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Relation of rock mass characterization and damage

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ABSTRACT: There are several empirical relations between the rock mass mechanical parameters (unconfined compressive strength, deformation modulus) and of the rock mass classification systems. These uniformly show increasing deformation modulus and compressive strength with the increasing quality of the rock mass. The goal of this paper is to put the deformation modulus—rock mass quality dependence into a framework of damage mechanics and to find a connection with the unconfined rock mass strength—quality relation. The rock mass characterizations are interpreted as damage classifications. Then thermodynamics reveals an interrelation between the deformation modulus and rock mass strength. This relationship is discussed from some points of view.

1 INTRODUCTION

Large scale rock mass characterization introduces several material parameters in relation to mechanical properties. Two of the most important ones are the deformation modulus and the unconfined rock mass strength. These material parameters are frequently related to laboratory data characteristics of intact rock samples and to the classical rock mass classification systems (e.g. RQD, RMR, Q, GSI or RMI methods). These rock mass quality measures quantify the relation of the rock mass to the intact rock. In a sense, they are structure parameters that measure the damage in the large scale rock samples for practical purposes.

In this paper we investigate the dependence of deformation modulus and unconfined strength of rock masses on the rock mass quality in a damage mechanics framework. Damage mechanics (more properly, non-equilibrium thermodynamics with internal variables) is a convenient and simple theoretical frame to understand this correspondence that can help to reveal some deeper relations of the material parameters.

1.1 *Deformation modulus of the rock mass*

In the literature several relationships have been suggested between the deformation modulus and the rock mass quality measures. We have collected some equations that are using the elastic modulus of the intact rock for calculating the deformation modulus of the rock mass (Table 1) using the RMR (or GSI) values.

Zhang & Einstein (2004) recommended a relationship between RQD and E_{rm}/E_i (i.e. the ratio of the deformation modulus of the rock mass and the intact rock):

$$E_{rm}/E_i = 10^{0.0186\,\mathrm{RQD}-1.91} \tag{1}$$

There is no mechanical (physical) interpretation of the above empirical equations. They were analyzed by Palmström & Singh (2001), Kayabasi et al. (2003) and Gokceoglu et al. (2003) showing their advantages and disadvantages, and suggested some corrections (e.g. influence of the weathering), too.

Table 1. Calculation the deformation modulus of the rock mass (E_{rm}) using the RMR value, with the elastic modulus of the intact rock (E_i).

Equation	Ref.
$E_{rm}/E_i = 1/100(0.0028\mathrm{RMR}^2 + 0.9\exp(\mathrm{RMR}/22.82))$	Nicholson & Bieniawski (1990)
$E_{rm}/E_i = (s^a)^{0.4}$, $s = \exp((\mathrm{RMR}-100)/9)$, $a = 0.5 + 1/6((\exp(-\mathrm{RMR}/15) - \exp(-20/3))$	Sonmez et al. (2004)
$E_{rm}/E_i = s^{0.25}$, $s = \exp((\mathrm{RMR}-100)/9)$,	Carvalcho (2004)

1.2 *Strength of the rock mass*

To estimate the unconfined compressive strength of rock masses (σ_{cm}), there are various suggested empirical correlations considering the discontinuity characteristics. Table 2 lists some of the widely used ones where the UCS of the intact rock (σ_c) appears. Their functional form is exponential but with different parameters.

Note, the equation of Hoek et al. (1995) was calculated from the Hoek-Brown strength criterion for rock masses, i.e. the unconfined compressive strength can be expressed as

$$\sigma_{cm} / \sigma_c = \sqrt{s} \tag{2}$$

2 USING THE DAMAGE VARIABLE

In what follows, we relate the empirical fracture and disturbance measures of rock mechanics (RQD, Q, RMR, GSI, etc.) to a damage measure D. According to the physical interpretation of damage, the value $D = 0$ characterizes the intact rock and $D = D_{cr}$ stands for the fractured rock mass at the edge of failure. As the rock mass quality measures are zero at maximal possible damage and are one hundred at the undamaged state, we suggest the simplest linear relationship interpreting them as damage measures

$$D_{RM} = 100\left(1 - \frac{D}{D_{cr}}\right) \quad \text{therefore}$$
$$D = D_{cr}\left(1 - \frac{D_{RM}}{100}\right). \tag{3}$$

Here D_{RM} is the damage of the rock mass. RM would be one of the rock mass classification systems (RQD, RMR or GSI value, i.e. between 0 and 100). As a simplification, hereafter we assume that $D_{cr} = 1$. In our case this is not a restriction because we do not associate a direct physical meaning (e.g. crack density, fractal dimension of the crack system) to damage and we accept the normalization and measurement methods of the rock mass quality measures as a proper characterization. We know that the dependence of both Q and RMi methods on the quality of the rock mass is nonlinear, therefore the empirical relationships applying these characterizations are not considered here.

It has been assumed that:

- The mechanical parameters of the rock mass in case of $D = 0$ are equal to the intact rock parameters
- The $D \neq 0$ relationships can be modeled by an empirical function.

Table 2. Empirical correlations between the unconfined compressive strength of the rock mass (σ_{cm}) as a function of the rock mass quality (RMR), knowing the UCS of the intact rock (σ_c).

Equation	Ref.
$\sigma_{cm}/\sigma_c = \exp(7.65((\mathrm{RMR}-100)/100)$	Yudhbir et al. (1983)
$\sigma_{cm}/\sigma_c = \exp((\mathrm{RMR}-100)/18.5)$	Ramamurthy et al. (1985)
$\sigma_{cm}/\sigma_c = \exp((\mathrm{RMR}-100)/25)$	Kalamaras & Bieniawski (1993)
$\sigma_{cm}/\sigma_c = \exp((\mathrm{RMR}-100)/18)$	Hoek et al. (1995)
$\sigma_{cm}/\sigma_c = \exp((\mathrm{RMR}-100)/20)$	Sheorey (1997)

2.1 *Damage model for deformation of rock mass*

We can transform the functions of Table 1, introducing the damage as an independent variable, into the form

$$\frac{E_{rm}}{E_i} = \exp(-AD). \tag{4}$$

The values of the material parameter A corresponding to the published equations are summarized in Table 3.

We recognize that the parameter A of Eq. (4) is different in the different functions: in case of

Table 3. The constant A of Eq. (4) for the various published relationships, collected in Table 2.

Equation	A
Nicholson & Bieniawski (1990)	4.358
Zhang & Einstein (2004)	4.440
Sonmez et al. (2004)	2.624
Carvalho (2004)	2.778

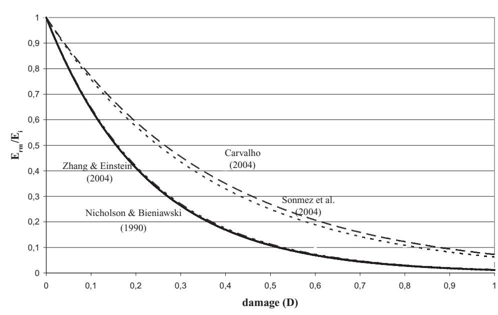

Figure 1. The empirical relationships between the ratio of the deformation modulus of rock mass and the intact rock and rock mass damage factor (D). (See Eq.(3) and Table 3.)

Nicholson & Bieniawski (1990) and the Zhang & Einstein (2004) it is around 4.4, while using the empirical relationships of Sonmez et al. (2004) and Carvalho (2004) it is approximately 2.7.

However, that both Sonmez et al. (2004) and Carvalho (2004) suggested the usage of the Hoek-Brown constant s for undisturbed or interlocking rock masses. If the constants A are calculated as disturbed rock masses (i.e. $s = \exp((\mathrm{RMR}-100)/6)$, see Hoek & Brown, 1988) then they are $A = 3.936$ and $A = 4.167$, respectively.

2.2 *Damage model for unconfined compressive strength of the rock mass*

Similarly to the deformation moduli of the rock mass, we recalculate the empirical equations of the unconfirmed compressive strength (σ_{cm}) for the following form, using the compressive strength of the intact rock (σ_c):

$$\frac{\sigma_{cm}}{\sigma_c} = \exp(-BD) . \tag{5}$$

The calculated parameters B are summarized in Table 4.

The average value of B is 5.542 (between 4.167 and 7.650; variance is 1.291). In Fig. 2. these equations are plotted.

Table 4. The constant B appearing in Eq. (5) when using the damage factor D for calculating the strength of the rock mass.

Ref.	B
Yudhbi et al. (1983)	7.650
Ramamurthy et al. (1985)	5.333
Kalamaras & Bieniawski (1993)	4.167
Hoek et al. (1995)	5.556
Sheorey (1997)	5.000

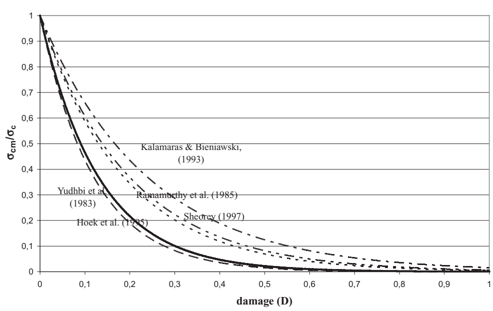

Figure 2. The empirical relationships between the ratio of unconfined strength of rock mass and the intact rock and the damage factor (D) (see Eq. (5) and Table 4).

3 THERMO-DAMAGE MECHANICS

Damage mechanics introduces additional physical quantities, namely, damage parameters, to characterize the failure (Krajcinovic, 1996). Thermodynamic theories of damage interpret damage parameters as internal thermodynamic variables and place damage mechanics into a thermodynamic framework. One of the advantages of this theoretical background is a connection between deformation/strain and failure by interpreting failure as a loss of thermodynamic stability (Ván, 2001, Ván & Vásárhelyi, 2001).

Several theories of damage mechanics introduce a polynomial dependence of the thermodynamic potentials (e.g. entropy or Helmoltz free energy) on deformation and damage. This dependence is supported by mesosocopic calculations (Papenfuss et al. 2007), as well as by an analogy with the theory of elasticity. Assuming a tensorial damage parameter (fabric tensor) one may apply similar principles determining the damage and deformation dependence of the free energy. For isotropic materials the most general second order polynomial expression of free energy characterizing the material was given by Papenfuss & Ván (2008). That model contains 11 parameters in addition to the classical elastic moduli. However, considering a tensorial damage measure is only one of the possibilities. Crack orientation and length averaging can lead to scalar and vectorial damage measures depending on the symmetries of the crack system. Rock mass characterization uses special damage measures that do not consider the cracking and fracture system in detail. Therefore here we construct the simplest possible model with a scalar damage measure and a minimal number of parameters. The robustness of our model is ensured by thermodynamic principles.

Our first observation is that damage quantity differs from the classical internal variables of non-equilibrium thermodynamics by its so-called unilateral property: as a function of time damage typically increases, and decreases only in rare, exceptional cases. Therefore one can investigate functional dependencies of the free energy that are not symmetric to zero in the sense that they are not interpreted in case of negative damage values. This observation leads to our primary assumption: damaging uniformly weakens the elastic bonds of the rock mass and the change in the energy content by damaging is proportional to the actual energy content of the body. That is we suppose that the effect of damage is *energetic*, the same damage degreases more the energy content of a more deformed rock mass. Therefore the damage dependence of the free energy

$F = F(\varepsilon, D)$ is related to the following differential equation:

$$\left.\frac{\partial F}{\partial D}\right|_{\varepsilon} = -\alpha F(\varepsilon, D). \tag{6}$$

Here ε is the strain of the damaged rock mass and α is a dimensionless material constant.

Our second theoretical assumption conventionally introduces a pressure which is proportional to the strain

$$\left.\frac{\partial F}{\partial \varepsilon}\right|_{D} = \sigma = E(D)\varepsilon, \tag{7}$$

where $E(D)$ is the damage dependent deformation modulus. Here the partial derivative of the free energy by the strain is the pressure according to the thermodynamic framework. Conditions Eqs. (6) and (7) determine the free energy in the following form:

$$F(\varepsilon, D) = e^{-\alpha D}\left(E_i \frac{\varepsilon^2}{2} + F_0\right). \tag{8}$$

Here the constant parameters E_i and F_0 can be interpreted as the elastic modulus of the intact rock ($D = 0$) and the free energy of the intact and undeformed rock $F_0 = F(\varepsilon = 0, D = 0)$.

Therefore the deformation modulus is calculated according to Eq. (7) as

$$\left.\frac{1}{\varepsilon}\frac{\partial F}{\partial \varepsilon}\right|_{D} = \frac{\sigma}{\varepsilon} = E_i e^{-\alpha D}. \tag{9}$$

This exponential dependence corresponds to the empirical data.

On the other hand we may also investigate the conditions of thermodynamic stability, that is the convexity of the free energy. If the convexity of the free energy is violated then the corresponding thermodynamic state is unstable. This condition is best investigated by the second derivative of the free energy

$$\partial^2 F = \begin{pmatrix} E_i e^{-\alpha D} & -\alpha E_i e^{-\alpha D}\varepsilon \\ -\alpha E_i e^{-\alpha D}\varepsilon & \alpha^2 e^{-\alpha D}\left(E_i \frac{\varepsilon^2}{2} + F_0\right) \end{pmatrix}. \tag{10}$$

Using Sylvester's criterion, this matrix is positive definite as long as its determinant is positive:

$$\det(\partial^2 F) = \alpha^2 E_i e^{-2\alpha D}\left(F_0 - E_i \frac{\varepsilon^2}{2}\right) \geq 0. \tag{11}$$

The vanishing of the determinant can be interpreted as a condition of failure. We can recognize that it is a critical mechanical energy condition where the critical deformation ε_{cm} is obtained as

$$\varepsilon_{cm} = \sqrt{2F_0}. \tag{12}$$

This condition can give the critical rock mass strength σ_{cm} according to Eq. (9) as

$$\varepsilon_{cm} = \frac{\sigma_{cm}}{E_i} e^{\alpha D} = \sqrt{2F_0}. \tag{13}$$

Recognizing that $\sigma_c = E_i \varepsilon_c$, Eq. (13) can be related to the unconfined compressive strength of the intact rock, σ_c, as

$$\frac{\sigma_{cm}}{\sigma_c} = e^{-\alpha D}. \tag{14}$$

This form corresponds to the known empirical relations (Table 2) and to Eq. (9) with the deformation modulus. Moreover, according to Eq. (13) the value of the material parameter α related to the deformation modulus is equal to the material parameter B of Eq. (5) related to the unconfined compressive strength.

4 CONCLUSIONS

The empirical relations for the deformation modulus of rock mass can be summarized as

$$\frac{E_{RM}}{E_i} = e^{\frac{RMR-100}{a}}, \tag{15}$$

where $22.5 < a < 38.1$ (see Table 5) and the average value is around 30 (variance: 8.3!), while using the disturbed s Hoek-Brown parameter, it is 23.7 (the variance is in this case only 1.3).

For unconfined compressive strength of the rock mass, the similar equation can be used

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{RMR-100}{b}}, \tag{16}$$

Table 5. The constant a in Eq. (15), with the data of Table 3, using the equations of Table 1. The value in the bracket is the calculated number using the disturbed rock mass Hoek-Brown parameter s.

Equation	a
Nicholson & Bieniawski (1990)	22.95
Zhang & Einstein (2004)	22.52
Sonmez et al. (2004)	38.11 (25.41)
Calvalho (2004)	36.00 (24.00)

where b is between 13.75 and 24.00 according to the published equations. The average value is 18.76 (variance: 3.94). Except using the equation of Yudhbi et al. (1983), it is 20.19 with a variance 2.67.

However, the simple damage model requires that $a = b$ in case of the RMR characterization of rock mass, not only in damage variables.

With $a = b$ the relation between deformation modulus and rock mass strength is

$$\frac{\sigma_{cm}}{\sigma_c} = \frac{E_{rm}}{E_i} = \mathrm{e}^{\frac{\mathrm{RMR}-100}{22}}. \tag{17}$$

The above analyzed empirical relations give different values for the parameters a and b (see Table 5 and Table 6). However, the difference is small considering the uncertainties in the definition, in the measurement and in the measurement methods of the deformation modulus and the damage measures. Accepting the theoretical result $a = b$ we obtain from Eq. (17)

$$\frac{E_{rm}}{\sigma_{cm}} = \frac{E_i}{\sigma_c}, \tag{18}$$

where E_i/σ_c is known as the modification ratio MR (see Palmström & Singh, 2001). It means, the ratio of the deformation modulus and the unconfined rock mass strength is independent the rock mass quality and it is equal to the ratio of the Young's modulus and compressive strength of the intact rock.

Note, according to this result, the Hoek-Brown strength criteria can be used for calculating the ratio of the deformation modulus of the rock mass and the intact rock. According to Eq. (2), this relation is:

$$\frac{\sigma_{cm}}{\sigma_c} = \frac{E_{rm}}{E_i} = \sqrt{s}\,. \tag{19}$$

where s is the Hoek-Brown constant, and according to the definition: $s = \exp((\mathrm{RMR}-100)/9$. Therefore, we obtain similarly to Eq. (17):

Table 6. The constant of b of Eq. (16), using the constants from Table 4.

Ref.	b
Yudhbi et al. (1983)	13.07
Ramamurthy et al. (1985)	18.75
Kalamaras & Bieniawski (1993)	24.00
Hoek et al. (1995)	18.00
Sheorey (1997)	20.00

$$\frac{\sigma_{cm}}{\sigma_c} = \frac{E_{rm}}{E_i} = \mathrm{e}^{\frac{\mathrm{RMR}-100}{18}}. \tag{20}$$

This result is near to the experimental data (basically within the experimental error).

On the other hand the exponential forms Eqs. (4) and (5) of the empirical relations ensure a correspondence of the deformation modulus E_{rm} and of the unconfined rock mass strength (σ_{cm}), without considering the presented damage model:

$$\frac{E_{rm}}{\sigma_{cm}} = \frac{E_i}{\sigma_c} e^{\frac{(B-A)(\mathrm{RMR}-100)}{100}}, \tag{21}$$

where A and B are the constants according to Eqs. (4) and (5).

Considering Tables 3 and 4 the differences $B - A$ are between 0 and 5, the average being around 2. Thus, applying this average value:

$$\frac{E_{rm}}{\sigma_{cm}} = \mathrm{MR} \times e^{\frac{2(\mathrm{RMR}-100)}{100}}, \tag{22}$$

where MR is the modification ratio, according to Palmström & Singh (2001).

Finally, we remark that thermodynamic stability of damage models may result in a three dimensional rock mass failure criteria, too. For intact rock and polynomial free energy that was shown by Ván & Vásárhelyi (2001) and it can also be used for modeling the influence of the water content for the strength of the rocks, as well (see e.g. Romana & Vásárhelyi, 2007).

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