

# Early exclusion leads to cyclical cooperation in repeated group interactions

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Explaining the emergence and maintenance of cooperation among selfish individuals from an evolutionary perspective remains a grand challenge in biology, economy, and social sciences. Ostracism or social exclusion is believed to be an answer to this conundrum, but related studies often assume one-shot interactions and ignore how free-riding is identified. In this work, we consider repeated interactions where excluders need to pay a monitoring cost to identify free-riders for exclusion and free-riders cannot participate in the following possible game interactions once they are identified and excluded by excluders in the repeated interaction process. We reveal that the introduction of such exclusion can prevent the breakdown of cooperation in repeated group interactions. In particular, we demonstrate that an evolutionary oscillation among cooperators, defectors, and excluders can appear in infinitely large populations when early exclusion is implemented. In addition, we find that the population spends most of the time in states where cooperators dominate for early exclusion when stochastic mutation-selection is considered in finite populations. Our results highlight that immediate exclusion is more successful than late implementation of exclusion in solving the mentioned enigma of cooperation.

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## 1. Introduction

Cooperative behaviour provides generous benefits for group members, but it is vulnerable to the exploitation of public goods by selfish individuals who contribute nothing to the joint venture [1–5]. In this way, cooperation will eventually collapse [6]. Conversely, cooperation described by providing public goods is universal ranging from microorganism to human society [7–10]. Thus what drives the emergence of cooperative behaviour has attracted widespread attention from broad range of disciplines [11–14].

In past decades, a substantial body of theoretical and experimental investigations have illuminated various solutions including norms and incentive mechanisms that can be used to promote the evolution of cooperation [15–26]. A frequently discussed incentive mechanism is punishment, with which some cooperators pay extra cost to punish selfish individuals [27–31]. As a special form of punishment, social exclusion has been introduced. Indeed this incentive mechanism has been found in realistic human and biological interactions [32–35]. For instance, cooperating groups of bacteria (wild-type *Pseudomonas aeruginosa*) growing on protein can restrict cheater emergence by producing costly cyanide to exclude LasR-null social cheaters from sharing the provided public goods [32, 33]. Recently, social exclusion has been investigated from the theoretical perspective recently [36]. It is often assumed that free-riders will be expelled from the beneficiaries and cannot get any benefit when exclusion is implemented, which is different from the setting of traditional punishment under which free-riders are imposed with a fixed penalty fine. Social exclusion strategy has been found to be more favourable to the evolution of cooperation compared with the traditional punishment, since its introduction into evolutionary games [37–39].

Although the aforementioned studies have enhanced our understanding about how social exclusion influences cooperative behaviour, they generally build on some limiting and questioning assumptions. To be specific, the evolutionary dynamics of exclusion strategy are always explored in ephemeral one-shot, but not repeated interactions. Furthermore, excluders only need to pay the exclusion cost for excluding defectors. Indeed in previous behavioural experiments about social exclusion, it is often required that excluders in the group must be able to identify “bad apples” before excluding them in repeated public goods games [40–42]. These excluders identify cooperators and defectors by recording players’ names and allocation decisions, which could require excluders to pay the extra cost of monitoring and information collecting in the interaction process.

31 Hence when the above mentioned restrictions are released, it is unclear whether exclusion is still  
 32 a viable strategy and how it can facilitate cooperation from the theoretical perspective, which are  
 33 worth investigating.

34 In this work, we thereby propose a social exclusion strategy and explore its evolutionary con-  
 35 sequence in repeated public goods games. We assume that excluders can monitor the decisions  
 36 of all group members permanently to identify free-riding behaviours in the repeated interaction  
 37 process by paying a monitoring cost, and subsequently expel these identified free-riders from  
 38 group benefits. Our results reveal that the exclusion round is crucial for cooperation to thrive,  
 39 since it determines how much defectors can benefit from previous repeated interactions. In infinite  
 40 well-mixed populations, we observe evolutionary oscillations among cooperators, defectors, and  
 41 excluders when free-riders can be expelled at the earliest stage. Similarly, stochastic evolution in  
 42 finite populations can favour cooperation over defection when implemented exclusion is timely.

## 43 2. Model and Methods

44 We consider a well-mixed population from which  $N$  individuals are selected randomly to form  
 45 an interaction group for playing a repeated public goods game. There are three game strategies:  
 46 cooperation ( $C$ ), defection ( $D$ ), and peer exclusion ( $E$ ). Both  $C$  and  $E$  individuals contribute  $c$  to  
 47 the common pool in every round, while  $D$  individuals do nothing. In every round the accumulated  
 48 contributions are multiplied by an enhancement factor  $F$  ( $1 < F < N$ ) and subsequently shared  
 49 equally among all group members, irrespective of whether they contributed or not. In the frame-  
 50 work of repeated group interactions, the above game process will be repeated with probability  $w$   
 51 ( $0 < w < 1$ ), resulting in an average number of  $\langle r \rangle = \frac{1}{1-w}$  rounds [43]. Here we assume that peer  
 52 excluders pay a permanent cost  $\sigma$  to monitor the game process for identifying  $D$  individuals and  
 53 subsequently exclude all identified defectors from the group at the  $\varsigma$ -th round by paying an addi-  
 54 tional cost  $c_E$ . The excluded free-riders cannot share the public goods of the  $\varsigma$ -th round and will  
 55 lose the opportunity to participate in the following possible game rounds. It is worth noting that if  
 56 the average number of rounds is smaller than the round in which defectors are excluded, namely,  
 57  $\langle r \rangle < \varsigma$ , then peer exclusion strategists completely degenerate into pure cooperators except the  
 58 permanent cost  $\sigma$  of monitoring. Thus in this case defection is a more advantageous strategy than  
 59 peer exclusion and cooperation [36].

60 Following the above description of game model, we can write the payoff of a cooperator, defec-  
 61 tor, and excluder playing the repeated public goods game in a group in which  $N_C$  cooperators,  $N_E$

62 excluders, and  $N_D$  defectors are present. Thus the related payoff values for cooperators, defectors,  
63 and excluders are respectively give as

$$\pi_C = \begin{cases} \frac{Fc(N_C + N_E + 1)}{N}(\varsigma - 1) + Fc(\langle r \rangle - \varsigma + 1) - \langle r \rangle c, & \text{if } \varsigma \leq \langle r \rangle \text{ and } N_E \neq 0; \\ \frac{Fc(N_C + N_E + 1)}{N} \langle r \rangle - \langle r \rangle c, & \text{otherwise.} \end{cases}$$

$$\pi_D = \begin{cases} \frac{Fc(N_C + N_E)}{N}(\varsigma - 1), & \text{if } \varsigma \leq \langle r \rangle \text{ and } N_E \neq 0; \\ \frac{Fc(N_C + N_E)}{N} \langle r \rangle, & \text{otherwise.} \end{cases}$$

$$\pi_E = \begin{cases} \frac{Fc(N_C + N_E + 1)}{N}(\varsigma - 1) + Fc(\langle r \rangle - \varsigma + 1) - \langle r \rangle c - c_E N_D - \sigma, & \text{if } \varsigma \leq \langle r \rangle; \\ \frac{Fc(N_C + N_E + 1)}{N} \langle r \rangle - \langle r \rangle c - \sigma, & \text{otherwise.} \end{cases}$$

64

65 We then apply evolutionary game theory to investigate the evolution of strategies in our  
66 model [55] . To be specific, we will respectively consider our model in infinite and finite  
67 well-mixed populations in the following.

68

#### (a) Infinite well-mixed populations.

We consider an infinite well-mixed population, in which  $N$  individuals are selected randomly to engage in the repeated public goods game. Since random sampling leads to groups with compositions that follow a multivariate binomial distribution, the average payoff  $P_i$  of one individual adopting a given strategy  $i$  can be calculated as

$$P_i = \sum_{N_C=0}^{N-1} \sum_{N_D=0}^{N-N_C-1} \binom{N-1}{N_C} \binom{N-N_C-1}{N_D} x^{N_C} y^{N_D} z^{N-N_C-N_D-1} \pi_i,$$

69 where  $i = C, D,$  or  $E$ .

70 In order to investigate the evolutionary dynamics in infinite well-mixed populations, we use the  
71 replicator equation approach [44, 45]. To do that, we use  $x, y,$  and  $z$  to denote the fraction of  $C, D,$   
72 and  $E$  players, respectively. Correspondingly, we have  $x + y + z = 1$  and the replicator equation  
73 can be written as

$$\begin{cases} \dot{x} = x(P_C - \bar{P}), \\ \dot{y} = y(P_D - \bar{P}), \\ \dot{z} = z(P_E - \bar{P}), \end{cases} \quad (1)$$

74 where  $\bar{P} = xP_C + yP_D + zP_E$  represents the average payoff of the whole population. More  
 75 detailed theoretical analysis of the replicator dynamics can be found in Electronic Supplementary  
 76 Material.

77 Furthermore, since replication is error-prone [56, 57], we also incorporate mutation into the  
 78 above deterministic replicator dynamics. The resulting replicator-mutator equation can be written  
 79 as

$$\begin{cases} \dot{x} = xP_Cq_{C \rightarrow C} + yP_Dq_{D \rightarrow C} + zP_Eq_{E \rightarrow C} - x\bar{P}, \\ \dot{y} = yP_Dq_{D \rightarrow D} + xP_Cq_{C \rightarrow D} + zP_Eq_{E \rightarrow D} - y\bar{P}, \\ \dot{z} = zP_Eq_{E \rightarrow E} + xP_Cq_{C \rightarrow E} + yP_Dq_{D \rightarrow E} - z\bar{P}, \end{cases} \quad (2)$$

80 where  $q_{U \rightarrow V}$  denotes the probability that strategy  $U$  generates an offspring using strategy  $V$   
 81 ( $U, V = C, D, \text{ or } E$ ). A detailed description and analysis of the replicator-mutator equation is  
 82 present in Electronic Supplementary Material.

83

#### 84 (b) Finite well-mixed populations.

85 We further consider our model in a finite well-mixed population in which the population size is  
 86  $Z$ . For finite well-mixed populations of size  $Z$  with  $i_C$  cooperators,  $i_E$  excluders, and  $Z - i_C -$   
 87  $i_E$  defectors, we know that the average payoffs of these three strategies in a configuration  $\mathbf{i} =$   
 88  $\{i_E, i_C\}$  can be respectively computed by using a hypergeometric sampling [18]. More precisely,  
 89 the average payoffs of cooperators, defectors, and excluders in the configuration  $\mathbf{i} = \{i_E, i_C\}$  can  
 90 be respectively given as

$$\begin{aligned} f_C &= \sum_{N_C=0}^{N-1} \sum_{N_E=0}^{N-N_C-1} \frac{\binom{i_C-1}{N_C} \binom{i_E}{N_E} \binom{Z-i_C-i_E}{N-N_C-N_E-1}}{\binom{Z-1}{N-1}} \pi_C, \\ f_D &= \sum_{N_C=0}^{N-1} \sum_{N_E=0}^{N-N_C-1} \frac{\binom{i_C}{N_C} \binom{i_E}{N_E} \binom{Z-i_C-i_E-1}{N-N_C-N_E-1}}{\binom{Z-1}{N-1}} \pi_D, \\ f_E &= \sum_{N_C=0}^{N-1} \sum_{N_E=0}^{N-N_C-1} \frac{\binom{i_C}{N_C} \binom{i_E-1}{N_E} \binom{Z-i_C-i_E}{N-N_C-N_E-1}}{\binom{Z-1}{N-1}} \pi_E. \end{aligned} \quad (3)$$

91 Subsequently, we consider the pairwise comparison rule combined with mutation to describe  
 92 how the number of individuals adopting a given strategy evolves in finite well-mixed populations  
 93 [48, 61, 62]. To be specific, with probability  $\mu$ , mutation occurs and one randomly selected player  
 94  $L$  picks a new strategy randomly from the remaining available strategies space. With probability  
 95  $1 - \mu$ , player  $L$  imitates the strategy of another randomly selected player  $R$  as follows. Player  $L$

96 adopts the strategy of player  $R$  with probability

$$\bar{p} = \frac{1}{1 + \exp[\beta(f_L - f_R)]}, \quad (4)$$

97 where  $\beta \geq 0$ , the so-called the intensity of selection, translates into noise associated with errors in  
 98 the imitation process [60]. In the limit case  $\beta \rightarrow \infty$ , player  $L$  always adopts the more successful  
 99 strategy of player  $R$ . While  $\beta \rightarrow 0$  indicates weak selection limit where strategy adoption becomes  
 100 random independently of the payoffs difference between these two players. In between these  
 101 extremes, for a finite value of  $\beta$ , a better performing strategy is more likely to be adopted. **With**  
 102 **probability  $1 - \bar{p}$ , player  $L$  remains his/her current strategy unchanged.**

103 Based on the above description, we can write the transition probability that an individual se-  
 104 lected from  $i_U$  players with a given strategy  $U$  changes into another different strategy  $V$  as [61]

$$T_{U \rightarrow V} = (1 - \mu) \left[ \frac{i_U}{Z} \frac{i_V}{Z-1} \frac{1}{1 + \exp(\beta(f_U - f_V))} \right] + \mu \frac{i_U}{(d-1)Z},$$

105 where  $U, V = C, D$ , or  $E$ , and  $d$  is the number of alternative strategies in the strategy space.

106 We stress that, in the presence of mutation, the population will never fixate in any of the ab-  
 107 sorbing states. Here we focus on finite populations and consider two different cases: the case of  
 108 sufficiently small mutation rates and the case of arbitrary mutation rates. For smaller mutation  
 109 rates, a mutant either goes extinct or takes over the whole population before next mutation occurs  
 110 [63–65]. Thus the population is homogeneous most of the time and only consists two strategies at  
 111 a given time. In this case, the dynamics can be described by an embedded Markov chain based on  
 112 the transition from one homogeneous state to another. The stationary distribution which describes  
 113 the percentage of time spent by the population can be calculated by searching the eigenvector  
 114 corresponding to the eigenvalue 1 of the transition matrix. The element of transition matrix, repre-  
 115 senting the probability of transition from  $AllU$  state to  $AllV$  state, can be given by  $\mu\rho_{UV}/(d-1)$ ,  
 116 where  $\rho_{UV}$  denotes the fixation probability and can be written as

$$\rho_{UV} = \frac{1}{1 + \sum_{q=1}^{Z-1} \exp[\beta \sum_{i_V=1}^q (f_U - f_V)]}.$$

117 It is worth noting that the imitation strength  $\beta$  plays a crucial role in the evolutionary dynamics  
 118 of the system. For theoretical analysis, we shall focus on the two limit cases, i.e., weak selection  
 119 limit ( $\beta \rightarrow 0$ ) and strong imitation strength limit ( $\beta \rightarrow \infty$ ). Detailed theoretical analysis can be  
 120 found in Electronic Supplementary Material.

121 For arbitrarily mutation rates, the evolutionary dynamics among  $C$ ,  $D$ , and  $E$  can be described  
 122 by an embedded Markov process over a two-dimensional space. Transitions take place between  
 123 different configurations of the system characterized by the vector  $\mathbf{i}(t) = \{i_E, i_C\}$ . The study of  
 124 Markov process is to mainly determine the evolution of its probability density function,  $p_{\mathbf{i}}(t)$ ,  
 125 which provides information on the prevalence of each configuration at time  $t$  [61]. The transition  
 126 probability and probability density function obey the following discrete time Master Equation [66],  
 127 described as

$$p_{\mathbf{i}}(t + \tau) - p_{\mathbf{i}}(t) = \sum_{\mathbf{i}'} \{T_{\mathbf{i}\mathbf{i}'} p_{\mathbf{i}'}(t) - T_{\mathbf{i}'\mathbf{i}} p_{\mathbf{i}}(t)\}, \quad (5)$$

128 where  $T_{\mathbf{i}\mathbf{i}'}$  denotes the transition probability from configuration  $\mathbf{i}$  to adjacent configuration  $\mathbf{i}'$  in  
 129 the time interval  $\tau$ . The so-called stationary distribution  $\bar{p}_{\mathbf{i}}$  is obtained by making the left side of  
 130 Eq. (5) zero, which transforms our calculation into an eigenvector search problem. Namely, the  
 131 eigenvector is associated with the eigenvalue 1 of the transition matrix (see Electronic Supplemen-  
 132 tary Material for details).

133 Furthermore, employing the Kramers-Moyal expansion of the above Master Equation, we can  
 134 obtain the gradient of selection, which provides information on the most likely direction of change  
 135 of the population configuration with time [49], as

$$g_{\mathbf{i}} = (T_{\mathbf{i}}^{E+} - T_{\mathbf{i}}^{E-})\mathbf{u}_E + (T_{\mathbf{i}}^{C+} - T_{\mathbf{i}}^{C-})\mathbf{u}_C, \quad (6)$$

136 where  $\mathbf{u}_E$  and  $\mathbf{u}_C$  are unit vectors defining as a basis of the two dimensional simplex. Besides,  $T_{\mathbf{i}}^{E+}$   
 137 ( $T_{\mathbf{i}}^{E-}$ ) represents the probability that the number of individuals adopting strategy  $E$  to increas-  
 138 es (decreases) by one, and  $T_{\mathbf{i}}^{C+}$  ( $T_{\mathbf{i}}^{C-}$ ) represents the probability that the number of individuals  
 139 adopting strategy  $C$  increases (decreases) by one, which can be respectively computed as

$$T_{\mathbf{i}}^{E\pm} = T_{\mathbf{i}\{i_E\pm 1, i_C\mp 1, i_D\}} + T_{\mathbf{i}\{i_E\pm 1, i_C, i_D\mp 1\}},$$

140 and

$$T_{\mathbf{i}}^{C\pm} = T_{\mathbf{i}\{i_E\mp 1, i_C\pm 1, i_D\}} + T_{\mathbf{i}\{i_E, i_C\pm 1, i_D\mp 1\}}.$$

141 Furthermore, to describe the average abundance of strategy  $U$  more intuitively, we provide  
 142 another important quantity, the so-called average strategy level  $\bar{\rho}_U$ , averaging over all possible  
 143 configurations, weighted with the corresponding stationary distribution. Accordingly, it can be

144 computed as

$$\bar{\rho}_U = \sum_{\mathbf{i}} \mathbf{i}_{i_U} \bar{p}_{\mathbf{i}} / Z, \quad (7)$$

145 where  $\mathbf{i}_{i_U}$  denotes the number of  $U$  strategists in the configuration  $\mathbf{i}$  and  $U = C, D$  or  $E$ .

146

147 By means of individual-based simulations, we also investigate social learning dynamics with an  
 148 arbitrary exploration rate  $\mu$  in finite populations [64]. Each individual obtains an average payoff  
 149 calculated by Eq. (3) based on the random sampling of the interaction groups. Strategies evolve  
 150 via a mutation-selection process that can be used to describe the evolution of strategies in discrete  
 151 time. At each time step, one player  $L$  is randomly selected to update its strategy in the following  
 152 way: with probability  $\mu$ , player  $L$  undergoes a mutation and randomly adopts one of the available  
 153 strategies. With probability  $1 - \mu$ , another player  $R$  is selected randomly to be a role model for  
 154 player  $L$ . Then player  $L$  imitates the strategy of player  $R$  according to Eq. (4).

### 155 3. Results

156 In the following, we respectively present our results of evolutionary dynamics of cooperators,  
 157 defectors, and excluders in infinite and finite well-mixed populations.

158

#### 159 (a) Evolutionary dynamics in infinite well-mixed populations.

160 Let us first focus on the evolutionary dynamics of the three strategies  $C$ ,  $D$ , and  $E$  described by  
 161 Eq. (1) in infinite well-mixed populations. According to the theoretical analysis, we distinguish  
 162 three different cases where the outcome changes with increasing exclusion round  $\varsigma$ . For small  
 163 exclusion round satisfying  $\varsigma < \frac{N[(F-1)\langle r \rangle c - \sigma - (N-1)c_E]}{(N-1)Fc} + 1$ , cooperation, defection, and exclusion  
 164 can form a cyclic dominance that resembles a rock-scissors-paper cycle on the boundary of  $S_3$ , as  
 165 shown in Fig. 1A and B. Here the evolution on the  $CD$  edge is unidirectional from  $C$  to  $D$ . It goes  
 166 from  $D$  to  $E$  on the  $DE$  edge and goes from  $E$  to  $C$  on the  $EC$  edge. [In Electronic Supplementary](#)  
 167 [Material, we theoretically prove that there is an asymptotically stable heteroclinic cycle on the](#)  
 168 [boundary of  \$S\_3\$ . We show that there exists a limit cycle in the interior of the simplex, in which all](#)  
 169 [interior trajectories of the state space form evolutionary oscillations around the interior equilibrium](#)  
 170 [when  \$\varsigma\$  is small \(see Fig. 1A, B, D, and E\). As  \$\varsigma\$  increases and satisfies  \$\frac{N\[\(F-1\)\langle r \rangle c - \sigma - \(N-1\)c\_E\]}{\(N-1\)Fc} + 1 <\$   
 171  \$\varsigma < \frac{N\[\(F-1\)\langle r \rangle c - \sigma\]}{\(N-1\)Fc} + 1\$ , an unstable equilibrium appears on the  \$DE\$  edge and the state where each  
 172 individuals choose to defect is the only stable equilibrium. Numerical calculations confirm that all](#)

173 trajectories started in the state space converge to a state where all  $D$  players appear (see Fig. 1C and  
 174 F). As  $\varsigma$  further increases and exceeds  $\langle r \rangle$ , defectors have more advantages over both cooperators  
 175 and excluders, so that the state of full  $D$  is globally stable.

176 Next, we study the effects of other parameters on the replicator dynamics of cooperators, de-  
 177 fectors, and excluders, as shown in Electronic Supplementary Material. We find that cooperative  
 178 behaviours can be maintained in the population when the exclusion round  $\varsigma$  is not too large. Partic-  
 179 ularly, in the specific case of  $\varsigma = 1$  we can reproduce previous findings that excluders can emerge  
 180 in a sea of defectors and dominate them [36]. Furthermore, we provide some numerical examples  
 181 to show the effects of different observation cost  $\sigma$  (Fig. S2) and discount factor  $w$  (Fig. S4) on  
 182 replicator dynamics. We find that our main results remain valid if the value of  $\sigma$  is approximate-  
 183 ly changed. Besides, the cyclic dominance among cooperators, defectors, and excluders can still  
 184 appear when the value of  $w$  is not too small.

185 In order to shed light on the details of the evolutionary oscillations dynamics shown in the  
 186 Fig. 1A and B, we depict the average levels of the mentioned three strategies over an approximate  
 187 periodic orbit for different exclusion rounds  $\varsigma$ . Fig. 2 illustrates that the highest cooperation rates  
 188 can be obtained for immediate exclusion. The intuitive explanation is that the maximal cooperation  
 189 level arises if excluders can quickly expel free-riders from the beneficiaries. Interestingly, although  
 190 the average level of pure cooperators decreases with the increase of  $\varsigma$ , it is always higher than that  
 191 of excluders and free-riders, as long as the exclusion is executed not too late.

192 Furthermore, we extend the replicator equation and consider the “replicator-mutator equation”  
 193 for frequency-dependent selection with mutation in infinite well-mixed populations. For small  
 194 mutation rate, we find that there is also an unstable interior equilibrium containing all three  
 195 strategies, and an evolutionary oscillation forms around this equilibrium (Fig. S5A in Electronic  
 196 Supplementary Material). If the mutation rate exceeds a critical value, the evolutionary oscillation  
 197 disappears and all interior trajectories converge to a stable fixed point. With the increase of  $\mu$ ,  
 198 the interior equilibrium point moves towards the inner part of simplex gradually (Fig. S5B-E in  
 199 Electronic Supplementary Material).

200

## 201 (b) Evolutionary dynamics in finite well-mixed populations.

202 We note that replicator equations cannot be used directly to describe the dynamics of a more  
 203 realistic system where the population size is finite. In the latter case, stochastic effects including  
 204 behavioural mutations or errors of imitation may play important roles in the evolutionary dynamics

205 [47, 48]. In the following, we investigate the stochastic dynamics of finite populations when  
 206 mutation rates are sufficiently small and arbitrarily large.

207 When mutation rate is arbitrarily large, the stochastic effects can drive the system from the  
 208 vicinity of one configuration of the state space to the other. The stochastic dynamics associated  
 209 with these three strategies are best characterized by the gradient of selection and the full station-  
 210 ary distributions, as shown in Fig. 3A-C and Figs. S8-S10. Each triangular simplex contains all  
 211 possible configurations of the finite population where each configuration is represented by a small  
 212 circle. The magnitude of stationary distributions is shown by using a grey scale where darker areas  
 213 indicate more often visited configurations. The arrows show the so-called gradient of selection,  
 214 which provides the most likely direction of evolution from a given configuration, which can be  
 215 calculated from the drift term of the Fokker-Planck equation [49].

216 Similarly to the evolutionary outcomes in infinite well-mixed populations, the qualitative be-  
 217 havioral dynamics in the finite population change with the exclusion round  $\varsigma$ . As shown in Fig. 3A-  
 218 C, three representative examples of the **behavioural** dynamics among  $C$ ,  $D$ , and  $E$  players for d-  
 219 ifferent exclusion rounds  $\varsigma$  are presented when mutation is not negligible. For small exclusion  
 220 rounds, if most players of the population are  $C$  players, it is better to be a  $D$  player due to social  
 221 dilemma. If  $D$  players are prevalent,  $E$  players can rapidly outcompete  $D$  players, leading the  
 222 population to the configuration that contains all- $E$  players. In the latter case, **behavioural** muta-  
 223 tions allow the emergence of  $C$  players in the population and  $E$  players have less advantage than  
 224  $C$  players since  $C$  players do not need to bear additional monitoring costs. Accordingly,  $C$  players  
 225 spread. As a result, a cyclical evolutionary scenario can be formed (see the direction of arrow  
 226 flow). This outcome is also verified by individual-based simulations (see Fig. 3D and E). Besides,  
 227 as shown by the stationary distributions, the population will spend most of the time in configu-  
 228 rations comprising a sizable amount of  $C$  players together with  $E$  players, which can prevent  $D$   
 229 players from invading (see Fig. 3A and B).

230 For high values of exclusion round ( $\varsigma = 9$ ), the **behavioural** dynamics are quite different.  
 231 As shown in Fig. 3C, the population spends a significant amount of time in configurations in  
 232 which defectors spread. Fig. 3E also reveals that the whole population is taken over completely  
 233 by defectors. It is not surprising that such outcome occurs when the value of exclusion rounds  
 234 is too large because  $D$  players have obtained a higher payoff before they are excluded from the  
 235 repeated group interactions by  $E$  players. Furthermore, exclusion strategy will be completely  
 236 ineffective in resisting defection when exclusion round exceeds the average game rounds, allowing

237 the population to spend longer periods of time in configurations with many defectors.

238 Finally, we present theoretical and numerical results for the stationary distribution when the  
 239 mutation rate is sufficiently small. In this case, the time scales of mutation and imitation are  
 240 separated, thus the population is homogeneous most of the time. The evolutionary dynamics are  
 241 determined by an embedded Markov chain where the state space is composed of homogeneous  
 242 states (more details can be found in Electronic Supplementary Material). Both theoretical and  
 243 numerical results underline that exclusion strategy can have more evolutionary advantages over the  
 244 other two strategies for weak imitation strength when the exclusion round is small (Fig. S10A). For  
 245 strong imitation strength, however, the long-run frequencies of the three competing strategies are  
 246 identical (Fig. S11A). A detailed analytical approximations of the transition matrix among these  
 247 three homogeneous states shows that the long-run frequencies in the  $(C, D, E)$  subpopulations are  
 248  $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$  in the strong selection (see Electronic Supplementary Material).

249 Similarly to the above described cases, when exclusion round is high, defectors can have more  
 250 advantages than the two other strategies for weak imitation strength (Fig. S10B). With the increase  
 251 of imitation strength, the advantage of defectors increases gradually. In the strong imitation limit,  
 252 defection is the most advantaged strategy, yielding  $(0, 0, 1)$  fractions for  $C, E,$  and  $D$  strategies,  
 253 respectively (Fig. S11B). A detailed theoretical analysis of the transition matrix of the system and  
 254 the linear approximation of the stationary distribution in the weak imitation strength can be found  
 255 in Electronic Supplementary Material.

256 In Fig.4, we further show the average levels of cooperation, exclusion, and defection for dif-  
 257 ferent values of  $\varsigma$  when mutation cannot be ignored. Interestingly, the average cooperation level  
 258 slightly increases first then decreases with the increase of the exclusion round  $\varsigma$ . The changing  
 259 trend about the cooperation level is opposite to that about the average level of excluders. Besides,  
 260 the level of defection increases with the increase of  $\varsigma$ . Altogether, Fig. 4 confirms that cooperation  
 261 can be maintained at a high level as long as free-riders can be executed in the early rounds. Indeed,  
 262 in the presence of mutation, the population will never fixate in any of the absorbing states. In addi-  
 263 tion, the intensity of selection incorporates noise associated with errors into the process of strategy  
 264 updates. Accordingly, we can find that there exists evolutionary oscillations for the frequencies of  
 265 cooperators, defectors, excluders, as shown in Fig. 3D and E. Since the cooperation level shown  
 266 in Fig.4 averaging over all possible configurations are weighted with the corresponding strategy  
 267 distribution, the oscillation effects has been involved and we can see that the cooperation level  
 268 slightly increases with the number of rounds even when the values of intensity of selection and

269 mutation rate are changed appropriately. However, we stress that our main conclusion that im-  
270 mediate exclusion is [favourable](#) to the evolution of cooperation is still valid in finite well-mixed  
271 populations.

#### 272 **4. Discussion**

273 In modern societies, it has been widely observed that ostracism or the exclusion of disapproved  
274 members from groups, teams, and communities can enforce norm conformity and cooperation  
275 [33, 50]. However, it still needs further theoretical efforts to understand its evolutionary advan-  
276 tage. In this work, we have developed an evolutionary game model to explore the dynamical  
277 consequences of social exclusion strategy in the public goods game where group members are  
278 engaged in repeated interactions. We assume that excluders need to monitor the whole group  
279 interactions for collecting relevant information before excluding free-riders. If the exclusion is ex-  
280 ecuted, then free-riders cannot obtain the benefits of common pool anymore, but they can have the  
281 incomes originated from previous rounds. We have found that the actual round at which defectors  
282 are excluded plays a decisive and highly non-trivial role in the general well-being. Interest-  
283 ingly, the evolutionary oscillation among the competing strategies can be detected if the exclusion  
284 is executed at an early stage in repeated interactions. More surprisingly, the time averages of  
285 the temporary fractions of strategies display the outcome of the dominance of cooperators. We  
286 have also investigated the stochastic dynamics of a finite population in the presence of [behavioural](#)  
287 mutations and found that immediate exclusion can lead to a high level of cooperation.

288 [In this work, we investigate the effects of ostracism on cooperation in the multi-period public](#)  
289 [goods game by comparing the treatments with and without exclusion, mainly inspired by some](#)  
290 [interesting behavioural experiments \[40–42, 51\].](#) In the former experiments, subjects are given  
291 the opportunity to exclude each other after they are informed about the individual contributions  
292 of other group members, and player who is ostracized cannot take part in the subsequent interac-  
293 tions. The experimental results show that subjects exhibit a willingness to exclude other members  
294 of their group and the threat of social exclusion for uncooperative [behaviour](#) can significantly im-  
295 prove the level of contribution [40]. It is worth emphasizing that two hypotheses have been con-  
296 sidered to identify uncooperative “bad apples” in previous experiment: cooperation decisions are  
297 not anonymous; group members can be identified by name [42]. [Inspired by the above-mentioned](#)  
298 [behavioural experiments](#), our mathematical model considers that excluders need to pay a perma-  
299 nent cost of monitoring to identify other players’ [behaviours](#) during the repeated group interaction

300 process. This realistic set-up makes our model more reasonable and brings a step closer towards  
301 understanding how social exclusion strategy works in the realistic case. Accordingly, our work  
302 is essentially different from the previously reported incidental ostracism [50]. First, the ways of  
303 implementing social exclusion and incidental ostracism are different. Previous study assumes that  
304 individuals can select partners and those who are not selected are excluded from social interactions  
305 [50]. However, in our work we regard social exclusion as instrumental punishment of free-riders.  
306 In addition, participants play an iterated prisoners dilemma or coordination game with partner  
307 choice in the previous study [50]. While in our model, we explore the evolutionary dynamics of  
308 social exclusion strategy in repeated group interactions. Our results underline that cooperative  
309 behaviour cannot be promoted when implementing exclusion on free-riders is too late, which is  
310 consistent with the experimental results that the introduction of ostracism increases contribution  
311 levels except in the last period [51]. Our calculation also reveals that too early exclusion could be  
312 a source of modest cooperation.

313 The positive role of peer exclusion in the evolution of cooperation has already been report-  
314 ed in previous theoretical model developed in large well-mixed populations [36]. However, the  
315 mentioned work mainly examined the dynamics of the system when interaction among group  
316 members is one-shot and defectors can be detected in a cost-free manner. In our model, we re-  
317 lease these limitations. In addition, in our model we assume that social exclusion are implemented  
318 simultaneously and faultlessly in all groups and we do not consider the case in which defectors  
319 will be excluded from the group probabilistically. The main reason is that our model takes in-  
320 to account that the social excluders have paid the observation cost in advance to identify all the  
321 free-riders during the repeated group interaction process. Interestingly, we explore new dynamical  
322 behaviours, such as evolutionary oscillations, as summarized in Fig. 1. It is worth noting that evo-  
323 lutionary oscillations can be also observed from the interactions of *Pseudomonas aeruginosa* when  
324 *Pseudomonas aeruginosa* uses the strategy of social exclusion by using cooperator-released toxic  
325 cyanide to exclude lasR mutant defectors lacking the LasR regulon for detoxification [32, 33].  
326 Besides, we can reproduce previous findings that rare excluders can emerge in a sea of defectors  
327 and subvert them [36] (also see Fig. S1). Our work may unveil the effects of social exclusion on  
328 the evolution of cooperation in a realistic manner.

329 As we have found, the application of different exclusion rounds can induce rather distinct dy-  
330 namical behaviours. We stress that the actual number of exclusion rounds is not the only factor  
331 that affects evolutionary dynamics. As Fig. S2 in Electronic Supplementary Material demonstrates

332 clearly, the cost of monitoring can modify the amplitude of oscillation significantly, thus altering  
333 the evolutionary advantage of this incentive. Further important parameter is the discount factor of  
334 repeated games, which strongly influences the cooperation level. Our results highlight that cooper-  
335 ative behaviours cannot emerge when the mentioned discount factor is lower than an intermediate  
336 value (Fig. S4). Here, we show that it is beneficial for excluders to manage to expel free-riders  
337 from the beginning even in situations complicated with high monitoring cost or intermediate dis-  
338 count factor. Our model thus predicts that quickly excluding of free-riders can maintain a high  
339 level of cooperation over a wide range of parameters. Our observations may offer a potential way  
340 for how to use exclusion strategy efficiently in real and complicated situations.

341 In this work, we stress that once defectors are excluded from the interaction group, they can-  
342 not participate in the interactions any more in our model. In reality, it is a frequently applied  
343 practice that individuals who violate social rules can regain their qualification to participate in  
344 activities after a certain period of time. For example, a drunk driver whose driver's license has  
345 been revoked can take a driving license again after some time. Also, this scenario has been ap-  
346 plied to a behavioural experiment in which one player who is ostracized can re-enter the next  
347 round unless ostracized again [52]. For future study, thus a potential extension of our present work  
348 could be to explore the possible consequence in the realistic scenario where the finite-time effect  
349 is considered for exclusion of free-riders in repeated interactions. Furthermore, it is also worth  
350 exploring the effect of different social structures on the dynamics of social exclusion in repeat-  
351 ed group interactions. Previous experimental evidences underlined that free choice of interaction  
352 links drastically increases cooperation in social networks. In experimental treatments, cooperators  
353 who refuse interacting with defectors on the basis of past behaviour account for most of social ex-  
354 clusion [53]. Besides, it is worth noting that in structured populations excluders may be separated  
355 from simple cooperators, hence they can fight against defectors more efficiently than those who  
356 behave as second-order free-riders [54]. In this way, there is an indirect territorial competition be-  
357 tween excluders and pure cooperators, but the competitive outcome is intrinsically unpredictable,  
358 which is worth investigating for future study. Furthermore, we stress that in our present model the  
359 existence of a single excluder can purge any number of defectors in the group. Thus, the larger  
360 the interaction group is, the more effective the exclusion strategy is in keeping defectors at check.  
361 Such an advantage provided to large groups contrasts the usually found benefit from small groups,  
362 which may lead to a nonlinear effect of the group size on the evolutionary dynamics, as found in  
363 previous studies [67–69]. In particular, the group size for players undergoing group interactions

364 in structured populations could be variable and diverse [70–72] , which can lead to more compli-  
365 cated effects on the evolution of cooperation. Thus, the effects of group size would deserve to be  
366 considered in our model for future study.

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522

523 **Author contributions**

524 X.C., L.L., and A. S. designed the research, L.L. and Z.X. performed the research, X.C., L.L., and  
525 A. S. wrote the manuscript, and all authors discussed the results and commented on and improved  
526 the manuscript.

527

528 **Competing financial interests**

529 The authors declare no competing financial interests.

530

531 **Data availability**

532 The raw data generated with these computer calculations is available from the corresponding au-  
533 thor upon reasonable request.

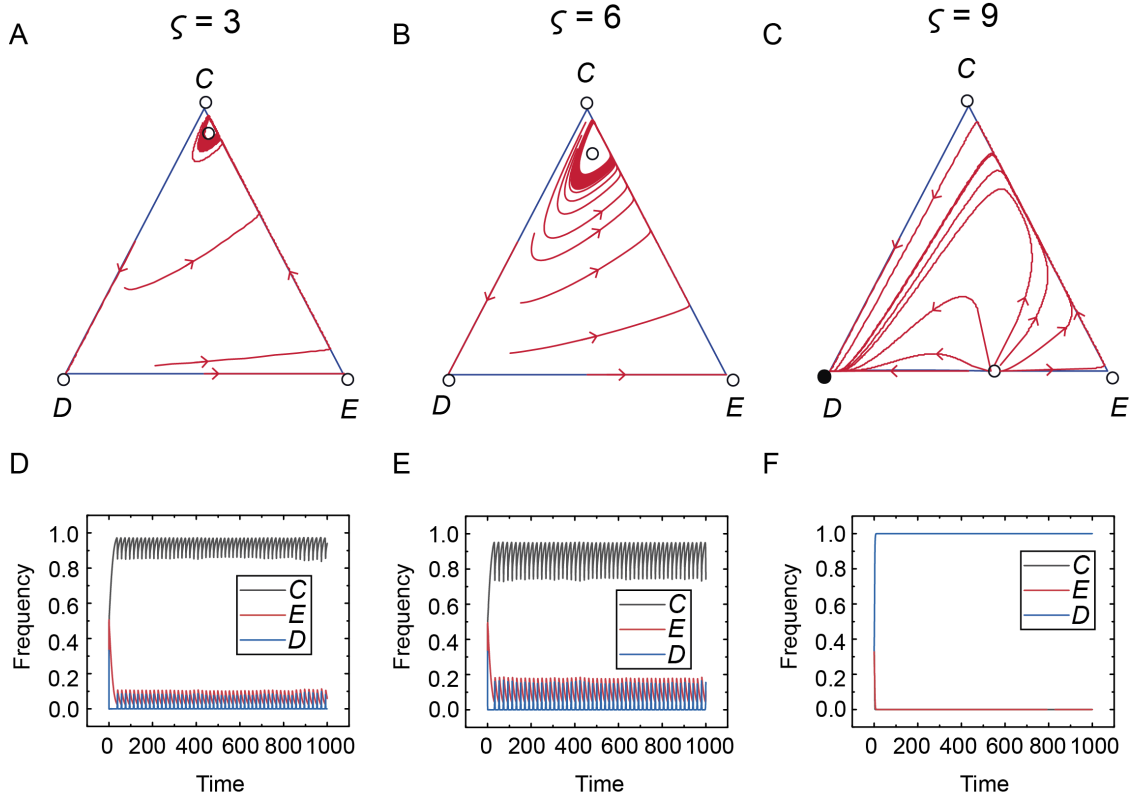


FIG. 1: Deterministic replicator dynamics among cooperators, defectors, and excluders in an infinite well-mixed population with different exclusion rounds  $\zeta$ . The triangle represents the state space  $S_3 = \{(x, y, z) : x, y, z \geq 0, x + y + z = 1\}$ , where  $x, y$ , and  $z$  denote the frequencies of cooperators, defectors, and excluders, respectively. The three vertices  $C, D$ , and  $E$  correspond to the three homogeneous states in which the population exclusively consists of cooperators ( $x = 1$ ), defectors ( $y = 1$ ), or excluders ( $z = 1$ ). The open circles represent unstable equilibria, and the filled circles denote stable equilibria. Panels D-F show the time evolution of fractions of three strategies  $C$  (black line),  $E$  (red line), and  $D$  (blue line) for three different  $\zeta$  values. Parameter values are:  $N = 5, F = 3, c = 1, c_E = 0.4, w = 0.9$ , and  $\sigma = 0.1$ .

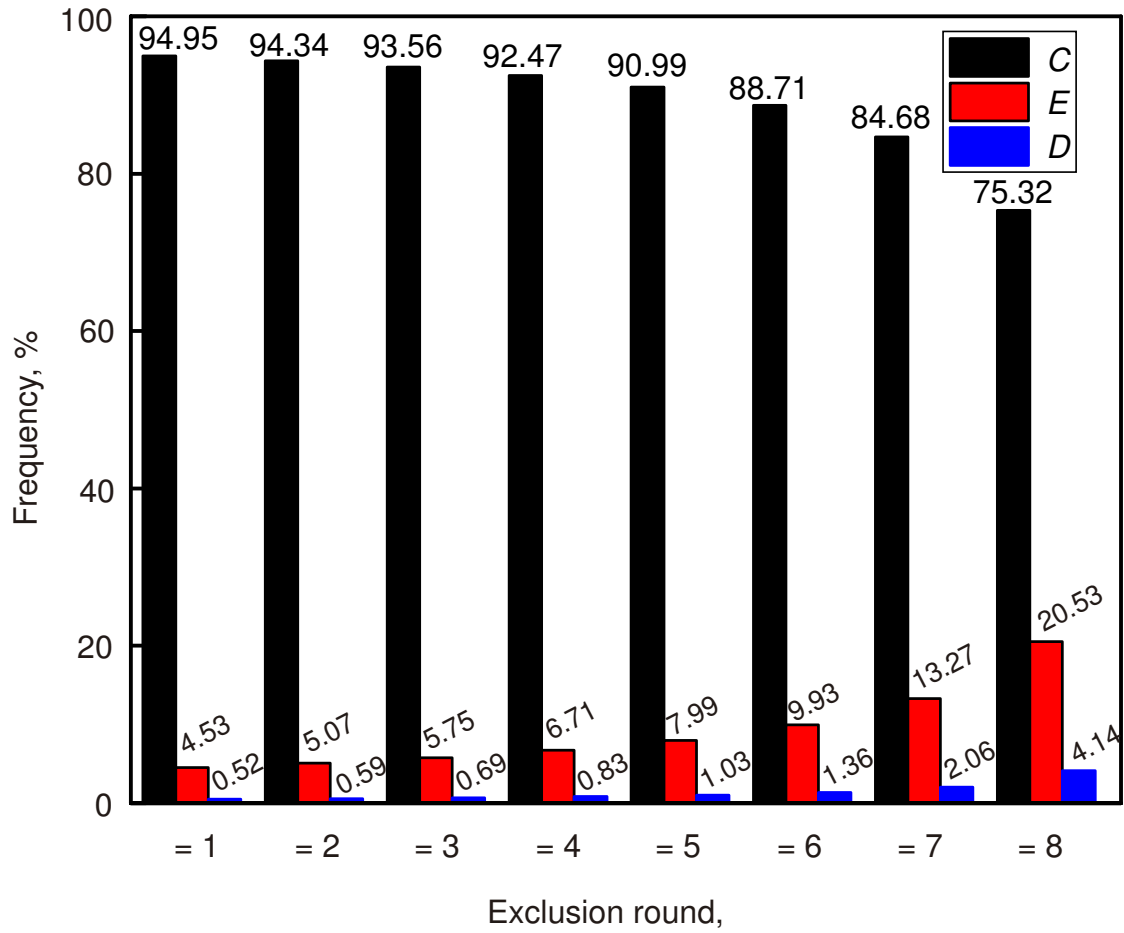


FIG. 2: Average levels of cooperation, defection, and exclusion for different  $\varsigma$  values. When  $C$ ,  $D$ , and  $E$  can form an evolutionary oscillation, the average level of each strategy can be calculated over an approximate periodic orbit. These frequencies of  $C$ ,  $D$ , and  $E$  strategies vary with different exclusion round values  $\varsigma$ , as indicated on the horizontal axis. Parameter values are:  $N = 5$ ,  $F = 3$ ,  $c = 1$ ,  $c_E = 0.4$ ,  $w = 0.9$ , and  $\sigma = 0.1$ .

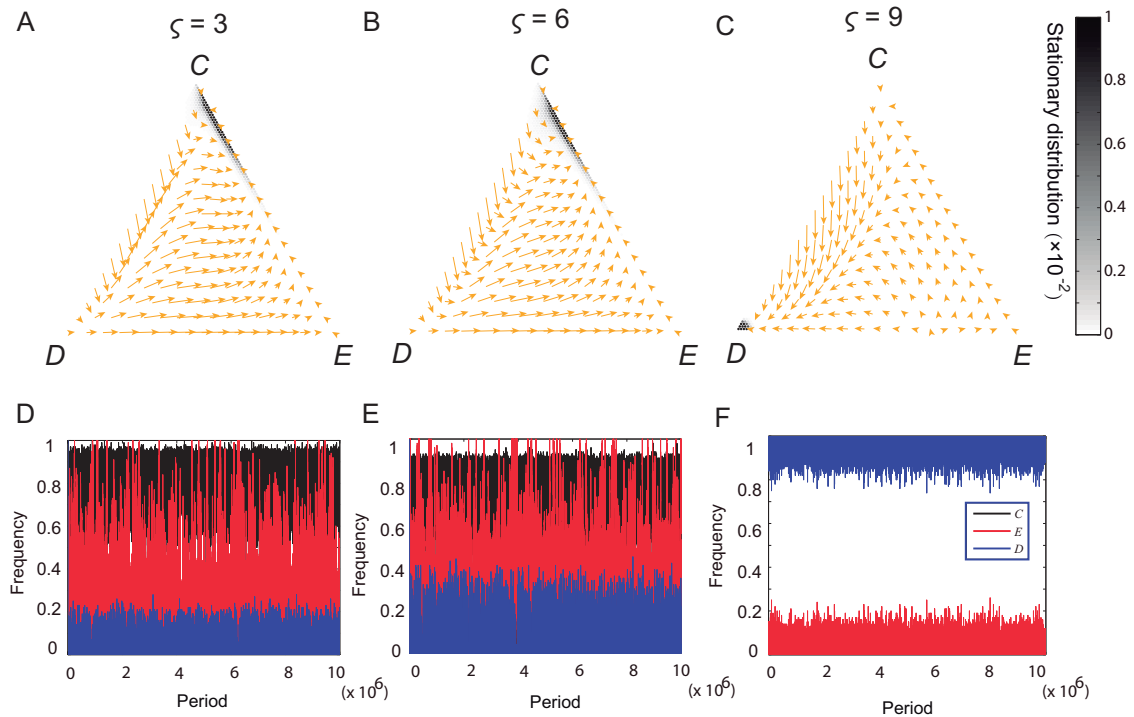


FIG. 3: Evolutionary dynamics of cooperation, defection, and exclusion in finite well-mixed populations for different exclusion round  $\zeta$  values. The dynamics are best characterized by the full stationary distribution, which is mapped onto the triangular simplex. Each simplex contains all possible configurations of the population, and each state is denoted by a small cycle. The value of the stationary distribution at each configuration is shown using a grey scale, where darker dots denote those configurations in which the population spends more time. The red arrows depict the so-called gradient of selection, which provide the most probable directions of evolution. Panels D-F show the time evolution of strategies by individual-based simulations for different  $\zeta$  values. Parameter values are:  $Z = 100$ ,  $N = 5$ ,  $F = 3$ ,  $c = 1$ ,  $c_E = 0.4$ ,  $\beta = 2$ ,  $\sigma = 0.1$ ,  $\mu = 0.01$ , and  $w = 0.9$ .

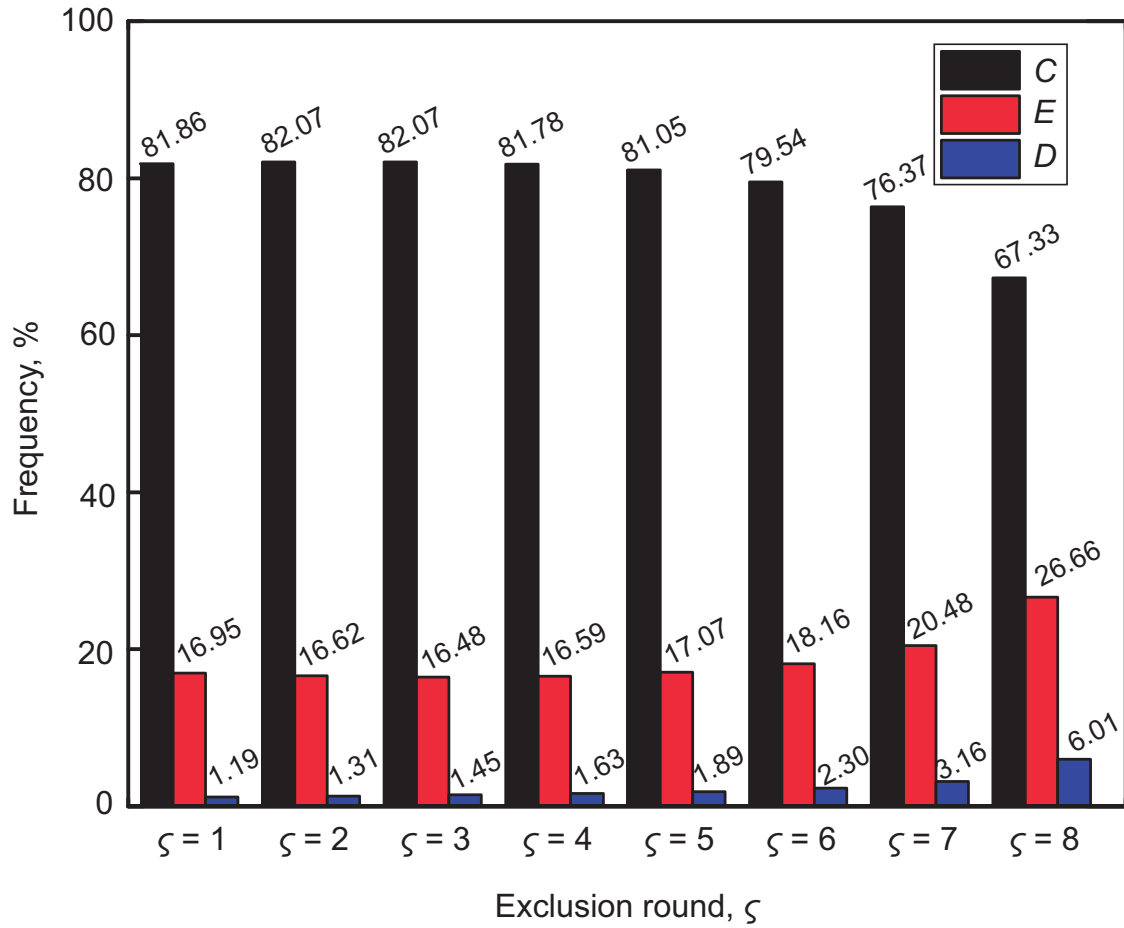


FIG. 4: The average levels of cooperation, defection, and exclusion in finite populations for different exclusion round values  $\zeta$ . These frequencies of  $C$ ,  $D$ , and  $E$  strategies vary with different exclusion round values  $\zeta$ , as indicated on the horizontal axis. Parameter values are:  $Z = 100$ ,  $N = 5$ ,  $F = 3$ ,  $c = 1$ ,  $c_E = 0.4$ ,  $\beta = 2$ ,  $\sigma = 0.1$ ,  $\mu = 0.01$ , and  $w = 0.9$ .