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EVOLUTION METHODS FOR DISCRETE MINIMAL WEIGHT DESIGN OF SPACE TRUSSES WITH STABILITY CONSTRAINTS

Anikó Csébfalvi
Department of Structural Engineering
University of Pécs
Boszorkány u. 2., 7624 Pécs, Hungary
csebfalv@witch.pmmf.hu

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Dedicated to István Páczelt on the occasion of his 65th birthday

Abstract. This paper provides a comparative study of evolution methods for minimal weight design of space trusses. Recently used genetic algorithms (GA), simulated annealing (SA) and tabu search (TS) methods are observed for metal structures where the truss member profiles are selected from available catalogue values. In this paper, global and local stability problems are considered using a path-following method for non-linear stability investigation. The results of the comparative study are presented for the commonly known numerical test problems. A twenty-four-member shallow dome structure was presented where structural instability constraints and member buckling are considered as well as using linear elastic material property. The effect of the nonlinear material law is compared in optimal design of the ten-bar truss structure and the twenty-five-bar transmission tower using an inverse Ramberg-Osgood material law.

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1. **Introduction**

One of the most important practical considerations in the optimal design of steel structures is the best selection of design variables from available catalogue values. Therefore, the design is formulated as a discrete optimization problem, searching for global or local optimal solution. However, most optimization methods are suited and developed for continuous design variables. A few procedures [1, 3, 4, 5, 9, 13] have been considered for discrete optimization including e.g. enumeration techniques, integer programming, branch and bound algorithms.

This paper provides a comparative study where simulated annealing (SA), genetic algorithms (GA), and tabu search methods (TS) are considered for discrete minimal weight design problems of shallow space trusses with stability constraints.

Simulated annealing as a heuristic algorithm is associated with its original use for solving metal models as they heat and cool. Kirkpatrick et al. [10] introduced it first for discrete optimization problems. In this paper a new SA algorithm is presented for shallow space trusses with stability constraints. The SA algorithm has proven to be a good technique [6, 11] for solving combinatorial optimization problems in particular for large flexible space structures. However, it seems sometimes less useful than some conventional algorithms. Consequently, simulated annealing has not been widely accepted in engineering optimization. In order to accelerate the overall convergence, it is proposed to use the best solution for a starting point every time that the temperature is reduced. The results show that simulated annealing algorithm provides a computationally efficient tool to find near optimal solutions to otherwise computationally intractable problems.

GA methods are search algorithms that are based on the concepts of natural selection and natural genetics. Recently GA methods are very popular and have been used for sizing, shape, and topology optimization of structures, e.g. [8, 12]. The core characteristics of GAs are based on the principles of survival of the fittest and adaptation. GA methods operate on population of design variable sets, with each design variable set defining a potential solution called a string. Each string is made up of series of characters as binary numbers, representing the discrete variables for a particular solution. The fitness of each string is a measure of performance of design variables defined by the objective function and constraints. GA methods consist of a series of three processes: coding and decoding design variables into strings, evaluating the fitness of each solution string, and applying genetic operators to generate the next generation of solution strings. Most GA methods are variations of the simple GA proposed by Goldberg [8], which consists of three basic genetic operators: reproduction, crossover, and mutation. By varying these parameters, the convergence of the problem may be altered. Much attention has been focused on finding the theoretical relationships between these parameters. Rajeev and Krishnamoorty [12] applied GA for optimal truss design. They presented all the computations for three successive generations in the form of tables for easy understanding of the problem. In this study a GA method is proposed for minimal weight design of trusses. According to the shallow space form, the instability consideration is required. The general procedure is described in subsection 3.2.

Tabu search (TS) is a computational process which attempts to solve difficult combinatorial optimization problems through controlled randomization. In other words, TS is a metaheuristic method designed to find near optimal solutions of combinatorial optimization problems. Basically it consists of several elements called the move, neighborhood, initial solution, searching strategy, intensification, diversification and stopping rules. For obtaining near optimum solutions of such problems, a better minimum of an objective function should be searched for among a huge number of local minimums, since it is almost impossible to find an exact optimum. Intensification' means decreasing of the objective function value to find a better solution closer to the local minimum. Diversification' means a jump from a searching region to other regions to avoid getting trapped in a single local minimum. The details of the TS process are presented in paper [7] and in subsection 3.3.

2. Structural optimization problem

The basic, initial equation system of discrete minimal weight design is the total potential energy function of the geometrically nonlinear truss structure.

$$V\left(u_i, a_q, \lambda\right) = U\left(u_i\left(a_q\right)\right) - \lambda p_i u_i\ , \qquad i = 1, 2, ..., n \qquad q = 1, 2, ..., e \tag{1}$$

The total potential energy function is formulated in terms of load intensity parameter λ, applied external load vector p_i, nodal displacement vector u_i, and vector of the member sizing a_q, where n is the number of nodes, e is the number of elements, and $U(u_i(a_q))$ is the non-linear strain energy function. In this study nonlinear material is supposed in comparison with the results obtained by using the linear elastic material law. In the case of the nonlinear (Ramberg-Osgood) material law, the strain energy function $U\left(u_i\left(a_q\right)\right)$ is replaced by the following expression:

$$U\left(u_i\right) = \frac{\sigma_y^2 \sqrt{\left(1 + \frac{E^2 \varepsilon_q^2(u_i)}{\sigma_y^2}\right)}}{E} - \frac{\sigma_y^2}{E}\ , \tag{2}$$

where E is the elasticity modulus, ε_q is the member strain, and σ_y is the yield stress of the materials applied.

The design variables are selected from a discrete set of the predetermined cross-sectional areas, such that minimize the total weight of the structure:

$$V\left(a_q\right) \longrightarrow \min \tag{3}$$

subject to

$$V_{,i} = 0 \tag{4}$$

$$\lambda\left(a_q\right) = 1 \tag{5}$$

$$i = 1, 2, ..., n \qquad q = 1, 2, ..., e$$

where $V_{,i} = 0$ is the equilibrium criterion, $\lambda\left(a_q\right) = 1$ the maximal locally and globally stable and stress feasible load intensity. The path-following procedure of instability investigation is terminated when the unit load intensity is reached without any constraint violation.

The proposed instability investigation [2] is based on the perturbation technique of the stability theory and on the non-linear modification of the classical linear homotopy method. With the help of the higher-order predictor-corrector algorithm, we are able to compute an arbitrary load deflection path and detect the different types of stability points. Within the predictor step, we compute the solution of an implicit ODE problem and the corrector phase is the solution of a nonlinear equation system. The first-order derivatives are obtained from the equation system by null-space computation of the augmented Hessian matrix. The higher order derivatives are obtained from the inhomogeneous equations using the Moor-Penrose pseudo-inverse.

The basic function of the stability investigation is the total potential energy function. The equilibrium equation system is obtained from the total potential energy function. Starting from the zero point of the equilibrium path assuming that the Hessian is positive definite, the solution is obtained in terms of the arch-length parameter of the equilibrium path t.

The stability investigation is based on the eigenvalue computation of the Hessian matrix $V_{,ij}$. In every step of the path-following process we get information about the displacement, stresses, local, and global stability of the structure. This higher order predictor-corrector method provides an accurate computation of the singular points. It is capable of computing not only points but also segments of the equilibrium path. The curve segment approximation is the basis for the identification of the singular points. Since we are concerned with finding feasible designs we must define a certain appropriate measure of performance. In the proposed path-following approach the applied measure of design infeasibility $\lambda\left(a_q\right)$ is defined as the solution of the following system:

$$\lambda\left(a_q, t\right) \longrightarrow \max \tag{6}$$

$$0 \leq \lambda\left(a_q, t\right) \leq 1 \tag{7}$$

$$\eta_i\left(a_q, t\right) > 0 \tag{8}$$

$$\underline{s} \leq s_q\left(a_q, t\right) \leq \overline{s} \tag{9}$$

$$i = 1, 2, ..., n \qquad q = 1, 2, ..., e$$

where t is the arch-length parameter of the equilibrium path, η_i is the vector of eigenvalues of Hessian matrix $V_{,ij}$., and $\underline{s}$, $\overline{s}$ are the lower and upper bounds of the stress constraints.

The path-following process is terminated at the first constraint violation.

3. Discrete optimization methods

In this study tree heuristic techniques are considered: simulated annealing, a genetic algorithm and a tabu search method to find a solution for the discrete minimal weight design problem of shallow space trusses.

3.1. **Simulated annealing.** Simulated annealing is a computational process, which attempts to solve difficult combinatorial optimization problems through controlled randomization. Simulated annealing emulates the physical process of annealing which attempts to force a system to its lowest energy state through controlled cooling.

In general, the annealing process involves the following steps:

- The temperature of the system is raised to a sufficient level.
- The temperature of the system is maintained at the level for a prescribed amount of time.
- The system is allowed to cool under controlled conditions until the desired energy-state is attained.

The initial temperature the time system remains at and the rate at which the system is cooled are referred to as the annealing schedule. If the system is allowed to cool too fast it may freeze at an undesirable high-energy state. In simulated annealing the process starts at a given feasible or unfeasible solution. To avoid freezing at a local optimum the algorithm walks very slowly through the solution space.

The general procedure for the simulated annealing algorithm can be described as follows:

```
MaxStep = 1000
MaxNode = 150
MaxNeighbourhoodSearch = 10
Call ProblemDefinition
Temperature = 1
CoolingRatio = 0.95
n = 0
Call RandomInitialStructure
Call PathFollowingMethod
Call CurrentNodeUpdate
Call BestSolutionUpdate
Call BestFeasibleSolutionUpdate
For s= 1 To MaxStep
ParentSolution=BestSolution
   For m = 1 To MaxNeighbourhoodSearch
      If         RandomNeighbourStructure(ParentSolution)
Then
         Call PathFollowingMethod
         If AcceptedSolution(Temperature) then
         n = n+ 1
                  Call CurrentNodeUpdate
                  Call BestSolutionUpdate
                  Call BestFeasibleSolutionUpdate
         Endif
      Else
         Exit
      End If
Next m
        Call TemperatureUpdate: If Temperature < 0.001
Then Exit
Next s
```

3.2. **Genetic algorithm.** The genetic algorithm (GA) is an efficient and widely applied global search procedure based on a stochastic approach. All of the recently applied genetic algorithms for structural optimization have demonstrated that genetic algorithms can be powerful design tools [8, 12]. The crossover operation creates variations in the solution population by producing new solution strings that consist of parts taken from selected parent solution strings. The mutation operation introduces random changes in the solution population. In GA, the mutation operation can be beneficial in reintroducing diversity in a population. In this study, a pair of parent solutions is randomly selected, with a higher probability of selection being ascribed to superior solutions. The two parents are combined using a crossover scheme that attempts to merge the strings representing them in a suitable fashion to produce an offspring solution. Offspring can also be modified by some random mutation perturbation. The algorithm selects the fittest solution of the current solution set, i.e. those with the best objective function values. Each pair of strings reproduces two new strings using a crossover process and then dies.

The steps of the algorithm:

PopulationSize = 50
NumberOfNewGenerations=50
CrossoverProbability=0.5
SwapProbability=0.1
MutationProbability=0.1
Call ProblemDefinition
Call RandomInitialPopulationGeneration
Call BestFeasibleSolutionUpdate
For n = 1 to NumberOfNewGenerations
    $\{i, j\} \longleftarrow$ *Call RandomFittestParentPairSelection*
        Call Crossover
        For Each Child: Call Mutation
                Call PathFollowingMethod
                Call BestFeasibleSolutionUpdate
        $\{i, j\} \longleftarrow$ *Call OffspringPairUpdate*
Next n

3.3. **Tabu search algorithm.** In the case of tabu search, diversification is introduced as follows: if there are no improving moves, the move that least degrades the objective function is chosen. In order to avoid returning to the local optimum just visited, the reverse moves are forbidden. This is realized by storing those moves in a data structure called the tabu list. This contains s elements, which define forbidden moves, where s is the tabu list size. Once a move is stored in the tabu list, it will become available s iterations later.

The steps of the algorithm:

MaxStep = 1000
MaxNode=150
MaxNeighbourhoodSearch = 10
MaxTabuListSize=50
Call ProblemDefinition
n = 0
Call RandomInitialStructure

Call CurrentNodeUpdate
Call BestFeasibleSolutionUpdate
Call BestNodeUpdate
For s= 1 To MaxStep
  ParentSolution = BestSolution
  For m = 1 To MaxNeighbourhoodSearch
    If RandomNeighbourStructure(ParentSolution) Then
      n=n+1
        Call PathFollowingMethod
        Call BestFeasibleSolutionUpdate
        Call CurrentNodeUpdate
        Call BestNodeUpdate
        If n = MaxNode then Exit
    Else
      Exit
    End If
  Next m
Next s

4. **Numerical example**

4.1. **The 24-member dome structure.** In this paper, one of the frequently used test examples is considered. The geometry of the 24-member is shown in Figure 1 and Table 1. According to the requirement of the symmetrical structure, the truss members were partitioned into linking groups. Group 1 includes bars 1-6, group 2 includes bars 7-12, and group 3 includes bars 13-24.

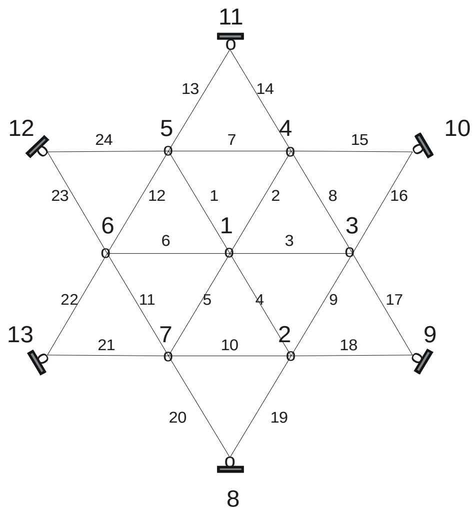

Figure 1. Layout of the 24-member dome structure

Nodal Points	X [m]	Y [m]	Z [m]
1	0.0	0.00	0.000
3	25.0	0.00	2.000
4	12.5	21.65	2.000
10	43.3	25.00	8.216
11	0.0	50.00	8.216

Table 1. Geometry of the 24-member dome structure

The elasticity modulus is $E = 7x1010N/m^2$. The stress constraints for tension and compression are $25x10N/m^2$. The density is $27500N/m^3$.

The cross-sectional areas of the truss-members with circular sections are selected from an available catalogue:

$$A_i = \{12.00; 12.25; 12.50; 12.75; 13.00; 13.25; 13.50; 13.75; 15.75\} * 10^{-4}\text{m}^2$$

The applied loads of the 24-member dome structure are $P_1 = 6kN$ at the nodal point 1, and $P_{2-7} = 12kN$ at the nodal points 2-7, which causes a bifurcation instability phenomenon. The results of the optimization process are shown in Tables 2-4.

Run	Weight	Cross-sections (catalogue values)	Relative error
1	258.506	{12.75,15.75,13.50}	1.408
2	258.013	{12.25,15.50,13.75}	1.215
3	256.994	{13.25,14.25,13.75}	0.815
4	256.940	{13.50,15.25,13.25}	0.794
5	256.549	{12.00,13.50,14.50}	0.641
6	256.447	{13.00,15.00,13.50}	0.601
7	256.440	{12.50,15.50,13.50}	0.598
8	256.440	{12.50,15.50,13.50}	0.598
9	255.953	{12.50,14.75,13.75}	0.407
10	255.953	{12.50,14.75,13.75}	0.407
11	255.950	{12.25,15.00,13.75}	0.406
12	255.899	{12.75,15.75,13.25}	0.386
13	255.463	{12.25,14.25,14.00}	0.215
14	255.409	{12.50,15.25,13.50}	0.193
15	255.406	{12.25,15.50,13.50}	0.192
16	255.406	{12.25,15.50,13.50}	0.192
17	255.402	{12.00,15.75,13.50}	0.191
18	254.916	{12.00,15.00,13.75}	0.000
19	254.916	{12.00,15.00,13.75}	0.000
20	254.916	{12.00,15.00,13.75}	0.000

Table 2. Results of genetic algorithm (GA)

Run	Weight	Cross-sections (catalogue values)	Relative error
1	260.697	{13.75,12.75,14.50}	2.268
2	260.044	{14.25,15.25,13.25}	2.012
3	259.557	{14.25,14.50,13.50}	1.821
4	258.526	{14.25,14.25,13.50}	1.416
5	258.468	{14.25,15.50,13.00}	1.393
6	258.465	{14.00,15.75,13.00}	1.392
7	258.084	{13.25,13.25,14.25}	1.243
8	257.978	{14.00,15.00,13.25}	1.201
9	257.536	{13.00,14.00,14.00}	1.028
10	257.488	{13.75,14.50,13.50}	1.009
11	257.488	{13.75,14.50,13.50}	1.009
12	257.430	{13.75,15.75,13.00}	0.986
13	257.046	{12.75,13.50,14.25}	0.836
14	257.036	{12.00,14.25,14.25}	0.832
15	256.991	{13.00,14.50,13.75}	0.814
16	256.978	{12.00,15.50,13.75}	0.809
17	256.450	{13.25,14.75,13.50}	0.602
18	259.557	{14.25,14.50,13.50}	1.821
19	255.950	{12.25,15.00,13.75}	0.406
20	255.406	{12.25,15.50,13.50}	0.192

Table 3. Results of simulated annealing method (SA)

Run	Weight	Cross-sections (catalogue values)	Relative error
1	264.304	{15.75,12.25,14.25}	3.683
2	263.807	{15.00,12.25,14.50}	3.488
3	262.232	{15.00,12.50,14.25}	2.870
4	262.228	{14.75,12.75,14.25}	2.868
5	262.228	{14.75,12.75,14.25}	2.868
6	260.697	{13.75,12.75,14.50}	2.268
7	260.598	{15.00,14.00,13.50}	2.229
8	260.540	{15.00,15.25,13.00}	2.206
9	259.701	{12.00,13.00,15.00}	1.877
10	259.615	{14.25,13.25,14.00}	1.843
11	259.016	{14.50,14.75,13.25}	1.608
12	259.016	{14.50,14.75,13.25}	1.608
13	258.577	{13.75,13.50,14.00}	1.436
14	258.125	{12.00,13.25,14.75}	1.259
15	258.125	{12.00,13.25,14.75}	1.259
16	258.032	{13.75,14.00,13.75}	1.222
17	256.994	{13.25,14.25,13.75}	0.815
18	256.450	{13.25,14.75,13.50}	0.602
19	255.460	{12.00,14.50,14.00}	0.213
20	254.916	{12.00,15.00,13.75}	0.000

Table 4. Results of tabu search method (TS)

The results obtained using simulated annealing (SA), genetic algorithm (GA), and tabu search (TA) methods have been illustrated in Tables 2-4.

The total number of the cross-sectional combinations for three member groups is 4096. The global optimal solution of the problem: $GW = 254.916$ the weight of the structure; $GC = \{12.00; 15.00; 13.75\}$. Using a standard implicit enumeration algorithm, 1615 node evaluations were needed to obtain this solution and to prove its global optimality. To compare the standard local search methods, we ran each method 20 times from a randomly selected design (population). In each case we stopped the searching process after 150 design evaluations.

4.2. **The 10-bar truss.** In this comparative study, according to the widely used dimension in the literature we adopted the same values and the same dimension system in our computation.

Load condition: $P = 100000\ lb$; Material density: $\rho = 0,1\ lb/in^2$;
Young modulus: $E = 10^7\ psi$; Yield stress: $\sigma = 40000\ psi\ \pm$.
The following conversion table gives us the International System of Units (SI):
1 inch $(in) = 25,4\ mm$;
1 pound $(lb) = 0,4536\ kg$;
1 pound per square in (lb/in^2) =6895 N/m^2;
1 $kips = 4448\ N$.

In this example, a genetic algorithm was applied for both cases using One-Point Crossover. The cross-sectional areas are selected from the given set of the catalogue values of $\{\ 36;\ \ 27;\ \ 19;\ \ 12;\ \ 7;\ \ 4;\ \ 2;\ \ 1\ \}$.

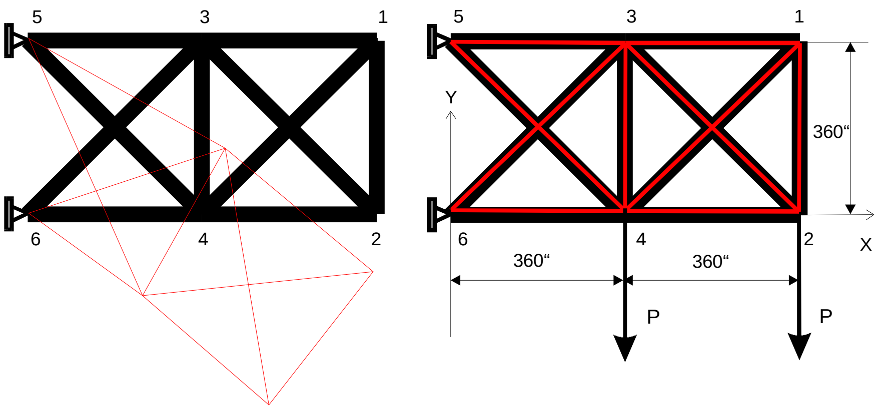

Figure 2. Layout and load condition of the 10-bar truss

The results of the ten-bar truss example (Table 5 and Table 6) demonstrate that the efficiency of GA strongly depends on the choices of population size and the crossover size. Using the nonlinear material law, we obtained much a lighter structure in both cases than in the case of a linear elastic material in the *paper quoted [3] (see table 10).*

However we have to note that the discrete solution method proposed was different *in paper [3].*

Population	Crossover	Best weight	Cross-sections
50	*50*	*3028.06*	*{19,4.,19,2,7,2,7,4,4,7}*
50	50	5833.94	{7, 27, 19, 36, 1, 7, 4, 19, 19, 4}
50	50	3819.00	{12, 2, 19, 19, 2, 4, 7, 7, 19, 1}
50	50	3552.09	{19, 4, 19, 7, 2, 1, 2, 12, 12, 7}
50	50	4154.29	{12, 4, 19, 7, 12, 2, 4, 12, 7, 19}
50	50	3727.41	{4, 12, 7, 1, 2, 4, 12, 2, 19, 19}
50	50	3466.23	{12, 27, 19, 7, 2, 1, 4, 7., 7, 2}
50	50	3682.23	{19, 12, 7, 7, 2, 27, 7, 2, 4, 7}
50	50	3725.91	{7, 7, 12, 12, 12, 4, 19, 2, 7, 7}
50	50	4841.91	{7, 12, 19, 4, 7, 36, 12, 19, 2, 2}

Table 5. Results of ten-bar truss

Population	Crossover	Best weight	Cross-sections
100	100	3045.53	{12, 2, 19, 4, 1, 7, 7, 12, 7, 2}
100	*100*	*2823.35*	*{7, 7, 7, 7, 4, 4, 7, 12, 4, 7}*
100	100	4539.00	{7, 7, 19, 7, 2, 36, 7, 1, 7, 19}
100	100	3343.32	{12, 12, 19, 4, 12, 7, 12, 1, 4, 2}
100	100	3708.44	{36, 4, 7, 12, 2, 1, 4, 12, 12, 1}
100	100	3930.62	{7, 7, 19, 27, 7, 4, 12, 4, 7, 4}
100	100	3750.62	{27, 1, 12, 12, 12, 2, 7, 4, 4, 12}
100	100	3908.47	{12, 4, 19, 4, 1, 12, 27, 2, 4, 7}
100	100	4588.41	{4, 19, 27, 12, 36, 4, 12, 1, 1, 4}
100	100	3157.15	{4, 2, 12, 2, 2, 36, 12, 4, 1, 4}

Table 6. Results for ten-bar truss

4.3. **The 25-bar truss.** Material density, Young modulus, and yield stress constraints are the same as for the ten-bar truss. The loads are given in Table 7. The relationship between the indices and the cross-sections is given in Table 8. The results obtained by using the nonlinear material law (see Table 9) are approximately half of the optimal weight obtained using the linear elastic material law (see Table 10).

In this example a genetic algorithm was also applied. The cross-sectional areas are selected from the given set of the catalogue values of

$$\left\{\ 3,5 \quad 3,4 \quad 3,3 \quad ...... \quad 1,0 \quad 0,9 \quad ..... \quad 0,2 \quad 0,1 \quad 0,01\ \right\}.$$

Nodal points	X [kips]	Y [kips]	Z [kips]
1	-	20	-5
2	-	-20	5

Table 7. Applied loads of 25-bar truss

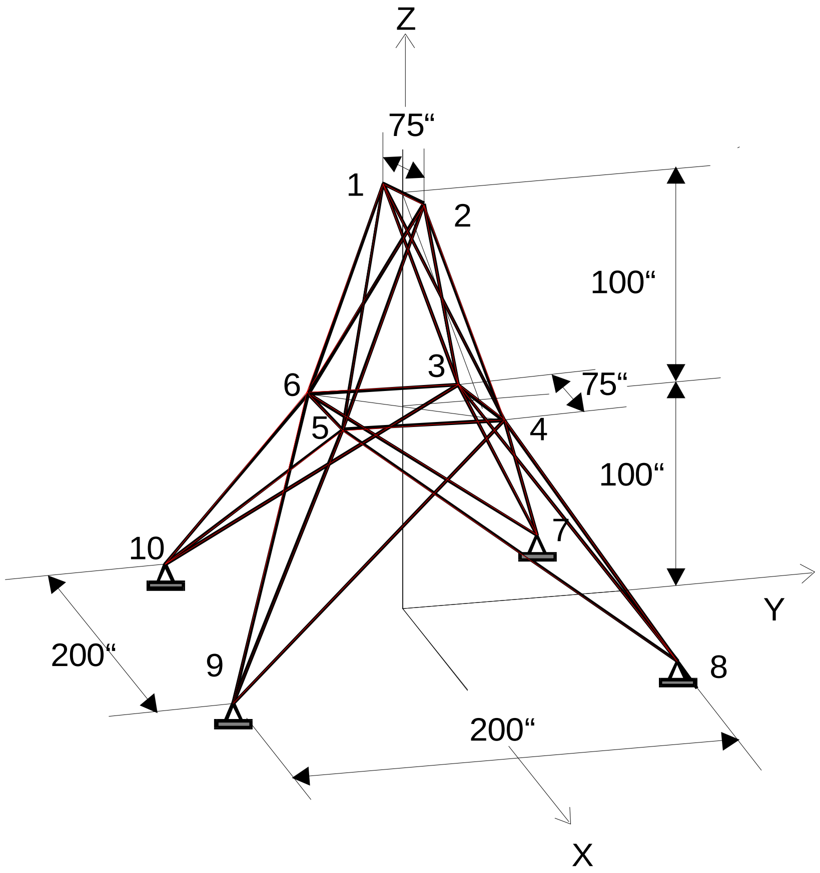

Figure 3. Geometry of the 25-bar truss

[1]=0.01	[2]= 0.1	[3]= 0.2	[4]= 0.3	[5]= 0.4	[6]= 0.5
[7]= 0.6	[8]= 0.7	[9]= 0.8	[10]= 0.9	[11]= 1.0	[12]= 1.1
[13]= 1.2	[14]= 1.3	[15]= 1.4	[16]= 1.5	[17]= 1.6	[18]= 1.7
[19]= 1.8	[20]= 1.9	[21]= 2.0	[22]= 2.1	[23]= 2.2	[24]= 2.3
[25]= 2.4	[26]= 2.5	[27]= 2.6	[28]= 2.7	[29]= 2.8	[30]= 2.9
[31]= 3.0	[32]= 3.1	[33]= 3.2	[34]= 3.3	[35]= 3.4	[36]= 3.5

Table 8. Relationship between the indices and cross-sections

Population	Crossover	Best weight	Cross-sections
50	50	201.678	{12, 5, 3, 14, 17, 5, 7, 10}
50	50	165.837	{3, 6, 12, 15, 13, 3, 4, 4}
50	50	204.183	{10, 11, 16, 3, 17, 3, 5, 3}
50	50	260.231	{15, 15, 10, 8, 3, 6, 11, 4}
50	50	188.972	{30, 8, 9, 5, 11, 5, 3, 7}
50	*50*	*153.736*	*{27, 4, 4, 12, 15, 3, 4, 7}*
50	50	166.926	{13, 3, 6, 6, 15, 5, 9, 3}
50	50	228.348	{8, 4, 13, 23, 16, 3, 7, 9}
50	50	160.656	{27, 4, 9, 3, 7, 3, 7, 5}
50	50	157.644	{15, 6, 4, 7, 13, 6, 5, 4}

Table 9. Results of 25-bar truss

Results of ten-bar truss			Results of twenty-five-bar truss		
Variables	Continuous solution	Discrete method	Variables	Continuous solution	Discrete method
1	30.4015	36.0	1	0.3	0.3
2	0.1	0.1	2	2.03572	2.00
3	23.1041	27.0	3	2.75761	2.80
4	15.2160	19.0	4	0.01	0.01
5	0.1	0.5	5	0.01778	0.01
6	0.6623	0.1	6	0.33511	0.40
7	7.5049	7.0	7	1.9	1.90
8	20.9631	19.0	8	0.1	0.20
9	21.5409	19.0			
10	0.1	0.1			
weight [lb]	5056.15	5273.32		394.027	403.897

Table 10. The optimum cross-sectional areas [in^2] using linear elastic material law

The optimal weight of the 25-bar truss structure is *153.736 lb* using the nonlinear material law instead of the linear elastic rod members resulting in *403.897 lb* for discrete design variables and resulting in *394.027 lb* in the case of continuous solution methods.

5. **Conclusions**

In this work, three different solution techniques are discussed and compared, simulated annealing (SA), genetic algorithm (GA), and tabu search (TS) methods, for solving a discrete minimal weight design problem for shallow space trusses. In each case we stopped the searching process after 150 design evaluations. The computational results reveal the fact that the GA method produces high quality results when the solution time is limited. Obviously, the performance of SA, TA, and GA depends on various parameter choices, such as the cooling parameter for SA, and the population size, frequency of mutation for GA. The TA and the SA methods are very sensitive to the starting (initial) design. When the solution time is limited, the likelihood that the TA and SA methods provide near optimal solutions is very low.

We compared the numerical results of two frequently used test problems obtained by using the linear and nonlinear material law. We obtained much lighter structure in both cases but we have to note that the solution method was different in the quoted paper [3] (see Table 7) and in the present study. In contradiction with paper [3], here nodal displacement constraints were not considered. However, the large deflection in the behavior of the initial structure might be significant. In the last two examples, a genetic algorithm (GA) was adopted for the discrete optimal design problem. In each case we stopped the searching process after 150 (300) design evaluations related to a population size of 50 (100).

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