{1}{\alpha \sqrt{f \cdot 4\pi \cdot 10^{-7}}}$$

where f is the frequency of magnetic field.

connections in case if the RMF inductor does not operate moreover the atmospheric pressure is identical in the gauges connections. It is the so-called “stationary-level” or “0-level”.

The $r = 0.2$ mm position was chosen for the measure of the pressure, because there is the max pressure difference, and so there is the minimum relative error of the measured pressure.

In another experiments [13], we examined the wall effect on the shape of the free surface of the rotated Ga. The shape was paraboloid (as this succeeds from the theory) except close to the wall where was a decrease of the height of the surface, but the thickness of it was only ~ 0.2 mm (grey belt near the wall in Fig. 2). The white belt in the photo shows the maximum height of the free surface. We measured there. The inside diameter of the measuring tube was 1 mm (Fig. 2), and it was 0.2 mm from the wall of the tank. So, we stated, that the wall effect is very small, and with this setting, we measured the maximum pressure difference with a small error. The material of the tank was TEFLON with very smooth wall. The time of one experiment was only 30 s, and the

time between two experiments was 15 min, then the alloy was not heated by the induction. The temperature of melt was 22 ± 1 °C in all experiments.

A level-difference having a value of Δh develops between the melt-levels in the “a” and “b” gauge connections in case if the melt is rotated (stirred) by the RMF inductor (see Fig. 1.b). The $\rho g \Delta h$ metal-lostatic pressure of the melt-column is in equilibrium with the pressure-difference (“ Δp_{max} ”) developing between the axis of the tank and the periphery of the tank in case if the free surface of the melt is at the atmospheric pressure in the gauge connections, i.e.:

$$\Delta p_{max} = \frac{\rho[v(R)]^2}{2} = \frac{\rho\omega^2 R^2}{2} = \rho g \Delta h \quad (7)$$

$$\omega = \frac{1}{R} \sqrt{\frac{2\Delta p_{max}}{\rho}} \quad (8)$$

where: $v(R)$ = peripheral speed developing at radius R .

The Δh height-difference could be measured directly as well so the developing Δp_{max} pressure-difference can be calculated by it. Because the value of Δh can be high (higher than 100 mm) a different solution was chosen (e.g. Δh can be 600 mm in case of 37500 Pa ($37500 / [6350 \cdot 9.81] = 0.6$ m).

The pressure-difference was determined in such a way that the melt-surface was set back to the “0” level in the two gauge connections by p_{comp} pressure of air used in the “b” gauge developed by a pneumatic system (Fig. 1.b.). The exactness of pressure measurement was 20 Pa that resulted in a significantly less relative error than the direct distance measurement. The details of measurement and equipment are described in [14].

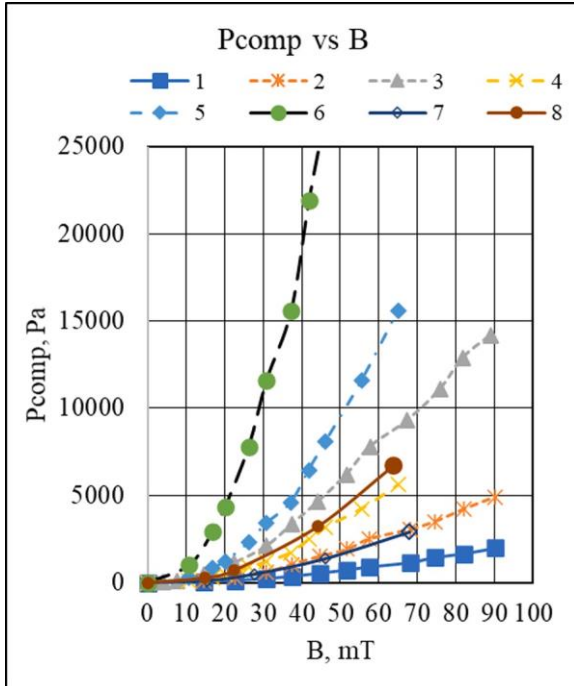
3. Experiments

The experiments were performed at room temperature therefore the Ga75wt%In25wt% alloy was chosen for the material of experiments.

Table 2

Calculated results.

No. exp.	R, mm	C (R) in Eq. (9).	ω_{cr} , rad/s at Re = 2320	ω_{cr} , rad/s at Re = 4000	B_{cr} (mT) at Re = 2320	B_{cr} (mT) at Re = 4000	Ta_m^{*cr} at Re = 2320	Ta_m^{*cr} at Re = 4000
50 Hz								
1	5	131	3.16E + 01	5.46E + 01	17.71	30.53	2.57E + 05	1.38E + 06
2	7.5	318	1.41E + 01	2.42E + 01	7.30	12.58	8.34E + 04	6.56E + 06
3	12.5	927	5.06E + 00	8.73E + 00	2.50	4.31	3.46E + 03	3.97E + 04
150 Hz								
4	5	301	3.16E + 01	5.46E + 01	7.71	13.29	6.1E + 04	3.73E + 05
5	7.5	742	1.41E + 01	2.42E + 01	3.13	5.39	6.64E + 03	6.03E + 05
6	12.5	2250	5.06E + 00	8.73E + 00	1.03	1.78	7.70E + 01	1.13E + 03
100 Hz								
7	5	206	3.16E + 01	5.46E + 01	11.26	19.42	1.47E + 05	8.04E + 06
200 Hz								
8	5	330	3.16E + 01	5.46E + 01	7.03	12.12	5.84E + 04	7.21E + 04

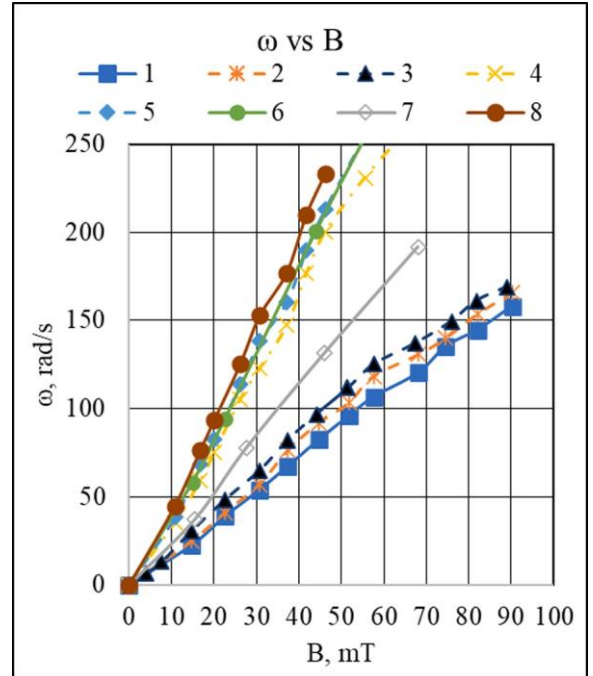
**Fig. 3.** Measured compensation pressure (P_{comp}) as a function of magnetic induction (B).

The physical parameters of alloy are shown in Table 1.

The experiments were performed by using melt cylinders with radii of 5, 7.5, and 12.5 mm at a frequency of 50, 100, 150, and 200 Hz in a magnetic induction range of 0 and 90 mT (Table 2.). The size of the closed tank and so the diameter of the melt cylinder ($2R$) was smaller in each case than the penetration distance except in case of 200 Hz and $2R=25$ mm. The height of the melt cylinder (L) was 100 mm; it was four times higher than the diameter of the melt cylinder in case of the highest diameter (25 mm) so the influence of penetration distance and “end-effect” on the measurement results could be neglected.

The lowest pressure-value that can well be measured by the equipment is 20 Pa (it can be decreased by using a more sensitive manometer). Based on the above facts, the lowest value of angular velocity (ω) that can be measured safely is 15.85 rad/s in case of a melt cylinder of $R=5$ mm, 10.57 rad/s in case of a melt cylinder of $R=7.5$ mm and 6.32 rad/s in case of a melt cylinder of $R=12.5$ mm. The relative error of the measured pressure was <2%.

Fig. 3. demonstrates the measured compensation pressure as a function of the magnetic induction at 50 and 150 Hz in case of 3 different melt cylinder radius, and in case of 100 and 200 Hz at 5 mm

**Fig. 4.** Angular frequency of melt (ω) as a function of magnetic induction (B).

melt cylinder radius.

4. Results and discussion

By using Equation (8), the real angular frequency of melt cylinder (ω) can be calculated from the P_{comp} value. The calculated values are shown in Fig. 4. as a function of B magnetic induction. Practically, the real angular frequency is the linear function of magnetic induction in the investigated range. Its value increases by the radius of the melt cylinder and it is following the fact that the Lorentz force causing the flow is proportional to the radius of the melt cylinder.

By using Equations (3) the real Ta_m^* which takes into consideration the real angular frequency of melt cylinder (ω) can be calculated (Fig. 5.).

In the References, the experiment is often characterized by the Ta_m value calculated only using the angular frequency of the magnetic field. Based on Equation (3) it is obvious that it is true merely now when the experiment starts, and the angular frequency of the melt cylinder is zero. In Fig. 6.a., the value of Ta_m/Ta_m^* is represented as a function of ω/ω_0 ; the value of Ta_m is 1.25 times higher than Ta_m^* at $\omega/\omega_0 = 20\%$, and 10 times higher at $\omega/\omega_0 = 90\%$. It can be seen in Fig. 6.b. that 1.25 times is obtained at ~30 mT in case of $f = 50$ Hz in our experimental

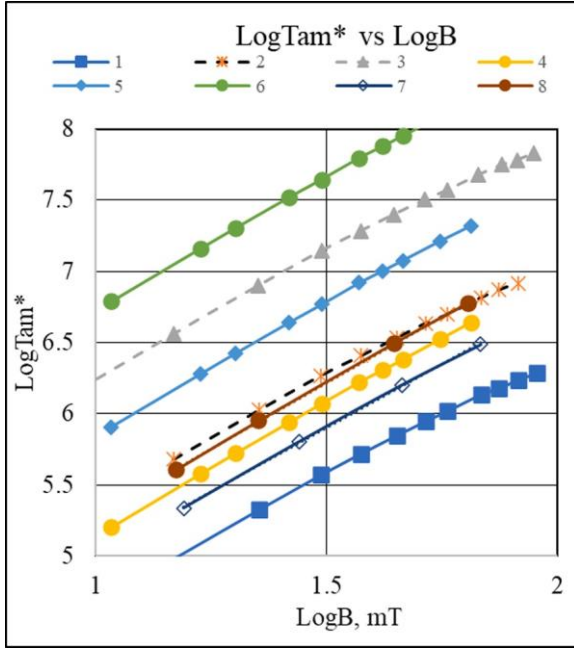


Fig. 5. Real Magnetic Taylor number (LogTam^*) as a function of magnetic induction (LogB).

circumstances, Ta_m is twice higher than Ta_m^* at $\omega/\omega_0 = 50\%$ (at 90 mT).

Based on the above-mentioned facts, it is obvious that the calculation of Ta_m value by the angular frequency of magnetic field can be allowed only in case of relatively low magnetic fields because a higher and higher value of error is obtained by increasing the magnetic induction and therefore used this Ta_m the different experiments cannot be compared.

By using Equation (4), the real Reynolds number (Re^*) can be calculated if the real (measured) angular frequency of melt cylinder is known. The calculated values are demonstrated in Fig. 7. as a function of the magnetic induction. The Re^* as a function of B can be described as follows:

$$\text{Re}^* = C(R)B \quad (9)$$

where C is a function of R (Table 2.) calculated by regression analysis used the data of Fig. 7.

Based on the Re^* , it can be decided that the flow is pure laminar, unstable, or pure turbulent. As was mentioned earlier the melt flow is pure laminar if the $\text{Re}_{cr} < 2320$, if $2320 < \text{Re}_{cr} < 4000$ there is an unstable flow where the vortices start to develop, and if $\text{Re}_{cr} > 4000$ the flow will be turbulent.

The critical angular frequency belonging to these two critical Re_{cr} are as follows:

$$\omega_{cr} = \frac{\text{Re}_{cr} \nu}{R^2} \quad (10)$$

The critical magnetic induction (B_{cr}) belonging to these two limits can be

$$\text{calculate used the Eq. (9): } B_{cr}(Re = 2320) = \frac{2320}{C(R)} \text{ and } B_{cr}(Re = 4000) = \frac{4000}{C(R)} \quad (11)$$

The calculate ω_{cr} and B_{cr} values are in Table 2.

By representing the values of critical magnetic induction (B_{cr}) as a function of the radius of melt cylinder, the experimental conditions – i.e.

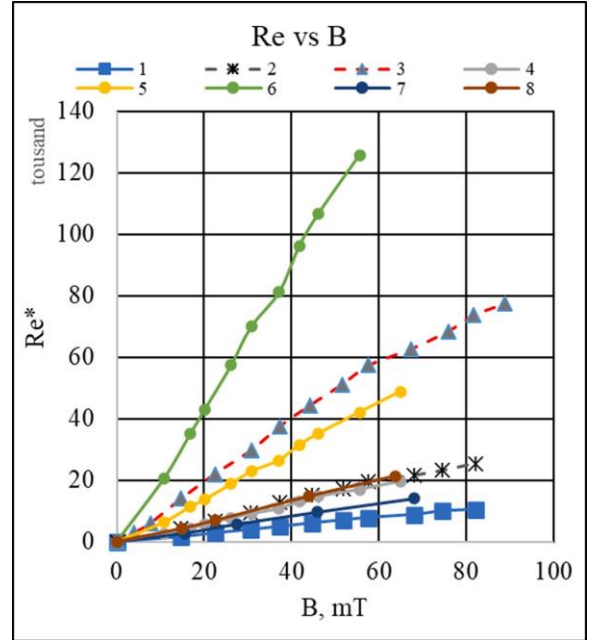


Fig. 7. The real Reynolds number (Re^*) as a function of magnetic induction (B).

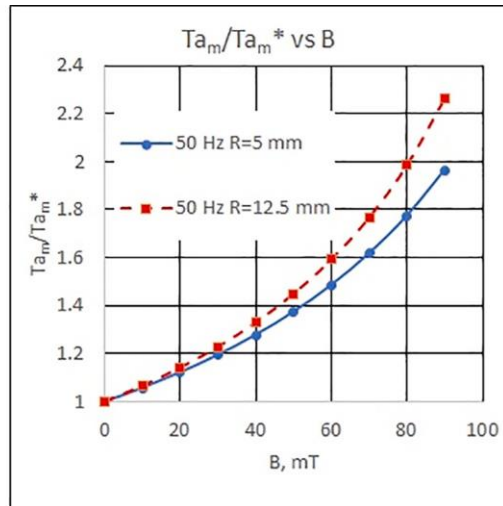
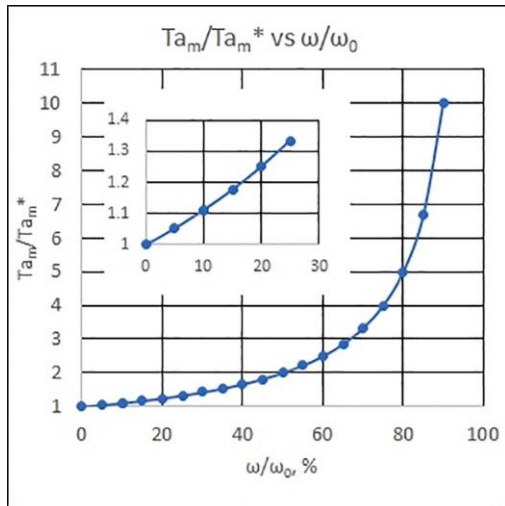


Fig. 6. (a) The ratio of the Magnetic Taylor number used in the literature (Ta_m) and the real one (Ta_m^*) as a function of the angular frequency of melt (ω)/angular frequency of magnetic field (ω_0) (b) Ratio of the Magnetic Taylor number used in the literature (Ta_m) and the real one (Ta_m^*) as a function magnetic induction (B).

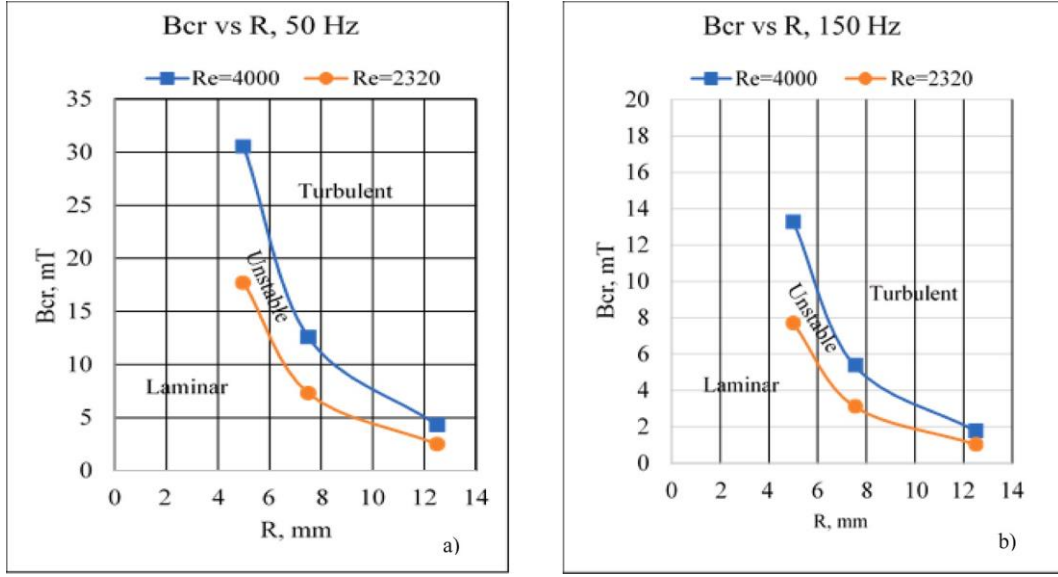


Fig. 8. (a) The critical magnetic induction (B_{cr}) as a function of sample radius (R) in case $f = 50$ Hz. (b) The critical magnetic induction (B_{cr}) as a function of sample radius (R) in case $f = 150$ Hz.

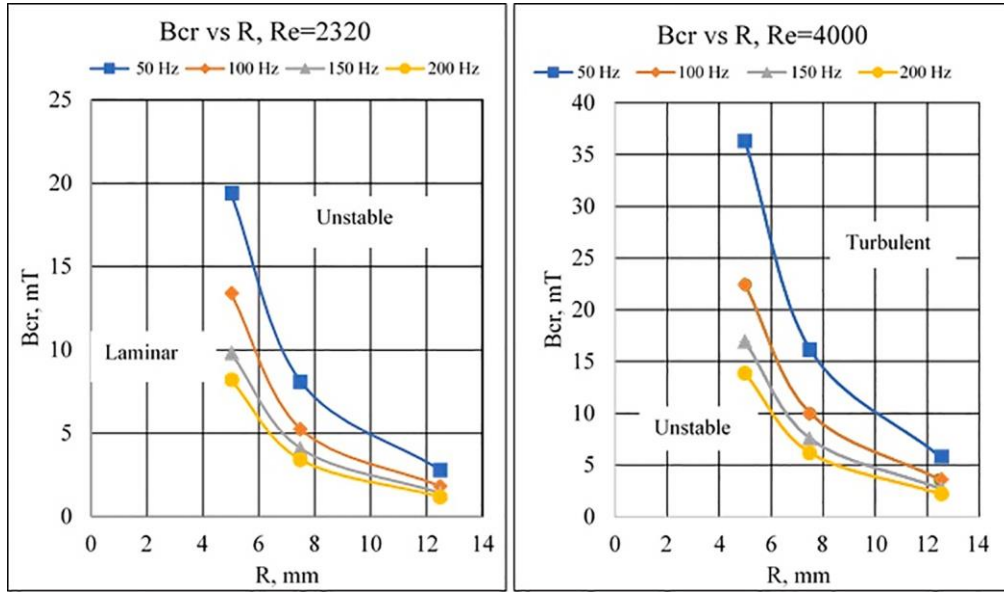


Fig. 9. (a) The critical magnetic induction (B_{cr}) as a function of sample radius (R) in case of laminar/unstable transition at 4 different magnetic field frequency (b) The critical magnetic induction (B_{cr}) as a function of sample radius (R) in case of unstable/turbulent transition at 4 different magnetic field frequency.

the radius of melt cylinder (R), the magnetic induction (B) – can be determined in case of which the flow will be pure laminar, pure turbulent or unstable (between the pure laminar and turbulent) in the melt. Fig. 8.a and 8.b show these results in cases of 50 and 150 Hz.

From these two figures, it is clear, that these transients depend on the frequency (f) of the magnetic field. Fig. 9.a shows the effect of the frequency on the transient of laminar/unstable flow, while Fig. 9.b. shows this effect on the transient of unstable /turbulent flow. It can be seen that the angular frequency of the magnetic field has a significant effect on the B_{cr} at a given radius.

Based on the value of $C(R, f)$ (Table 2) the critical magnetic induction as a function of the radius of the melt cylinder (R , mm) and the frequency of magnetic induction (f) can be given by the following function:

$$B_{cr} = A f^n R^{-2} \quad (12)$$

where $A = 7769$, $n = -0,628$ at $Re = 2320$, and $A = 16591$, $n = -0,695$ at $Re = 4000$ were calculated by multi-regression analysis of values of C (R). The correlation of measured and calculated (Eq. (12)) B_{cr} is shown in Fig. 10.

Used the determined B_{cr} values it is possible to calculate the $Ta_m^*_{cr}$ values by the Eq. (9) as a function of the radius which belongs to the laminar/unstable and unstable/turbulent transition. The calculated values of $Ta_m^*_{cr}$ as a function of radius are shown in Fig. 11.a. and 11.b. at 50 and 150 Hz, respectively. Similarly, to the values of B_{cr} $Ta_m^*_{cr}$ also depends on the frequency of the magnetic field. Fig. 12.a shows the effect of the frequency on the transient of laminar/unstable flow, while Fig. 12.b. shows this effect on the transient of unstable /turbulent flow as a function of R . These $Ta_m^*_{cr}$ values are like the values which can be found in the literature [15].

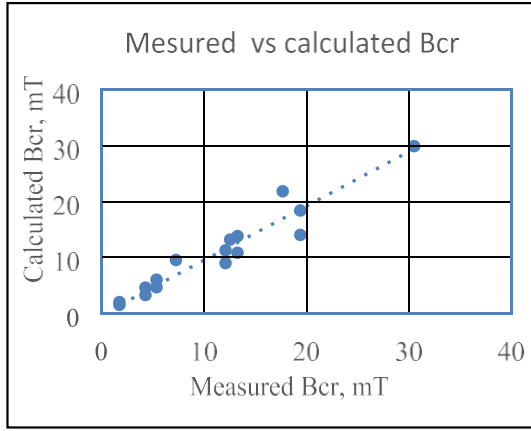


Fig. 10. Correllation of measured and calculated B_{cr} .

5. Summary and conclusions

In several cases, the effect of melt flow on the microstructure developing during the solidification process is investigated by the uni- directional solidification of melts with cylindrical sample geometry

(melt cylinders) while melt-flow is generated by a rotating magnetic field (RMF). The experiments have got a double aim: on the one hand, it is necessary to obtain direct information about the effect of melt flow on the solidified microstructure, and on the other hand, it is necessary to verify the results obtained by the simulation method. In both cases, it is very important to know the real flow rate of melt (in this case the angular frequency of rotating melt cylinder) which cannot be calculated directly from the angular frequency of the magnetic field. Similarly, it is also very important to know if the flow is laminar or turbulent at the given experimental parameters (radius of melt cylinder (R), magnetic induction (B), angular frequency of magnetic field (ω)). To compare the different experiments characterized by the magnetic Taylor number, we must know the real value of it which takes into account the real angular frequency of the melt.

A relatively simple, so-called pressure compensation method (PCM) was described in one of our earlier papers [12]. Using this method, the angular frequency (revolution number) of melt cylinder of a melt Ga75In25 alloy was determined as a function of the magnetic induction, the angular frequency of the magnetic field, and the radius of melt cylinder at room temperature ($22 \pm 1^\circ\text{C}$). By using and completing the published results,

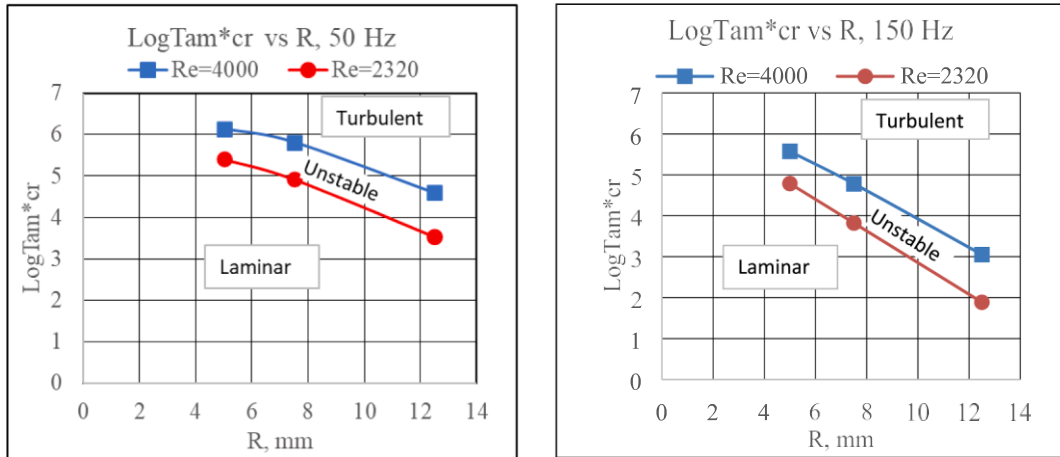


Fig. 11. (a) The critical real Magnetic Taylor number (LogTam^*_{cr}) as a function sample radius (R) in case of $f = 50$ Hz. (b) The critical real Magnetic Taylor number (LogTam^*_{cr}) as a function of sample radius (R) in case of $f = 150$ Hz.

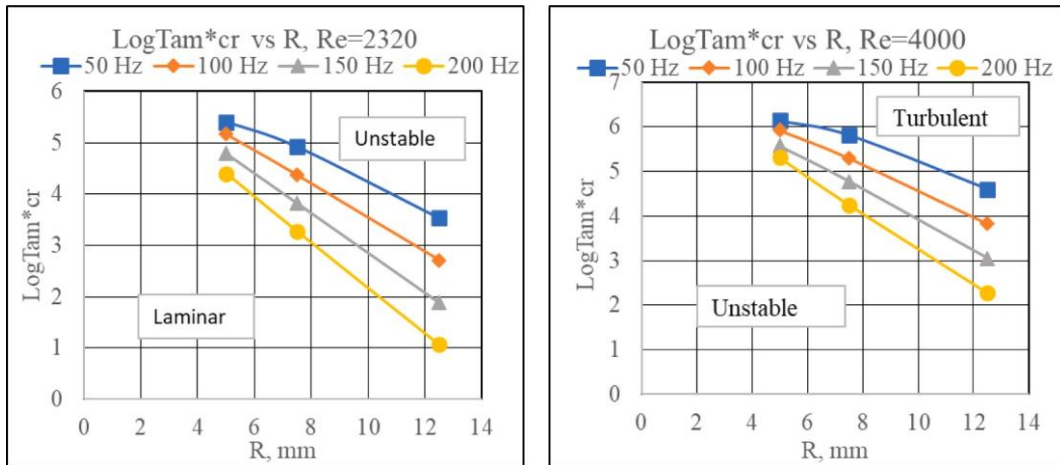


Fig. 12. (a) The real critical Magnetic Taylor number (LogTam^*_{cr}) as a function of sample radius (R) in case of laminar/unstable transition at 4 different magnetic field frequency; (b) The real critical Magnetic Taylor number (LogTam^*_{cr}) as a function of sample radius (R) in case of unstable/turbulent transition at 4 different magnetic field frequency.

- (i) the real angular frequency of melt flow was calculated from the measured pressure values,
- (ii) the real magnetic Taylor number was determined by knowing the real angular frequency; this number could be used when comparing the results of different experiments,
- (iii) it was proved that the magnetic Taylor number described in the References which does not take into consideration the real angular frequency of melt cylinder can be used for characterizing the experiment in case of a relatively low value of magnetic induction where the angular frequency of melt cylinder does not exceed 20% of the angular frequency of magnetic field,
- (iv) the real Reynolds number was determined by knowing the real angular frequency
- (v) the value of two critical magnetic inductions (B_{cr}) under which the flow is pure laminar and above pure turbulent were determined as a function of the diameter of melt cylinder by using the real Reynolds number
- (vi) the values of two B_{cr} can be described as a function of R radius of melt cylinder and the frequency of the magnetic field by the following function:

$$B_{cr} = \frac{A f^n}{R^2}$$

where A = 7769, n = 0.628 at Re = 2320, and A = 16591, n = 0.695 at Re = 4000 as determined by experiments, and R the radius of melt cylinder (mm).

Declaration of Competing Interest

The authors declared that there is no conflict of interest.

Acknowledgments

The authors are grateful for the Hungarian National Research, Development and Innovation Office for the subservience with the title of: "Formation of as-solidified structure and macrosegregation during unidirectional solidification under controlled flow conditions" and with the number of ANN 130946.

Appendix A

$$Ta_m^* = (Ha)^2 Re \quad (A1)$$

where Ha: Hartman number, Re: Reynolds number

$$Ha = \frac{\sqrt{F_L}}{F} \quad (A2)$$

The Lorenz Force (F_L) depend on the difference between the angular frequency of magnetic field (ω_0) and the melt (ω) and the friction force depend on the angular frequency of melt:

$$F_L = \frac{1}{2} \sigma \omega_0 B^2 R S \quad \text{and} \quad \omega_0 = 2\pi f$$

The S slip at the asynchronous motor (the RMF is an asynchronous motor), so $S = \frac{\omega_0 - \omega}{\omega_0}$, so

$$F_L = \frac{1}{2} \sigma B^2 R (\omega_0 - \omega) \quad (A4)$$

The Friction Force (F_F) depend on the angular frequency of melt:

$$F_F = \frac{\nu \rho \omega}{R} \quad (A5)$$

The Reynolds number depends on the angular frequency of melt:

$$Re = \frac{\omega R^2}{\nu} \quad (A6)$$

The modified (correct) Ta_m^* number is:

$$Ta_m^* = \frac{1}{2} \frac{\sigma B^2 R (\omega_0 - \omega)}{\frac{\nu \rho \omega}{R}} \frac{\omega R^2}{\nu} = \frac{1}{2} \frac{\sigma B^2 R^4 (\omega_0 - \omega)}{\nu^2 \rho} \quad (A7)$$

If ω is very small ($\omega \ll \omega_0$) we get back the originally used Taylor number

$$Ta_m^* = \frac{1}{2} \frac{\sigma B^2 R^4 \omega_0}{\nu^2 \rho} \quad (A8)$$

References

- [1] B. Frago, H. Santos, Effect of a rotating magnetic field at the microstructure of an A354, J. Mater. Res. Technol. 2 (2013) 100–109.
- [2] S. Nafisi, D. Emadi, M.T. Shehat, R. Ghomashchi, Effects of electromagnetic stirring and superheat on the microstructural characteristics of Al–Si–Fe alloy, Mater. Sci. Eng., A 432 (2006) 71–83.
- [3] S.S.C. Lim, E.P. Yoon, The effect of electromagnetic stirring on the microstructure of Al-7 wt %Si alloy, J. Materials Letters 16 (1997) 104–109.
- [4] J.C. Jie, et al., Separation mechanism of the primary Si phase from the hypereutectic Al–Si alloy using a rotating magnetic field during solidification, Acta Mater. 72 (2014) 57–66.
- [5] B. Willers, et al., The columnar-to-equiaxed transition in Pb–Sn alloys affected by electromagnetically driven convection, Mater. Sci. Eng., A 402 (2005) 55–65.
- [6] J. Kovács, et al., Quantitative Characterisation of Macroseggregation Produced by Forced Melt Flow, Trans. Indian Inst. Met. 60 (2007) 149–154.
- [7] J. Kovács, A. Rónaföldi, Á. Kovács, A. Roósz, Effect of the rotating magnetic field on the unidirectionally solidified macrostructure of Al6Si4Cu alloy, Trans. IndianInst. Metals 62 (2009) 461–464.

- [8] A. Rónaföldi, J. Kovács, A. Roósz, A suggested effective method for unidirectional solidification under rotating magnetic field in the space experiments, *Trans. Indian Inst. Metals* 62 (2009) 475–477.
- [9] O. Budenkova, et al., Simulation of a directional solidification of a binary Al-7wt% Si and a ternary alloy Al-7wt% Si-1wt% Fe under the action of a rotating magnetic field. *IOP Conf. Series, Mater. Sci. Eng.* 33 (2012), 012046.
- [10] H. Song, *Engineering Fluid Mechanics*, Jointly published with Metallurgical Industry Press, Beijing, China.
- [11] F. M. White, *Fluid Mechanics*, 4th edition. McGraw-Hill Higher Education, 2002, ISBN: 0-07-228192-8.
- [12] <https://byjus.com/physics/derivation-of-reynolds-number/>.
- [13] A. Rónaföldi, J. Kovacs, A. Roósz, Investigation and Visualisation of Melt Flow Under Rotating Magnetic Field *Trans. Indian Inst. Met.* 60 (2007) 213–218.
- [14] A. Rónaföldi, J. Kovács, A. Roósz, Revolution Number (RPM) Measurement of Molten Alloy by Pressure Compensation Method *Materials Science Forum*, Online649 (2010) 275–280.
- [15] J.S. Walker, L.M. Witkowski, Linear stability analysis for a rotating cylinder with a rotating magnetic field, *Phys. Fluids* 16 (2004) 2294–2299.