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Some Remarks on the Velocity of Flow with Friction

Faragó T.

Néhány megjegyzés a súrlódásos áramlás sebességéről. A súrlódásos horizontális áramlás sebességére vonatkozó, részben általánosabb levezetés lehetőségét *Rákóczi* (1976) mutatta ki. Jelen tanulmányban a szerző tovább általánosítja e mozgásforma leírását és egy kvázigeosztrófikus közelítéssel kellően reális becslést ad a sebesség időbeli változására.

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What are the influences that maintain the atmospheric phenomena of different form for a longer period? If we take into consideration the vector equation of the horizontal motion

$$\frac{d\vec{V}}{dt} + f\vec{k} \times \vec{V} = -\frac{1}{\varrho} \nabla p + \vec{S} \qquad (1)$$

and following *Defant-Defant* (1958), as it was also done by *Rákóczi* (1976), the frictional force is put proportional with the velocity

$$\vec{S} = -k\,\vec{V} \qquad (2)$$

then for the total temporal variation of the kinetic energy we shall have

$$\frac{dV^2}{dt} = -\frac{1}{\varrho} \vec{V} \cdot \nabla p - kV^2. \qquad (3)$$

In these equations V, V, ρ, p, k are the horizontal velocity, its magnitude, the density, the pressure and the frictional coefficient.

Let us omit the advection of the velocity, then only the local time derivative of the kinetic energy remains on the left-hand side of (3)

$$\frac{\partial V^2}{\partial t} = -\frac{1}{\varrho} \vec{V} \cdot \nabla p - kV^2. \qquad (4)$$

It is well-known that the pressure gradient perpendicular to the velocity vector in the case of the geostrophic approximation, so this first estimation provides the following solution

$$V = V_0 e^{-kt}. \qquad (5)$$

This formula results in the unnaturally fast dissipation of the kinetic energy. In the special case of $k = 1{,}2\text{x}10^{-4}$ sec^{-1} being valid at middle latitudes and over the continents, the velocity will have the 10 p. c. of its initial value at 5,3 hr.

Rákóczi (1976) partially extended the above deduction. Using the gas equation the substituted the advection of the pressure force with terms related to the advection of the temperature and that of the density, then he assumed the stationarity of these later advection terms and at last he fixed he angle e between wind velocity vector and the isobars. These suppositions give

$$V = \left(V_0 + \frac{A}{k}\right) e^{-kt} - \frac{A}{k}, \qquad (6)$$

where *A* is a function, among others, of the temperature and density gradients.

Now one must remark that (6), i. e. the integration of (4) in such a way can easily be generated.

Denoting the angle between the velocity vector and the pressure force by ε_1 (thus $\varepsilon_1 = 90° - \varepsilon$) the scalar product in (4) may be written in the form

$$\bar{V} \cdot \nabla p = -V \cdot |\nabla p| \cos \varepsilon_1. \qquad (7)$$

Fig. 1 shows a possible arrangement of the above-mentioned vectors. For the motion with friction the velocity vector V deviates from the geostrophic one V_g to the lower pressure area (Dési-Rákóczi, 1970). Eliminating the singular solution $V = 0$ (4) with (7) gives

$$\dot{V} = 1/\varrho \, |\nabla p| \cos \varepsilon_1 - kV. \qquad (8)$$

This is a linear inhomogen differential equation of first order. For the initial values $t = 0$, $V = V_0$ we shall have

$$V = (V_0 + \int_0^t B(s) \, e^{ks} \, ds) e^{-kt}, \qquad (9)$$

where

$$B = \frac{1}{\varrho} |\nabla p| \cos \varepsilon_1$$

Therefore the temporal variation of the wind velocity is a function of the frictional coefficient, the density and the advectional

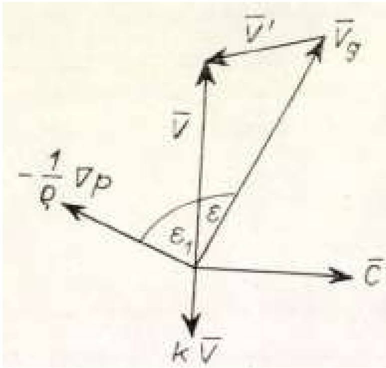

Fig. 1: Arrangement of acting forces and velocities.
C denotes the Coriolis force, V' is the izallobaric wind velocity,
other denotations are defined in text

conditions of the pressure (namely, the magnitude of the gradient and its position with respeect to the velocity vector). If particularly V independent of time

$$V = (V_0 - \frac{B}{k}) e^{-kt} + \frac{B}{k}, \qquad (10)$$

which is just equivalent to (6), because using the gas equation it can be proved that $B = -A$.

A futher simplification gives an approximation of the time-independence of the wind velocity, which is rather real as to its order of magnitude.

For this purpose we express the pressure gradient as follows

$$\nabla p(t) = \nabla p(t \pm \tau) \mp \nabla \dot{p}(t) \tau, \qquad (11)$$

where Vp is the izallobaric gradient. With reference to Dési-Rákóczi (1970) we may follow Ertel's (1938) principle, which states that the right-hand and left-hand sides of the geostrophic (horizontal) equations of motion (i. e. the expressions of the geostrophic wind) should not be given for the same time instant but as the variation of the velocity may answer the variation of the pressure only after a time, we have

$$f(\varrho V)_t = -k \times \nabla p(t \pm \tau). \qquad (12)$$

Moreover the ageostrophic wind determines the izallobaric gradient in (11)

$$f(\varrho V')_t = \mp \tau k \times \nabla \dot{p}(t). \qquad (13)$$

So (11)-(13) give

$$|\nabla p| = f\varrho V_g, \qquad (14)$$

which exactly corresponds to the geostrophic approximation. The behaviour described by (11)-(14) just expresses the tendency of the atmosphere to fall in a ballanced motion. Taking into account that the izallobaric wind is only 10-20 p. c. of the actual wind in average (Götz, 1967) now we shall use the estimation

$$|\nabla p| = f\varrho V. \qquad (15)$$

In addition we remark that e. g. for the gradient wind $V<V_g$ in general (Wiin-Nielsen, 1973). In this case (8) alters into

$$\dot{V} = -(k - f\cos \varepsilon_1)\ V \qquad (16)$$

whose solution is

$$V = V_0\ exp\ [(f\cos\ \varepsilon_1 - k)\ t]. \qquad (17)$$

The declination of V from V_g is about $\varepsilon = 30° - 50°$ at the middle latitudes, over the continents (Dési-Rákóczi, 1970) so $\cos \varepsilon_1 > -0$. This means that

$$-k + f\ \cos\ \varepsilon_1 > -k,$$

consequently in this case the decrease of the velocity is slower in comparison with the simplest estimation (5). Moreover $f \cos \varepsilon_l - k > 0$ involves its (temporal) increase. (However the elementary approximation of the frictional force (2) becomes less appropriate the larger V.) Particularly, at the middle latitudes over continents we may derive

$$-2\,.\,10^{-5} \geq -k + f\cos e_1 \geq -6\,.\,10^{-5},$$

which means in other words that the wind velocity takes the 10 p. c. of its initial value at about 10,6 - 31,8 hr.

More exact estimation of the temporal variation of the velocity may be computed by the direct use of (9) or by generalization of the above given solution with retaining the advection of the velocity in (3).

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