

A finite difference method for flows past mechanically vibrated circular cylinders

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ABSTRACT: A finite difference solution is presented in this paper for the 2-D, unsteady incompressible Navier-Stokes equations for laminar flow about fixed and oscillating cylinders. Boundary-fitted coordinates are used and the governing equations are transformed to a non-inertial system fixed to the accelerating cylinder. Convective terms are handled by a third order modified upwind difference, other space derivatives are by fourth order central differences and time derivatives by forward differences. The computed Strouhal numbers and time-mean drag coefficients for fixed cylinders compare well with experimental results. Amplitude bounds of locked-in vortex shedding due to forced transverse oscillation are determined for a fixed Re number. Computations were carried out for a cylinder in orbital motion placed in a uniform flow.

1. INTRODUCTION

The vibration of structures in a fluid flow has received much experimental and numerical study due to its practical importance. Flow-induced vibrations have often led to the damage of the structure. Examples of this are the collapse of the Tacoma-Narrows Bridge in the USA in 1940, which was triggered by periodic vortex shedding, and the damage to a thermometer case leading to the shut down of Monju fast breeder nuclear power plant in Japan in 1996. Numerical studies of vortex shedding have dealt with the flow of a uniform stream normal to a fixed cylinder (e.g., Karniadakis & Triantafyllou, 1989). If the cylinder is vibrating, either in forced or natural motion, a non-linear interaction occurs as the cylinder frequency approaches that of vortex shedding. In this case vortex shedding occurs at the cylinder vibration frequency over a range of flow velocities. This phenomenon is called lock-in. The numerical simulation of lock-in has been the subject of several papers (e.g. Hulbrut et al., 1982; Menighini & Bearman, 1995; Baranyi & Shirakashi, 1999).

The present study transforms the Navier-Stokes equations to a non-inertial reference frame fixed to the oscillating cylinder. The transformed equations are solved by the finite difference method. Computational results for flow about fixed cylinders are compared with those of experiments. Amplitude bounds of locked-in vortex shedding due to forced transverse oscillation of a circular

cylinder are determined for $Re=180$. Initial remarks on the flow around a circular cylinder in orbital motion are included.

2. PROBLEM FORMULATION

Incompressible laminar flow past a circular cylinder undergoing in-line and transverse harmonic oscillation is considered. Primitive variable formulation is used for the solution of the problem. The two components of the non-dimensional Navier-Stokes equations in a non-inertial system fixed to the cylinder can be written as follows:

$$\begin{aligned}\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{1}{Re} \nabla^2 u - a_{0x}; \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \frac{1}{Re} \nabla^2 v - a_{0y}\end{aligned}\quad (2.1)$$

where ∇^2 is the 2-D Laplacian operator. The equation of continuity has the form

$$\Theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (2.2)$$

The body force due to gravity is included in the pressure p . In these equations Re is Reynolds number based on cylinder diameter d ; x , y are Cartesian co-ordinates; u , v and a_{0x} , a_{0y} are the x , y components of velocity and cylinder acceleration

in the non-inertial or relative system, respectively; Θ is dilation; t is time.

It has been suggested (Roache, 1982) that instead of solving equations (2.1) and (2.2) for unknowns u , v and p , it is advisable to use a separate equation for pressure p , obtainable by taking the divergence of the Navier-Stokes equations and neglecting all but one term of dilation Θ , giving the Poisson equation

$$\nabla^2 p = 2 \left[\frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \right] - \frac{\partial \Theta}{\partial t}. \quad (2.3)$$

Strictly the dilation $\Theta=0$ by equation (2.2), but the finite difference scheme does not always conserve mass due to truncation errors. The reason that one term containing Θ is kept in equation (2.3) is to give dilation correction and to avoid instability.

Equations (2.1) - (2.3) naturally remain valid for flows when the cylinder is fixed ($a_{0x} = a_{0y} = 0$).

2.1. Boundary conditions and mapping

The physical domain is shown on the left-hand side of Figure 1.

Let us investigate the boundary conditions now.

On the surface of the cylinder (R_1) (see Figure 1):

Velocity: no-slip condition $u = v = 0$.

Pressure:

$$\frac{\partial p}{\partial n} = \frac{1}{Re} \nabla^2 v_n - a_{0n}$$

where n refers to components in the direction of the outer normal.

Far from the cylinder (R_2):

Velocity: uniform flow in the inertial system

$$u = u_{pot} - u_0; \quad v = v_{pot} - v_0$$

where u_0 , v_0 are the x , y components of cylinder velocity, and subscript 'pot' refers to potential flow.

Pressure:

$$\frac{\partial p}{\partial n} \cong \left(\frac{\partial p}{\partial n} \right)_{pot}$$

It should be noted that the assumption of uniform flow along R_2 is reasonable except for the narrow wake since the outer boundary of the physical domain is very far from the cylinder.

The physical domain and governing equations are transformed into a computational plane (see Figure 1). Since a boundary-fitted co-ordinate system is used, boundary conditions (BCs) can be

imposed accurately. In this way interpolation leading to inaccurate solutions can be avoided.

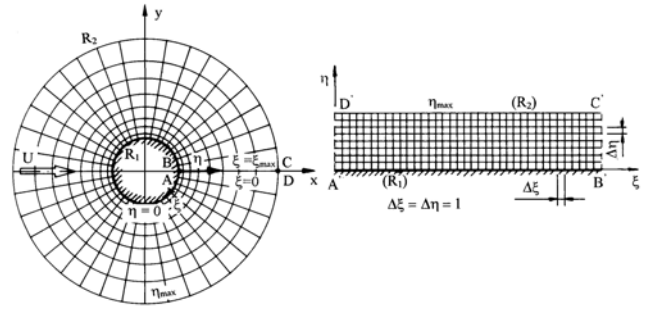


Figure 1: Physical and computational planes

A unique, single-valued relationship between the co-ordinates on the computational domain (ξ, η, τ) and the physical co-ordinates (x, y, t) can be written as

$$\begin{aligned} x(\xi, \eta) &= R(\eta) \cos [g(\xi)]; \\ y(\xi, \eta) &= -R(\eta) \sin [g(\xi)]; \quad (2.1.1) \\ t &= \tau \end{aligned}$$

where τ is time on the computational plane, and the dimensionless radius is

$$R(\eta) = R_1 \exp [f(\eta)]. \quad (2.1.2)$$

The structure of the mapping assures that the grid is orthogonal on the physical plane for the arbitrary functions $f(\eta)$ and $g(\xi)$. By choosing these functions properly a very fine grid can be obtained in the vicinity of the cylinder and a coarse grid far from the body. Transformations (2.1.1) and (2.1.2) are unique and single-valued only for a non-vanishing Jacobian.

Due to lack of space the transformed equations and BCs will not be included here. This non-inertial system formulation, in which both the co-ordinate system and the grid are fixed to the accelerating cylinder, has triple benefits:

- 1) the computational grid has to be generated only once;
- 2) there is no need for interpolation of the initial conditions at the beginning of each time step;
- 3) the transformed equations have simpler forms than those in the inertial system.

Since the mapping is given by elementary functions, the metric parameters and co-ordinate derivatives can be computed in closed forms. In this way the numerical error that arises from numerical differentiation of co-ordinates can be avoided.

3. RESULTS AND DISCUSSION

A computational code was developed for the solution of the problem. The transformed equations are solved by the finite difference method. The time derivatives are approximated by forward differences, fourth order central difference scheme is used for the diffusion terms and for pressure derivatives. The modified third order upwind scheme (Kawamura, 1984) proved to be successful in handling the convective terms in the Navier-Stokes equations. The equations of motion are integrated explicitly giving the velocity distribution at each time step. After determining the velocity distribution at an arbitrary time step, the pressure is calculated from the transformed Poisson equation by using the successive over-relaxation (SOR) method. At each time step, the dilation Θ is chosen to be zero, and the pressure on the cylinder surface is calculated by a third order formula derived from the Taylor series.

The computational grids used are 145x79 and 241x131 O-meshes. The number of grid points was chosen to assure the conformal property of the transformation. The diameter of the outer boundary of computation was $30d$. Dimensionless time steps used were 0.001 or 0.0005.

Computations were carried out for the flow around a fixed circular cylinder for different Reynolds numbers. The computed Strouhal numbers S are shown in Figure 2 compared with experimental results (Roshko, 1954). Measured and computed time-mean drag coefficients can be seen in Figure 3 (Schlichting, 1965). From a glance at Figures 2 and 3, we can see that the agreement between experimental and computational results is very good up to about $Re=200$. This is in accordance with previously published results (Henderson & Barkley, 1996), in which it was found that the flow becomes unstable, and 3-D effects start to appear at about $Re=190$.

Several other quantities were calculated, e.g., the instantaneous lift (C_L) and drag (C_D) coefficients; the distribution of velocity, vorticity, pressure and stream function; the location of front stagnation point, the lower and upper separation points changing with time. By applying the Fast Fourier Transform (FFT) to these oscillating signals their spectra can be obtained, and the frequency of vortex shedding can thus be determined.

Computations were carried out for flows past cylinders which are vibrated mechanically either in transverse or in-line directions. The amplitude

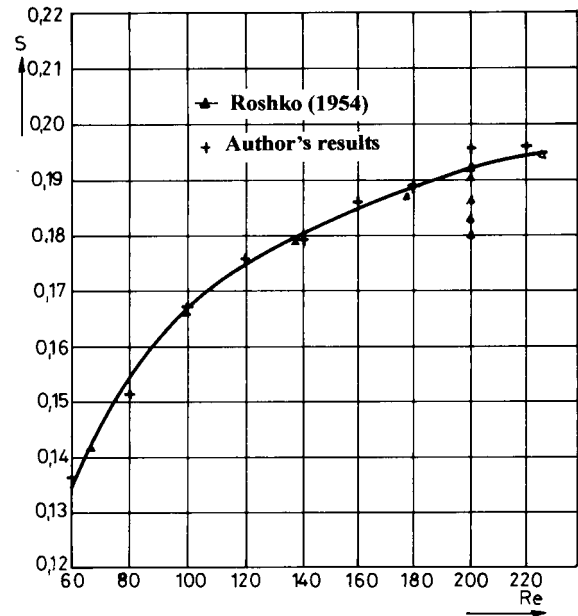


Figure 2: Strouhal number vs. Re number

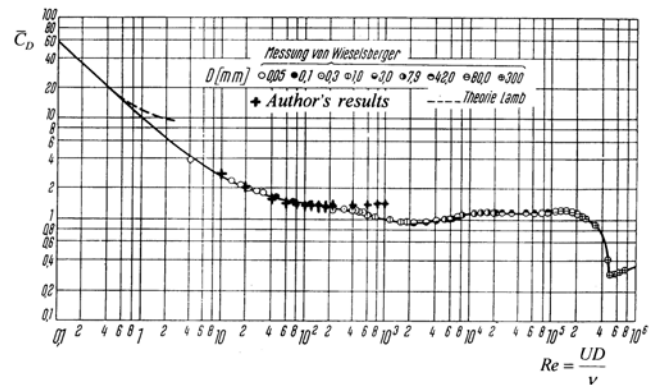


Figure 3:
Time-mean drag coefficient vs. Re number

threshold values A for locked-in vortex shedding due to forced transverse oscillation of a circular cylinder for $Re=180$ were also investigated, and are shown in Figure 4 as a function of the dimensionless frequency of cylinder oscillation S_c / S . Here S is the Strouhal number for fixed cylinder at $Re=180$, S_c is the Strouhal number based on the frequency of cylinder oscillation.

The author investigated the flow around a cylinder in orbital motion placed in an otherwise uniform viscous fluid flow. Identical frequencies were considered both in transverse and in-line directions. The amplitude of cylinder oscillation in x direction was kept constant while the amplitude was varied in y direction, making the path of the centre of the cylinder an ellipse. The variation of

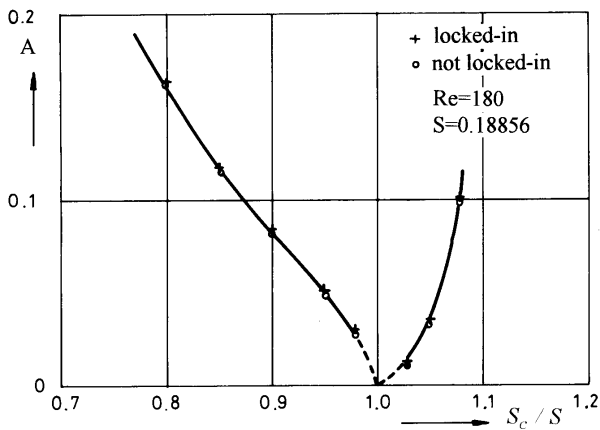


Figure 4: Amplitude threshold values for transverse cylinder oscillation

time-mean and root-mean-square (r.m.s.) values of lift and drag coefficients were investigated against the amplitude of vibration in y direction. The Reynolds number was kept constant at $Re=180$. It was found that, in the domain of $0.33 < A_y/A_x < 1$, while both the time-mean value of the drag coefficient and $C_{Lr.m.s.}$ increase with increasing oscillation amplitude in y direction, $C_{Dr.m.s.}$ decreases. In addition, an abrupt change can be seen in all of the three curves in the vicinity of $A_y/A_x=0.33$. A more careful investigation of these phenomena is planned in the future.

4. CONCLUSIONS

The finite difference method has been applied for the numerical simulation of unsteady, laminar incompressible fluid flow about fixed and oscillating circular cylinders placed in otherwise uniform flows.

By writing the governing equations in a non-inertial system fixed to the accelerating cylinder and using boundary fitted co-ordinates, more accurate computational results can be obtained. The choice of a grid fixed to the moving cylinder eliminates the need for interpolation of the initial values at each time step.

Good agreement was found between experimental and computational results for fixed cylinders in terms of Strouhal number and the time-mean drag coefficient up to $Re=200$. This indicates that extension of the method to oscillating cylinders is promising.

The amplitude bounds of locked-in vortex shedding due to forced transverse oscillation of a circular cylinder for $Re=180$ were determined.

Preliminary computational results for flow around a cylinder in orbital motion revealed an unexpectedly abrupt change in the root-mean

square values of lift and drag coefficients. Further investigation is needed in this field.

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