Energy Economics 111 (2022) 106084

Contents lists available at ScienceDirect

Energy Economics

journal homepage: www.elsevier.com/locate/eneeco

Convex combinatorial auction of pipeline network capacities

Dávid Csercsik *

Centre for Economic and Regional Studies, Institute of Economics, Tóth Kálmán u. 4., H-1097 Budapest, Hungary
Pázmány Péter Catholic University, Faculty of Information Technology and Bionics, Práter u. 50/A 1083 Budapest, Hungary

ARTICLE INFO

JEL classification:
C54
C57
C61
C63
D44
D47

Keywords:
Natural gas networks
Capacity allocation
Modeling
Mechanism design

ABSTRACT

In this paper we propose a mechanism for the allocation of pipeline capacities, assuming that the participants bidding for capacities do have subjective evaluation of various network routes. The proposed mechanism is based on the concept of bidding for route–quantity pairs. Each participant defines a limited number of routes and places multiple bids, corresponding to various quantities, on each of these routes. The proposed mechanism assigns a convex combination of the submitted bids to each participant, thus its called convex combinatorial auction. The capacity payments in the proposed model are determined according to the Vickrey–Clarke–Groves principle. We compare the efficiency of the proposed algorithm with a simplified model of the method currently used for pipeline capacity allocation in the EU (simultaneous ascending clock auction of pipeline capacities) via simulation, according to various measures, such as resulting and net utility of players, utilization of network capacities, total income of the auctioneer and fairness.

1. Introduction

1.1. Abbreviations and notations used in the paper

Table 1 summarizes the abbreviations and notations used throughout the paper.

1.2. Natural gas pipelines in the EU and third party access

The European natural gas network represents an enormous infrastructure system, which is also constantly in the focus of geopolitics (Bilgin, 2009; Ericson, 2009). In the traditional model national or multinational energy companies built their own pipelines requiring huge investments and expected that their latter trade transactions using the pipeline will provide them with sufficient returns. Nowadays exclusive ownership is not the general institutional setting. Many pipelines within the European Union are subject to regulated third party access (TPA). Since the early 1990s the EU have adopted a number of increasingly assertive directives and regulations to develop the common market for gas by ensuring fair TPA access to the transportation system within the Union — see EU (1991, 1998, 2003), European Comission (2005), EU (2009b). According to this scheme the member countries have established a system of transport fees overseen by a regulatory authority. Under such a regime the owner of a pipeline no longer enjoys exclusive right over the transport capacities. Instead, he has to grant access, provided he is compensated according to the regulated tariff. Cooperative game theoretic analysis of TPA and the implied transfer profits in natural gas networks has been proposed in Csercsik et al. (2019).

1.3. Motivation: Current practice of pipeline network capacity allocation in the EU

More than a decade ago regulations of the third energy package (EU, 2009a) basically separated the network operation from the trading and supply, expanded the rights of regulation authorities, and created the Agency for the Cooperation of Energy Regulators (ACER) and the European Network of Transmission System Operators for Gas (ENTSOG). As a result, in the last 10 years the bias of trading already significantly shifted from long term (usually fixed-price) contracts to more liquid trading platforms (markets corresponding to so called *gas hubs*). In this framework, to provide infrastructure for such increasingly interactive trading, the transmission system operators (TSOs) market the transfer capacities of pipelines as standardized products of variable time-frames (from yearly to intra-day intervals). According to the reports of Merino (2016), the volume of engagements corresponding to short-time products constantly increases.

As long as the capacities required for the planned trade transactions do not exceed the pipeline capacities, allocation is simple and it practically means only administration. However, if the available capacities are not enough to satisfy all participants aiming to allocate capacities in

* Correspondence to: Centre for Economic and Regional Studies, Institute of Economics, Tóth Kálmán u. 4., H-1097 Budapest, Hungary.
E-mail address: csercsik.david@krtk.hu.

https://doi.org/10.1016/j.eneco.2022.106084
Received 28 July 2021; Received in revised form 8 April 2022; Accepted 18 May 2022
Available online 31 May 2022

Table 1
Abbreviations and notations used in the paper.

Abbreviation/Notation	Meaning
TPA	Third party access
TSO	Transmission system operator
ACA	Ascending clock auction
CA	Combinatorial auction
CCA	Convex combinatorial auction
CDA	Combinatorial double auction
MA	Market area
VCG	Vickrey–Clarke–Groves
n	Number of nodes in the network
n_s	Number of source nodes in the network
m	Number of edges in the network
$\overline{q}_j$	Maximal transfer capacity of pipeline j
c^t_j	Transfer cost for pipeline j
c^s_k	Source cost for source k
UC^A_i	Consumption utility of player i in the case of the mechanism A
CT^A_i	Transfer payments for player i in the case of the mechanism A
CS^A_i	Source payments for player i in the case of the mechanism A
CC^A_i	Capacity payments for player i in the case of the mechanism A
UR^A_i	Resulting utility of player i in the case of the mechanism A
UN^A_i	Net utility of player i in the case of the mechanism A
Y^A_i	Consumption of player i in the case of the mechanism A
$B^i_{j,k}$	kth bid for route j of player i in the CCA
$q^i_{j,k}$	Quantity of the kth bid for route j of player i in the CCA
$p^i_{j,k}$	Value of the kth bid for route j of player i in the CCA
uF^A	Unfairness measure of results in the case of the mechanism A
r^A_{AC}	Ratio of allocated capacities in the case of the mechanism A
r^A_{UC}	Ratio of used capacities in the case of the mechanism A
r^A_{-UN}	Ratio of negative net utility in the case of the mechanism A
IA^A	Income of the auctioneer in the case of the mechanism A
P^i_j	Price parameter of the jth step of the inverse demand function of player i
Q^i_j	Quantity parameter of the jth step of the inverse demand function of player i
f_{aac}	Flows on already allocated capacities
f_{cau}	Flows on capacities under auction
L	Vector of inlet values
AAC^i	Vector of capacity products already allocated to player i
CAU	Vector of capacity products under auction
NW	Network configuration code used in the simulations

the network, some kind of capacity-allocation method must be used to distribute the available pipeline capacities among participants (players) who apply for them. The first auction, which coordinated the long-term bookings of available existing and future pipeline capacities on the EU-level has been held in 2017 March on the PRISMA auction platform. During this auction, yearly, quarterly and monthly pipeline capacity products have been auctioned simultaneously using an ascending clock auction (ACA). Altogether 2165 unique auctions took place on 6 March for each point and each year. As pointed out by Takácsné Tóth et al. (2017), in most of the cases no real competition emerged, and as the result of this auction, the dominant market player (GAZPROM) was able to acquire the great majority of high-importance capacity licenses for in some cases as long as 20 years (for example, *all* interconnection capacities on the border of Slovakia have been booked for 20–25 years by GAZPROM).

Several factors may be identified as underlying causes for this result. First, Russia, unlike other suppliers of Europe like Algeria or Norway, typically delivers gas to the border of the importer country, thus countries which import gas from Russia do have modest interest in acquiring transport routes. The reasons for this are partially historical — in deals of the former decades the market power of GAZPROM was very high, so importers were compelled to agree with such details of bargains.

Second, if a large producer supplying a significant number of clients aims to buy capacities for his deliveries, all the delivery paths in question originate from the production site, and they have potentially large overlaps (see e.g. the Nord Stream I and II and their connected pipelines, which are built practically to supply the majority of Europe). In this case, it is easy to identify pipelines and interconnection points which are critical for these delivery projects, and thus represent high value for the player. In other words, the optimal bidding strategy of such large producers is quite straightforward in the current framework, while they typically also have the resources to obtain capacity licenses for long periods.

In contrast, the optimal bidding strategy in the current framework is not trivial for smaller consumers. Consumers, in addition to long term contracts, typically buy gas the on various established hubs, the prices of which may be different and also uncertain regarding longer periods (e.g. years). In such cases, capacity products have to be booked in order to ensure connected paths to the *market areas* (MAs) hosting these hubs. Inside market areas, the physical transportation of gas is the responsibility of the local TSO. The article of Keller et al. (2019) discusses the implications of these market areas in Germany, and analyzes the efficiency of inter-area capacity bookings.

Overall, it can be said that the current allocation practice and the respective algorithms do have their pros and cons, but in general it is reasonable to ask if there is any alternative to the current method of capacity allocation. In the current paper we propose exactly such an alternative approach, called convex combinatorial auction (CCA) of pipeline capacities. As a first step, we define and test this method on an abstract model under several simplifying assumptions (see Section 2.1).

We consider a scenario where, under the principle of regulated third party access, local (national) TSOs have the right to determine transfer fees for their pipelines. We assume that these fees are prior defined, i.e. TSOs are not strategic players in the model. In addition, we assume that the pipeline capacity licenses are allocated by a central authority via auction. We compare the newly proposed CCA allocation to the allocation based on the simultaneous ACA (model of the currently used method), assuming a simple but reasonable optimal bidding strategy of the participants of the ACA (see Section 2.5.2). We use various measures for the comparison, such as resulting utility and net utility of players, utilization efficiency of network infrastructure, total amount of payment for the capacity rights (i.e. the income of the auctioneer) and fairness.

1.4. Contextualization in the mechanism design aspect

The proposed CCA mechanism is based on the principles of the combinatorial auction (De Vries and Vohra, 2003) (CA). Participants submit bids for the bundles of goods (capacity products in our case, the bundles of which are routes in the network), and the clearing aims to maximize the value of accepted bids, while taking into account various constraints. The most important difference in the case of auctioning pipeline capacities compared to the original model of the combinatorial auction is that while the basic combinatorial auction model assumes indivisible goods, pipeline capacities are considered as divisible items (thus the *convex* term in the CCA).

The principle of the CA may also be applied in the case of two-sided auctions, where both demand and supply bids are considered (double auctions). In this case, the objective is to maximize the resulting welfare, and the setup is called combinatorial double auction (CDA) (Xia et al., 2005). The earliest proposals of the CDA include stock exchange trading solutions (Fan et al., 1999), but the scope of applications became impressively wide in the past decades regarding the allocation of divisible and indivisible goods as well. The CDA framework has been applied for spectrum auctions (Chen et al., 2015), for the allocation of resources in cloud- and grid-computing environments (Li et al., 2009; Xu et al., 2014; Baranwal and Vidyarthi, 2015; Samimi et al., 2016), for matching production and demand in local electricity markets (Kiedanski et al., 2021), as well as for the carpooling and ride-sharing problems (Hsieh et al., 2019; Hsieh, 2020, 2021).

1.5. Structure of the paper

The structure of the paper is as follows. Section 2 details the methodology used in the paper. In Section 2.1 we define the principles of the used model of the network, demonstrate the concepts on a simple example, highlighting the differences between the two methods and their operation. In Section 2.5 we discuss the additional details of the simulations, and of the algorithms modeling the ACA and CCA based allocations. In Section 3 we present the simulation results originating from high numbers of randomized scenarios to get statistical data about the performance of the two methods. Section 4 evaluates the respective results and includes a general discussion, while Section 5 concludes.

2. Materials and methods

In the following, after introducing some basic concepts of the model, a simple example is introduced for the demonstration of the principles, fundamental properties and operation of the two analyzed methods (ACA and CCA). Following the example, which aims to give a reader a basic impression about the used models of the two auction processes, we present the details of the modeling methodology used.

In this paper we focus on consumers and we assume that they are the only participants of the capacity auction. Regarding realistic scenarios, at least in addition to local gas distribution companies, who may be considered as consumers on the level of the continent-wide network, multinational energy companies and international traders are also present as typical bidders of such auctions. The benefit of considering only consumers as bidders is that if we use a simple demand characterization, which is still able to capture demand-elasticity, their rational bidding strategy (under a few additional assumptions in the case of ACA) may be easily derived — this task would be much more harder in the case of agents representing multinational companies with more complex incentives.

In the current work we focus on long-term capacity rights. The regulation (EU, 2017) defines yearly, quarterly, monthly, daily and within-day capacity products, from which the first three are sold via the ACA algorithm. In other words this means that if one is willing to allocate capacities for example for the first month of the year, he/she has 3 opportunities to do it.

Motivated by this our model of the ACA process will have three rounds. In the first round we assume that all capacities of the network in question are subject to auction. After the first round has finished, remaining (not-allocated) capacities are subject to the second round of ACA auctions, and so similarly, following the second round, the remaining not-allocated capacities are subject to the last round of ACA auctions.[1] In contrast, as we will see later, the proposed CCA allocation method executes the allocations in a single step.

2.1. Modeling assumptions and an introductory example

2.1.1. Market areas and their representation in the model

As discussed by Keller et al. (2019) market areas (MAs) are sets of physical network nodes between which the transportation of gas is the responsibility of the TSOs. Network users are able to inject gas at any entry point of the MA and withdraw gas at any exit point that belongs to the same MA, if they have the capacity rights for the respective entry and exit points. Some entry points of a MA may correspond to production sites, while others may represent incoming pipelines. Let us note that multiple such pipelines exists (see Fig. 3 in Keller et al., 2019).

In the terms of our model the nodes represent MAs and the edges represent the capacities connecting them. According to the above considerations it is possible that more than one edge is present between two nodes. Although we do not consider such cases in the paper the model is capable of handling these scenarios.

2.1.2. Bundled products

Similar to transfer fees, capacity products in the practical European system (PRISMA — see https://www.prisma-capacity.eu/) are also considered corresponding to the entry and exit points of MAs. The basic reason for this is that the transfer capacities are managed locally by the TSO's of the respective price zone. To make the life of bidders easier, the capacity allocation platform defines so called bundled products composed of an exit and an entry capacity. This means that if I would like to transfer from node A to node B, I have the possibility to bid for a bundled AB product, in which the exit capacity of A and the entry capacity of B are included. These bundled products are handled in the PRISMA system in a way, which ensures that the total entry and exit capacities of price zones are respected.

For the aim of simplicity, we consider only such bundled capacity products in the used modeling framework. Let us note however that the used methodology may be easily generalized to a more detailed scenario. The current model takes capacity from A to B into account as a product, if node A is connected to node B. If one would like to consider entry and exit capacities distinctly, an intermediate node X may be introduced on the edge A–B. In this case the edge A–X represents the entry and exit capacities of A, while the edge X–B represents the entry and exit capacities of zone B.

2.1.3. The example network

Topology and pipelines. Let us consider the network depicted in Fig. 1. Let us denote the number of nodes/market areas by n (in this case $n = 10$) and the number of pipelines/edges by m (= 15 in this case).

We assign a direction to each edge, as denoted in Fig. 1, to account for the positive and negative direction of flows (according to the reference direction of the edge), but we assume that the pipelines corresponding to the edges are bi-directional, and their maximal transfer

[1] Let us note that according to the current practice, 10% of the available transfer capacity is reserved for short term trading.

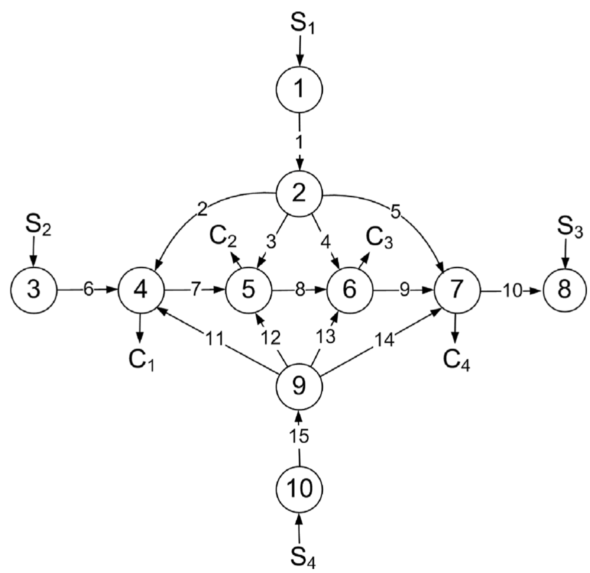

Fig. 1. Example network. Each node and edge is labeled with its ID. S_i denote the sources present, while C_i denote the consumers. Nodes 2 and 9 do not have any sources or consumers.

capacity ($\overline{q}_j$ for pipeline j) is the same in both directions. In the case of realistic scenarios, the potential bi-directional usage of pipelines depends on the presence of compressor stations. In the current example we assume that the transfer capacity of each edge is one unit (i.e. $\overline{q}_j = 1 \quad \forall j$).

For the aim of simplicity, we also assume that the transfer costs for pipelines (c^t_j for pipeline j) are also the same in the positive and in the negative direction. Furthermore, in the current example we assume that $c^t_j = 0.01$ for edges $6, 7, 8, 9$ and 10, and $c^t_j = 0.02$ for all other edges (horizontal transfers are cheap). In practice, local TSOs set entry and exit fees at interconnection points, but from these values transfer fees of a certain line i in the context of the model may be easily derived as the exit fee of the source point and entry fee of the destination point.

Modeling the sources and source costs. In the proposed model framework it is assumed that natural gas is available at distinguished nodes (representing market areas), from where consumers must ensure themselves routes to transport it to consumption sites. In the current model we assume that these sources are able to provide arbitrary quantities on prices, which are fixed for the period in question (for which we consider the allocation of transfer capacities). Let us note that in the case of realistic scenarios, the price of natural gas at the trading hubs may be significantly volatile, and depends on the nature of the source as well (obtained e.g. from actual transports, gas reservoirs or from LNG terminals).

According with the recent line of EU regulations, we assume that no price differentiation is allowed at the market, thus the sources S_1, S_2, S_3 and S_4, located in nodes 1, 3, 8 and 10 respectively provide gas for any consumer at fixed prices, namely $c^s_1 = 0.3$, $c^s_2 = 0.1$, $c^s_3 = 0.2$ and $c^s_4 = 0.4$.

Modeling the demand. Furthermore, in this simple example we assume that each of the consumers aims to get 1 unit of gas. The implied utility values are however not the same. Consumer 1 (located in node 4) is ready to pay 4 units of money for a single unit of gas, while consumers 2 3 and 4 (located in nodes 5, 6 and 7) are ready to pay 2 3 and 1 for it respectively.

2.2. Capacity allocation — general assumptions

We assume a central regulatory authority who has the exclusive right to sell pipeline capacity licenses for market participants, who, according to their individual positions and demand, have different evaluations for particular routes and products in the network.

Participants do have a strategy space — they decide which bids they would like to submit, thus they can be considered as players of the game. For the clarification of the terminology, we will use 'participants' and 'players' as synonyms in the rest of the paper. In this paper we will assume that the exclusive participants of the capacity allocation game are consumers of the model, since, as mentioned earlier, their optimal bidding strategy may be plausibly derived from the modeling assumptions via simple computations in both of the mechanisms analyzed.

We assume furthermore that participants do not have any information about each other's consumption parameters (e.g. demand and willingness to pay for certain products or product combinations), however they have full information abut the network parameters (topology, inlet- and source costs).

2.3. Bidding and capacity allocation in the ascending clock auction (ACA) framework

In the following we summarize the assumptions by which the bidding behavior in our model is described, and evaluate the ACA auction based on the principles laid down in EU (2017). The ACA auction process is carried out simultaneously for each line. Let us summarize the most important points of the regulation, which our model of the ACA process is based on.

- *Ascending clock auctions shall enable network users to place volume bids against escalating prices announced in consecutive bidding rounds, starting at the reserve price P_0.*
- *The volume bid per network user at a specific price shall be equal to or less than the volume bid placed by this network user in the previous round.*
- *If the aggregate demand across all network users is less than or equal to the capacity offered at the end of the first bidding round, the auction shall close.*
- *If the aggregate demand across all network users is greater than the capacity offered at the end of the first bidding round or a subsequent bidding round, a further bidding round shall be opened with a price equal to the price in the previous bidding round, plus the large price step.*
- *If a first-time undersell occurs, a price reduction shall take place and a further bidding round shall be opened. The further bidding round will have a price equal to the price applicable in the bidding round preceding the first-time undersell, plus the small price step. Further bidding rounds with increments of the small price step shall then be opened until the aggregate demand across all network users is less than or equal to the capacity offered, at which point the auction shall close.*

In the above mechanism the clear aim of the large and small price steps is to reduce the number of bidding rounds (the auction switches to small price steps before the undersell). For the aim of simplicity, in our simulations we use only one step size, which is small enough to capture the details of the change of individual evaluations as the price of a certain line increases (in this case 0.05).

In our simulation we assume that in the beginning, no capacities are allocated, every transfer capacity is subject to the auction. This means that in the case of the example network depicted in Fig. 1, we will have 30 products: capacities corresponding to positive and negative directions for each edge. According to the principles described in EU (2017), the initial price of capacity products are set to cover only the expanses of the TSO. As the proposed model considers transfer prices separately (not included in capacity prices), this implies that we assume that the initial price of the capacity products is 0.

According to the considerations discussed before in Section 1.3, we simulate 3 rounds of ACA auctions, each with the starting price of 0 for each active product. Products, which are not fully allocated in a round are active in the next rounds, considering their remaining capacity (all products start as active).

To clarify the terminology, each auction *round* of the ACA begins with the declaration of capacities under auction (and the announcement of initial prices), and after the submission of bids to every active product, the prices of overbidded products (products for which the sum of bids exceeds the capacity) increase in every *step* of the actual auction by 1 unit. If the total amount of submitted bids for a certain capacity are less than the volume of the particular capacity, the product becomes passive, and capacities are allocated according to the last submitted bids. The auction round ends, if all products become passive.

2.3.1. Bid format in the ACA auction

The bid format in the ACA is simple. In each bidding round, we have active products, for which bids may be still placed and closed products, for which the capacity is over. In the initial round all products are active. In our case the products are the + and – directional transfer capacities of the pipelines.

Each player must define the bid quantities he/she places on the active products in each round, in other words the overall bid of a player is defined by a vector.

In the next subsection we describe how we model the optimal bidding strategy of participants in these framework.

2.3.2. Principles of bidding strategy in the ACA auction model

Since in the ACA framework the bids for individual capacities are evaluated simultaneously, and the prices rise in the same time for items where the total amount of bids exceeds the capacity, bidding for multiple routes in the same time is risky, since it may happen that the participant ends up with overcapacity (which must be paid as well). Let us note however that, if these different routes are partially overlapping, this risk may be reduced (in addition, in realistic cases these unused capacities may be traded in secondary markets as well — this is however out of the scope of the current model).

In the current model a simple strategy is implemented for players in the ACA bidding. In each round, each player chooses the best route or routes (which cost the less for him/her considering the inlet cost at the source and the transfer costs as well), which is still accessible, and bids on its components, as long as its net utility remains positive considering the implied potential allocation costs. If the cheapest route provides not enough capacity to cover all the demand of the player, the second cheapest one is also considered for bidding, etc.

Let us note two thoughts here. First, the actual network and the parameters are constructed to serve as a simple example. In general, nothing guarantees that one route allows the fulfillment of the demand of the player. If the demand of P1 would be 2 units, the player would place bids for the two cheapest routes, etc. Second, it is possible that a participant ensures some capacities for him/herself in a certain round of the ACA, but these capacities may not be used (since they do not form a full path to a source). In this case, in the consecutive rounds the player may utilize these already allocated capacities, when determining the actually best route.

The principles above may be formalized via the concept of *optimal potential flows*, which is described in Section 2.5.2 in detail.

2.3.3. The ACA process in the case of the simple example

If we return to the setup depicted in Fig. 1 and consider all the network and consumption parameters, we can observe that the transfer costs of lines are significantly lower compared to source costs. This means that the optimal route for which players will place bids will be solely determined by the price of the source at the start of the route (in general, this is not necessarily true). The non-uniform transfer costs on the other hand imply that if a consumer (e.g. C_2 in node 5) has multiple routes to a source (e.g. to S_3 in node 8, which is accessible e.g. via the routes 3 – 5 – 10, 8 – 9 – 10 and 12 – 14 – 10), more and less preferred routes may be distinguished.

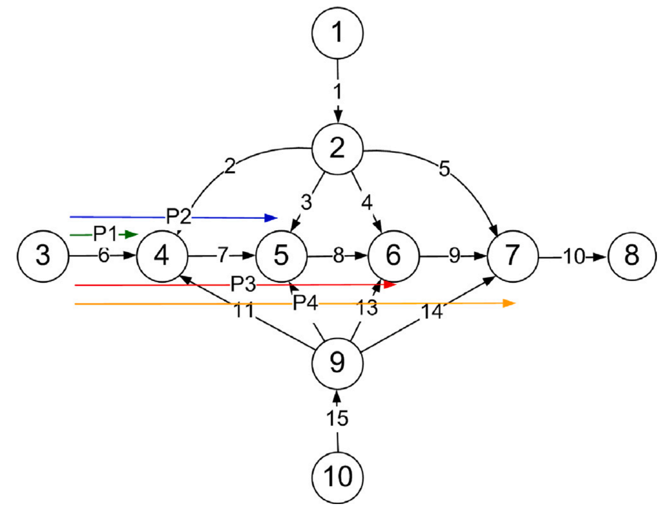

Fig. 2. Routes on which the participants bid in round 1 of the ACA. *Pi* denotes the route of player *i*.

Round 1. According to the above discussed considerations, in the first round, when all capacities are free and accessible, all players will aim for the cheapest source (S_2), located in node 3, considering the low-cost horizontal edges. The implied routes, on which the participants will place bids, are depicted in Fig. 2.

The ACA rules imply that the positive directional capacity of edge 9 (denoted by 9+) will be allocated to player 4 (P4) in the first step of the auction at the price of 0, since there is no competition for it (no other players submit bids for it), thus no overbidding arises. For the products 2+, 2–, 3+, 3–, 4+, 4–, 5+, 5–, 9–, 10+, 10–, 11+, 11–, 12+, 12–, 13+, 13–, 14+, 14–, 15+ and 15– the auction ends as well for round 1, since no bids are submitted for them. These products will be auctioned again in round 2. For the products 6+, 7+ and 8+ there is overbidding, thus prices will begin to rise (in steps of 0.05).

The first product for which the auction ends is 8+. As the price reaches 0.3, the route will be no longer profitable for P4, since considering the sum of the source costs (0.1), transfer costs (0.04) and the actual price of the route (3*0.3 – the prices of 6+, 7+ and 8+ are already at 0.3), it is no longer rational to maintain the bids for the route. As P4 withdraws its bids for the products 6+, 7+ and 8+ (9+ is already allocated to it), P3 is the only remaining active bidder, thus 8+ is allocated to it at the price of 0.3. The next product, for which the auction ends is 7+. In this case the price of 0.95 becomes too high for P2 (the source and transfer costs are equal to 0.11, while the cost of the route equals 1.9, and P2's willingness to pay is 2 units), and the product is allocated for P3 at the price of 0.95. Finally, P1 and P3 compete for the product 6+, which results in the final price of 2.9 units, when P3 withdraws, and the product is allocated to P1 at the price of 2.9.

In the following round, the remaining products are auctioned. Before round 2, the demands are updated. P1 already has a route (6+) via its demand (1 unit) may be completely fulfilled, so P1 does not submit any bids in the following. The rest of the players did not succeed in ensuring any connected route from any source to them, so their demand is unchanged.

Round 2. In round 2, the cheapest source S_2 is unavailable for the participants still active (since the product 6+ has been allocated to player 1 completely), thus all players aim for the next cheapest source, S_3. The course of events in round 2 is similar to round 1. First, 8– is allocated to player 2 at the price of 0 (no other bidders), then 9– and 10– are allocated to P3 at the price of 0.9, which is too high for P2 (P4 quitted earlier). As a result, P3 is able to cover its demand from S_3, and quits from the rest of the process.

Table 2
Results of the 3-round ACA process in the case of the simple example.

	Y_i^{ACA}	CT_i^{ACA}	CS_i^{ACA}	CC_i^{ACA}	UC_i^{ACA}	UR_i^{ACA}	UN_i^{ACA}
P1	1	0.01	0.1	2.9	4	3.89	0.99
P2	1	0.04	0.3	0.7	2	1.6	0.96
P3	1	0.02	0.2	3.05	3	2.7800	−0.27
P4	0	0	0	0	0	0	0

Round 3. In the final round, the remaining players P2 and P4 compete for the routes to S_1, which overlap in the product 1+. After 3+ and 5+ are allocated in the first step at the price of 0 to P2 and P4 respectively, the price of 1+ is increased to 0.7, where P4 quits (thus 1+ is allocated to P2 at the price of 0.7). As a result, P2 is able to cover its demand from S_1.

Summary of ACA results for the example. Let us evaluate the final results of the ACA process. First, let us consider P1 as an example. P1 managed to ensure a route via 6+ to S_1, fully covering its demand from this source. This implies the consumption utility $UC_1^{ACA} = 4$. The resulting utility of P1 in the case of the ACA (UR_1^{ACA}) is defined as described in Eq. (1), where CT_1^{ACA} denotes the total resulting transfer costs of P1 (= 0.01) and CS_1^{ACA} denotes the total resulting source costs of P1 (= 0.1).

$$UR_1^{ACA} = UC_1^{ACA} - CT_1^{ACA} - CS_1^{ACA} \tag{1}$$

In this case, $UR_1^{ACA} = 3.89$ units. This value has high importance, since it can be considered as the final evaluation of the received bundle of products by the participant. An allocation mechanism is considered to be efficient if it allocates the items to participants, who value them most (Börgers and Krahmer, 2015; Roughgarden and Talgam-Cohen, 2019), in other words the sum of UR values over the players denoted by UR_T^A in the case of the mechanism A, described by Eq. (2), may (an will) be considered as a measure for the efficiency of the allocation mechanism used. The lower index T stands for 'total'.

$$UR_T^{ACA} = \sum_i UR_i^{ACA} \tag{2}$$

In this particular case $UR_T^{ACA} = 8.27$.

In addition, we may also consider the net utility of P1 in the case of the ACA (denoted by UN_1^{ACA}), which is the value of UR_1^{ACA} decreased by the capacity payments (CC_1^{ACA}), equal to 2.9 in this case, implying a net utility of $UN_1^{ACA} = 0.99$ for P1.

If we repeat these calculations for all of the players, we may summarize the results as in Table 2, where Y_i^{ACA} denotes the resulting consumption value of player i, according to the final result of the ACA process.

We may observe two anomalies here. First, the resulting net utility of P3 is negative. The reason for this is that in the first round the products 8+ and 7+ have been allocated to P3 at the price of 0.3 and 0.95 respectively, but these products cannot be used in the following to constitute a continuous route for P3. At this point, the net utility of P3 is −1.25. In round 2, P3 receives the products −9 and −10 at the price of 0.9 each, which on the other hand allow P3 to buy gas from $S3$, implying a consumption utility of 3 units (and transfer and source costs of 0.02 and 0.2 respectively). Thus although the balance of round 2 is positive for P3 ($3 - 0.02 - 0.2 - 1.8 = 0.98$) it cannot fully compensate the loss implied by the results of round 1. The second anomaly is that even three rounds of auction are not able to cover the demand of P4, although the capacities of the network would allow this.

As a very simple measure of fairness, we may calculate the difference between the net utility value of the player who ended up with the highest UN, and the UN value of the player with the lowest resulting UN value. We denote this 'unfairness' measure with uF, and in this case $uF^{ACA} = 0.99 - (-0.27) = 1.26$.

A further important characterization of the allocation mechanism is the income of the auctioneer (IA^A in the case of the mechanism A), which is equal to the sum of capacity payments, as described by Eq. (3). In this case $IA^{ACA} = 6.65$.

$$IA^{ACA} = \sum_i CC_i^{ACA} \tag{3}$$

In addition, the allocation may be characterized by the ratio of allocated capacities (r_{AC}^A), and by the ratio of capacities, which are also used by participants after the allocation ($r_{AC}^A \leq r_{AC}^A$ – the upper index A refers again to the mechanism A). In this case $r_{AC}^{ACA} = 0.3333$, while $r_{UC}^{ACA} = 0.1667$.

Table 3
Routes of players considered in the proposed example. Every route is a set of products connecting a source to the player.

R_1^1	6+	R_1^3	6+, 7+, 8+
R_2^1	10−, 9−, 8−, 7−	R_2^3	10−, 9−
R_3^1	1+, 2+	R_3^3	1+, 4+
R_4^1	15+, 11+	R_4^3	15+, 13+
R_1^2	6+, 7+	R_1^4	6+, 7+, 8+, 9+
R_2^2	10−, 9−, 8−	R_2^4	10−
R_3^2	1+, 3+	R_3^4	1+, 5+
R_4^2	15+, 12+	R_4^4	15+, 14+

2.3.4. Notes on modeling of the ACA bidding strategy

In the proposed model of the ACA process, regarding the bidding strategies we assume that players place bids only for routes, which allow exactly to cover their demand (in this simple example, this means only one route in each case). One may ask that why a participant does not allow itself to have alternatives and submit bids for more routes, as long as the prices are low. This is a valid question, and a more complex strategy might be modeled as well, where the player starts with bidding on multiple routes, with more overall capacity than needed to cover its demand, and decreases the number of considered possible routes as the prices rise (or/and as capacities are potentially allocated). However such more complex strategies call for more detailed assumptions, parameters and more sophisticated algorithms, so we restrain ourselves to the simplification of 'minimal bidding' in this work, and leave the modeling of complex bidding in the ACA for further studies.

2.4. Bidding and capacity allocation in the convex combinatorial auction (CCA) framework

The principles of the CCA are as follows. Each player may place multiple bids on multiple routes, each described by the route and the quantity (i.e. a route–quantity ordered pair), and the evaluation of the player. Considering all bids of all players, the algorithm allocates a convex combination of the submitted bids to each player, which maximizes the resulting value of the accepted bids, while taking into consideration the network capacity constraints as well. CCA calculations may be formalized as a linear optimization program, the details of which are described in Section 2.6. Capacity payments in this case are determined according to the Vickrey–Clarke–Groves (VCG) principle (Vickrey, 1961; Groves et al., 1973; Scherr and Babb, 1975), according to which each participant pays the sum equal to the harm they cause to other players by participating. These values may be calculated easily, by iterative re-runs of the CCA, excluding players one-by-one from the process.

2.4.1. The CCA process in the case of the simple example

The CCA framework uses a route-centered formalism. Our first task in this approach is to define the routes of players, via which they are potentially able to transport the gas for themselves. The considered routes for player 1, 2, 3 and 4 are summarized in Table 3.

Table 4
Bids submitted in the CCA by participants in the case of the simple example. The first parameter is the quantity of the bid for the given route ($q^i_{j,k}$), while the second parameter is the value of the bid ($p^i_{j,k}$).

$B^1_{1,1}$	(1, 3.89)	$B^3_{1,1}$	(1, 2.87)
$B^1_{2,1}$	(1, 3.76)	$B^3_{2,1}$	(1, 2.78)
$B^1_{3,1}$	(1, 3.66)	$B^3_{3,1}$	(1, 2.66)
$B^1_{4,1}$	(1, 3.3.56)	$B^3_{4,1}$	(1, 2.56)
$B^2_{1,1}$	(1, 1.88)	$B^4_{1,1}$	(1, 0.86)
$B^2_{2,1}$	(1, 1.77)	$B^4_{2,1}$	(1, 0.79)
$B^2_{3,1}$	(1, 1.66)	$B^4_{3,1}$	(1, 0.66)
$B^2_{4,1}$	(1, 1.56)	$B^4_{4,1}$	(1, 0.56)

Table 5
Acceptance indicators of the submitted bids resulting from the CCA in the case of the simple example.

$B^1_{1,1}$	1	$B^3_{1,1}$	0
$B^1_{2,1}$	0	$B^3_{2,1}$	0
$B^1_{3,1}$	0	$B^3_{3,1}$	0
$B^1_{4,1}$	0	$B^3_{4,1}$	1
$B^2_{1,1}$	0	$B^4_{1,1}$	0
$B^2_{2,1}$	0	$B^4_{2,1}$	1
$B^2_{3,1}$	1	$B^4_{3,1}$	0
$B^2_{4,1}$	0	$B^4_{4,1}$	0

Table 6
Products allocated by the CCA to players in the case of the simple example (in general, the acceptance indicators are $\in [0,1]$).

Player	Allocated products	Player	Allocated products
P1	6+	P3	13+, 15+
P2	1+, 3+	P4	10−

2.4.2. Bid format in the CCA auction

As mentioned earlier, in the CCA framework participants of the auction may submit bids for route–quantity pairs. Let us denote the kth bid for route j of player i with $B^i_{j,k} = (q^i_{j,k},\ p^i_{j,k})$ where $q^i_{j,k}$ is the quantity of the bid, and $p^i_{j,k}$ is the price offered for the route–quantity pair on the route in question. Let us note that in the case of the simple example, it is not straightforward why the participants would ever submit multiple bids for one route. Indeed, in the case of this example, the demand of participants is characterized only by a quantity (which is universally equal to 1), and a price value, describing the willingness to pay for the given amount. This may be regarded as a one-step inverse demand function. In the following however, we will use a more detailed description of consumer demand and price elasticity, considering multi-step demand functions (see Section 2.5.1). If these multi-step inverse demand functions are considered, multiple bids for single routes are reasonable to submit.

2.4.3. Submitted CCA bids in the case of the simple example

In the case of the simple example, the players submit the bids for the CCA summarized in Table 4.

2.4.4. Outcome of CCA results for the example

The result of the CCA process is an acceptance indicator vector for each player, which describes a convex combination of their submitted bids. Bids with nonzero coefficient are partially or fully accepted. In the case of this simple example only full or zero acceptance is arising (however, in general this is not the case).

The results in Table 5 imply the product allocations summarized in Table 6.

Summary of CCA results for the example. Similarly to the ACA case, let us evaluate the final results of the CCA process. The results are summarized in Table 7, where Y_i^{CCA} denotes the resulting consumption value of player i, according to the final result of the CCA process.

Table 7
Results of the 3-round ACA process in the case of the simple example.

	Y_i^{CCA}	CT_i^{CCA}	CS_i^{CCA}	CC_i^{CCA}	UC_i^{CCA}	UR_i^{CCA}	UN_i^{CCA}
P1	1	0.01	0.1	0.32	4	3.89	3.57
P2	1	0.04	0.3	0.1	2	1.66	1.56
P3	1	0.04	0.4	0	3	2.56	2.56
P4	1	0.01	0.2	0.22	1	0.79	0.57

Table 8
Values of characteristic measures for the CCA results in the case of the simple example network.

UR_T^{CCA}	8.9	r_{AC}^{CCA}	0.2
uF^{CCA}	3	r_{UC}^{CCA}	0.2
IA^{CCA}	0.64		

The other measures calculated in Section 2.3.3 for the ACA case are detailed for the CCA results in Table 8.

If we compare Tables 7 and 8 to Table 2, and the other ACA results, we can make the following observations. First, the CCA does not allocate capacities, which are later unused by the participant. This is due to the fact, that in the CCA, bids are submitted (and accepted) for routes, not line capacity products. Second, the value of UR_T^A is higher in the case of the CCA (8.9 vs the value of 8.27 of the ACA case), which shows that the CCA allocation is more efficient in this case. Third, the income of the auctioneer is significantly lower in the CCA case (0.64 compared to 6.65 in the ACA case), also implying higher net utility values in the case of the CCA.

Let us emphasize that the aim of this simple example was to demonstrate the differences in the operation and results of the two allocation methods, and further calculations are needed to draw any conclusions about trends describing the differences between the two methods in general. From an other perspective, the presented example was also designed do shed light on some potential flaws of the ACA (allocated unused capacities, participant with no resulting access to sources), but it does not tell anything about how typical these phenomena are in general.

To compare the performance of the two methods in general, we use simulation. We generate randomized problems, perform both the ACA and the CCA, and compare the results. In some aspects (e.g. the description of demand and demand elasticity), these problems will be defined in a more complex form to get more general results. In the following subsection we describe the details of the modeling formalism and algorithms, which serve as basis for the simulations.

2.5. Detailed description of the modeling formalism used in the simulations

In the proposed simple example, a very simple demand description was used. In the simulations, we generalize the description of the demand, in order to account for demand elasticity and the implied strategic decisions of players.

2.5.1. Consumer demand

In the simulations, we use piecewise constant inverse demand curves for the description of demand elasticity as depicted in Fig. 3. Each piecewise constant part has two parameters: A price (P) and a consumption quantity (Q). In this formalism P_j^i denotes the price level of the jth step of the inverse demand function of player i. The inverse demand curve describes, that the consumer i is ready to pay a relatively high price P_1^i for the first Q_1^i units of gas, a somewhat lower ($P_2^i < P_1^i$) price for the next additional Q_2^i units, and even lower price (P_3^i) for additional quantities, up to the total demand.

The parameters of the demand function depicted ion Fig. 3 are summarized in Table 9.

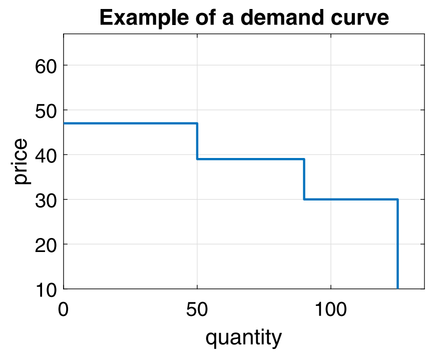

Fig. 3. An example of an inverse demand function used in the simulations.

Table 9
Parameters of the inverse demand function depicted in Fig. 3.

P_1^1	47	Q_1^1	50
P_2^1	39	Q_2^1	40
P_3^1	30	Q_3^1	35

2.5.2. Modeling of optimal bidding in the ACA framework

In this subsection we describe the details of calculations used in the simulation of the ACA mechanism.

Calculations between the steps of the ACA. As we already mentioned earlier, we assume that players have perfect information about source and transfer costs of the network (and of course about the actual prices of capacities in the auction), but they have no information about other player's utility functions. In other words, in the current simulation framework players do not make any speculations of other player's bids to optimize their own bidding strategy, they just consider the network parameters, the actual capacity prices in the particular step of the actual auction, and their own inverse demand function to determine their bids for the actual bidding step.

In the calculations we distinguish between *already allocated capacities*, and *potential capacities* of *capacities under auction*. Capacity products already allocated to player i are denoted by AAC^i, while the capacity products actually under auction are denoted by CAU. The size of these vectors depend on the number of edges in the actual network: AAC^i, $CAU \in \mathcal{R}^{2m}$, where m is the number of edges. In a general case (e.g. if we are not considering the first step of the first bidding round), players may hold already allocated capacities. To be more precise, we assume that after the first round finishes, each player assigns flows to the capacities, he/she obtained at the end of the auction to maximize its utility via fulfilling the demand in the respective consumer node. However, after the determination of these flows, some capacities may remain unused — these are considered as free-to-use *already allocated capacities* in the following (since the payment for them has already been completed). A simple example for such a scenario may be considered, if a player bids for several components (e.g. 2 line capacities) of a route, receives one of them in the early steps of the auction, but during the following steps the price of the other one increases so much that it does not make sense for the player to maintain its bid anymore. This way the player will be probably not able to assign any flow to this single capacity, thus he/she will have unused capacity at the end of the first round, which will be considered as *already allocated capacity* in the second bidding round. Capacities still under auction are considered differently, since the payment for them has not been completed yet (using them implies additional cost, since they must be acquired first at their actual price).

To exactly determine the actual bids of a player in a given auction step, we use the principle of *optimal potential flows*. This approach means the following. In each step of any auction round (1 2 or 3), players determine their optimal flows, which maximizes their resulting utility UR, assuming that they will receive the capacities on which they place bids. As discussed earlier, the resulting utility (UR) may be calculated as the utility of consumption at the consumer node UC, minus the cost of transfers (CT) and sources (CS), minus the payment for the capacity rights (CC). In this calculation the player takes into account that flows planned on already allocated capacities do not imply furthers costs in addition to the transfer cost, in contrast flows planned on potential capacities imply extra cost in addition to c^t, namely the actual capacity price (which must be paid if the products corresponding to the edge are allocated in the actual step).

In our model, players submit bids according to these optimal actual flows, namely we assume that they submit a bid vector, which is able to ensure the flows on potential capacities calculated in the optimal actual flows problem.

Formal determination of optimal flows in the ACA framework. The formalism of the approach is the following. Let us consider a linear programming problem, where f_{aac} denotes the vector of flows on already allocated capacities, and f_{cau} denotes the flows on capacities under auction (potential capacities), $L \in \mathcal{R}^{n_s}$ denotes the vector of inlet values (n_s stands for the number of source-nodes), while Y describes the consumption.

$$\max \; UR(x) \quad \text{where} \quad x = \begin{pmatrix} f_{aac} \\ f_{cau} \\ L \\ Y \end{pmatrix}$$

$$s.t \quad \begin{array}{c} 0 \le f_{aac} \le AAC^i \\ 0 \le f_{cua} \le CUA \\ 0 \le f_{cua} \le PB \\ 0 \le L \\ 0 \le Y \le \overline{Y} \end{array} \qquad A_{eq}x = 0 \tag{4}$$

f_{aac} may be decomposed as

$$f_{aac} = \begin{pmatrix} f_{aac}^+ \\ f_{aac}^- \end{pmatrix} \tag{5}$$

where f_{aac}^+ stands for the positive directional flows (according to the directions of edges) and f_{aac}^- denotes the negative directional flows. f_{cau} has the same structure as f_{aac}^-. The vectors f_{aac}^+, f_{aac}^-, f_{cau}^+, f_{cau}^- are $\in \mathcal{R}^m$, where m is the number of edges.

In addition, while the optimal actual flows in the first step of any round are bound only by the value of the capacity products under auction, in the following steps, the bids submitted in previous steps (PB) also bound these values (since bids may be only decreased in the next step of any round of the ACA).

To each step of the inverse demand function of the respective player, we assign a variable y_k, so $Y \in \mathcal{R}^{n_p}$, where n_p is the number of steps (in the case of the simulations we assume $n_p = 3$, as depicted in Fig. 3). The components of Y are bounded by the Q^i parameters of the inverse demand functions as described by Eq. (6).

$$\overline{Y} = \begin{pmatrix} Q_1^i \\ Q_2^i \\ Q_3^i \end{pmatrix} \quad \text{so} \quad Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \le \begin{pmatrix} Q_1^i \\ Q_2^i \\ Q_3^i \end{pmatrix} \tag{6}$$

where the Q_1^i values depend on the actual player i.

The equation $A_{eq}x = 0$ formalizes the nodal balances. For each node, the inlets plus the inflows must be equal to the outflows plus the consumption. Using the variables in x, and the network topology, the n equations corresponding to the rows of the A_{eq} matrix may be easily derived (n is the number of nodes).

It is clear that UC, CT and CS are linear functions of the variables of x: The coefficients of the linear functions may be derived from the P_j^i parameters of the inverse demand functions, the c_j^t transfer costs of edges and from the source costs (c^s). As mentioned earlier, in the case of optimal flow calculations, the costs of already allocated edges is equal to the transfer cost, while the costs of edges corresponding to capacities under auction is determined as the sum of the transfer cost and the actual auction price.

After each step, according to the potential allocations (if for any product, if there is no overbidding, the auction for that product is finished, it will be allocated at the actual price to the actual bidders), the values of AAC_i (for each player) and CUA are updated.

Calculations between the rounds of the ACA. After a round of ACA auction has finished, each player calculates the optimal flows on the capacities actually allocated to him/her. This can be done easily by solving the problem (4), under the assumption that there are no capacities under auction, only already allocated capacities. According to the results of this calculation, every player performs the two following operations:

1. First, the player in question determines the quantity ensured by the calculated flows. As this quantity is ensured for him/her in the following, the player updates its inverse demand function for the remaining auction steps accordingly (demand is reduced by the already accessible quantity).
2. Second, the player divides the capacities allocated to him/her into two groups: Capacities which are used by the flows are considered 'out of the game' in the following, thus they are allocated to the flows fixed after this round. Allocated capacities on the other hand which are not used in the flows are considered as already allocated capacities (AAC_i) in the next auction round (payment for them has been already completed, thus they may be used in the design of potential optimal flows in the following).

2.6. The formal details of the CCA

In the CCA setup we suppose that players submit bids for route–quantity pairs, according to the principle that in the outcome of the auction a convex combination of their submitted bids will be assigned to them. This assumption allows bidding for alternative routes: If a consumer needs 1 unit of gas and there are 2 alternative sources in the network, corresponding to two different routes, he/she can submit two bids for the capacity licenses of the two distinct routes, both with the quantity of 1 unit. At the end of the auction a convex combination of the two bids will be assigned to him, which means that he/she will not get more network capacity towards the sources than 1 unit, but this maximally 1 unit may be composed of arbitrary proportion of the two routes. Of course this line of thought applies for multiple potential alternative sources with multiple access routes to each as well.

2.6.1. Variables and constraints of the clearing problem

The variables of the linear program underlying the CCA are the acceptance indicators of bids. $x^i_{j,k} \in [0,1]$ denotes the acceptance indicator of the bid $B^i_{j,k}$.

Network constraints. To formulate the constraints which describe the limited capacity of pipelines, we need to decompose the routes considered in the auction to their components — to edges which correspond to capacity products. Furthermore we take into account the possibility that counter-directed flows cancel each other.

Let us denote the set of (directed) edges in the network with E, while $e \in E$ denotes a single edge. Each route j (of player i) may be represented as an $R^i_j \subseteq E$ subset of edges, where each element is signed, according to whether the direction of the route coincides with the direction of the included edge or not.

Let us suppose furthermore that edge e_m has different maximal capacity in the positive and negative direction (think of one-directional pipelines), denoted by $\bar{q}^+_m$ and $\bar{q}^-_m$ respectively.

In this case the maximal capacity constraints may be formulated as described in Eq. (7), where $q^i_{j,k}$ denotes the quantity corresponding to bid $q^i_{j,k}$ and $s^i_{j,m}$ is an indicator variable, which equals to 1 if edge e_m has positive sign in route j of player i, and -1 otherwise.

$$\sum_{i,j,k} \sum_{e_m \in R^i_j} s^i_{j,m}\, x^i_{j,k}\, q^i_{j,k} \le \bar{q}^+_m \qquad \sum_{i,j,k} \sum_{e_m \in R^i_j} -s^i_{j,m}\, x^i_{j,k}\, q^i_{j,k} \le \bar{q}^-_m \qquad \forall m \quad (7)$$

In this paper we assume unlimited quantity of gas at sources, however if one aims to take into account source constraints as well, then the implied constraints may be derived very similarly by constraining the total outflow of the edges connected to the source in question.

Convexity constraint. By definition, the auction assigns to each player a convex combination of his/her submitted bids. This consideration is formalized as

$$\sum_{j,k} x^i_{j,k} \le 1 \qquad \forall i \quad (8)$$

2.6.2. The optimization problem of the CCA framework

The objective of the optimization process is to maximize the total value of the accepted bids, under the previously detailed constraints. The parameter $p^i_{j,k}$ in Eq. (9) denotes the value of bid $B^i_{j,k}$.

$$\begin{array}{l} \max_x \quad x^i_{j,k} p^i_{j,k} \qquad \text{s.t.} \\ \sum_{i,j,k} \sum_{e_m \in R^i_j} s^i_{j,m}\, x^i_{j,k}\, q^i_{j,k} \le \bar{q}^+_m \quad \forall m \\ \sum_{i,j,k} \sum_{e_m \in R^i_j} -s^i_{j,m}\, x^i_{j,k}\, q^i_{j,k} \le \bar{q}^-_m \quad \forall m \\ \sum_{j,k} x^i_{j,k} \le 1 \quad \forall i, \quad 0 \le x^i_{j,k} \le 1 \quad \forall (i,j,k) \end{array} \quad (9)$$

The above problem falls into the class of linear programming problems. Let us recall that regarding the ACA framework, we only used linear programming to model optimal bidding behavior, but the allocation itself in that case has been performed by a logical algorithm described in Section 2.3. In contrast, in the case of CCA, the allocation process itself relies on solving a linear programming problem. Let us point out here however that in other auction framework related to energy economics as electricity auctions linear programming, and even more computationally demanding programming problems (as integer and quadratic programming) are routinely used in practice (see e.g. Madani, 2017).

2.7. Payments in the CCA

After the optimization process has been completed and the bid acceptance ratios have been determined, the payments of the players have to be calculated. To determine payments in the proposed framework, we use the Vickrey–Clarke–Groves (VCG) mechanism (Vickrey, 1961; Groves et al., 1973; Scherr and Babb, 1975), which charges each individual the harm they cause to other bidders with their participation. The VCG mechanism gives bidders an incentive to bid their true valuations, by ensuring that the optimal strategy for each bidder is to bid their true valuations of the items. It is a generalization of a Vickrey auction (Vickrey, 1961) for multiple items. The harm is measured via the total value of accepted bids for the rest of the players. After the optimization problem (9) has been solved, the total value of accepted bids for all players is evaluated (reference case). Following this, to determine the payment of any participant i, a modified version of the problem (9) is solved, excluding the bids of i. According to the solution of this modified problem, the total value of accepted bids of other participants may be evaluated and compared with the reference case. The VCG payment of player i is the difference of the two values.

Table 10
Invariant parameters used in the simulations.

Par.	Value	Par.	Value
$\overline{q}^{min}$	10	$\overline{q}^{max}$	90
c^t_{min}	3	c^t_{max}	11
c^s_{min}	20	c^s_{max}	30

2.8. Simulations

A computational approach has been used to compare the performance of the ACA end CCA methods. Random setups have been generated and the two capacity allocation processes were simulated in each case.

Each setup was generated as follows. Input parameters were:

- The number of vertices (nodes) n
- The number of edges m
- The number of sources n_s
- Upper and lower bounds for edge capacities $\overline{q}_{max}$ and $\overline{q}_{min}$
- Upper and lower bounds for pipeline transfer costs c^t_{max} and c^t_{min}
- Upper and lower bounds for source costs c^s_{max} and c^s_{min}

In the first step, graph of the network was generated. The first edge was placed randomly, the second was placed randomly among unconnected node-pairs and so on, until all n_e edges have been placed (see Erdős–Rényi graphs Erdős and Rényi, 1960). At the end, connectedness and planarity of the resulting graph was checked, and if any property did fail, the process was started over. Once the graph proved to be appropriate, n_s sources were picked at random from the set of nodes, and they were defined as source nodes (the rest were considered as consumer nodes).

In the second step, the parameters of edges were determined. Transfer costs for edges were assumed to be identical in any direction, thus m random values from a uniform distribution between $\overline{q}_{min}$ and $\overline{q}_{max}$ were picked, and rounded to the closest integer value to determine maximal edge capacities, and similarly, random values from a uniform distribution between c^t_{min} and c^t_{max} were picked, and rounded to the closest integer value to determine edge transfer costs. Furthermore, n_s random values from a uniform distribution between c^s_{min} and c^s_{max} were picked, and rounded to the closest integer value to determine the source costs for source nodes.

Following this step, the (maximum) 10 cheapest source–consumer routes were determined for every consumer, considering transfer costs. The minimal and maximal values of these routes (c^{Route}_{min} and c^{Route}_{max}) were calculated from the results.

Inverse demand functions were determined as follows. We assumed the three-step piecewise constant form as in Section 2.5.1, where the quantity (i.e. the width) of each step was determined by picking a random integer value from the interval [10, 50]. The price of each step was determined by picking a random integer value from the interval $[c^{Route}_{min} + c^s_{min}, 1.4 \cdot c^{Route}_{max} + c^s_{max}]$.

We considered various network sizes each with different node (vertex), edge and source node numbers (n, m, n_s), but the other parameters were fixed as summarized in Table 10. For each network size, 1000 setups were generated, on which the ACA (with 3 rounds) and CCA methods have been evaluated.

The considered network configurations (NW) are detailed in Table 11.

3. Results

Tables 12 and 13 summarize the simulation results. Table 12, holds the average total resulting and net utility values (UR and UN respectively) of the players in the ACA and in the CCA case, the average income of the auctioneer (IA) and the average value of the introduced simple unfairness measure (uF).

Table 11
Network configurations. n denotes the number of vertices (nodes), m is the number of edges, while n_s stands for the number of source nodes.

NW	n	m	n_s
1	6	8	1
2	9	12	2
3	15	20	3
4	20	30	4

Regarding the notation, UR^A and UN^A denote the sum of resulting/net utility values of players in the case of the auction method A, i.e.

$$UR^A = \sum_i UR^A_i, \quad UN^A = \sum_i UN^A_i$$

Additional details of the distribution of UR^{ACA}, UR^{CCA} and of UN^{ACA}, UN^{CCA} may be found in Appendix.

In Table 13, r_-^{ACA} stands for the negative utility ratio of the ACA in %, in other words, the proportion of cases, when the process resulted in negative total UN value for the players. As we have seen in Section 2.3.3, in the ACA framework it is possible that in the process of capacity allocation such capacities will be allocated to players, which will be useless for them as later they are determining their optimal flows on the capacities assigned to them. Capacity payments for these unused capacities which do not form a full route at the end of the process imply negative net utility components for these players. If these negative components outweigh the positive ones in the context of all players, UN^{ACA} may be negative as well.

$r_{CCA>ACA}$ denotes the ratio of cases in %, when the CCA proved to be more efficient than the ACA (i.e. $UR^{CCA} > UR^{ACA}$). r^{ACA}_{AC} and r^{CCA}_{AC} stand for the ratio of allocated capacities in %. Similarly, r^{ACA}_{UC} and r^{CCA}_{UC} denote the ratio of used capacities in %, i.e. the proportion of capacity products, which are used after the allocation by players to transport gas.

4. Discussion

4.1. Evaluation of the results

One of the most importantly required characteristics of a capacity allocation method is the efficiency in the terms of the total resulting utility (UR) of players. This corresponds to the less formal principle of 'items (capacities) shall be allocated to those who value them most'. As the simulation results show, regarding this aspect, the proposed CCA method outperforms the ACA in the majority of cases. The expected total resulting utility of players over the analyzed high number (1000) of random scenarios was 1.5–4.05% higher in the case of CCA.

As the CCA method always assigns network capacities to players in a way which ensures connected paths, and is able to consider multiple alternative routes, it seems reasonable to presume that these properties of the method result in higher gains in the case of larger networks and more consumer–producer pairs. This explanation is supported by Table 12, which shows that the average relative difference in the efficiency increases with the network size (from 1.5% to 4.05% as NW is increased from 1 to 4).

If we analyze the results in the terms of allocation efficiency, we can see that the ACA method always produces a higher allocation rate ($r^{ACA}_{AC} > r^{CCA}_{AC}$), but the capacities allocated this way can be only partially utilized by the players (see the r^{ACA}_{UC} values and their relation to the r^{ACA}_{AC} values). In contrast, if we consider the utilized allocated capacities, the CCA method performs better in every case ($r^{CCA}_{UC} > r^{ACA}_{UC}$).

Regarding the resulting relatively low values for the utilization of network capacities, let us note that while real-world networks are engineered, thus the network capacities match the expected flows in the network (no extra capacities are present in most of the time), the

Table 12
Simulation results 1: Values of UR, UN, IA and uF values for the ACA and the CCA case. Average results of 1000 simulations.

NW	UR^{ACA}	UR^{CCA}	UN^{ACA}	UN^{CCA}	IA^{ACA}	IA^{CCA}	uF^{ACA}	uF^{CCA}
1	7770.94	7886.32	1919.08	3338.94	5851.86	4547.38	1650.02	1634.75
2	14888.16	15267.13	5661.71	8228.40	9226.45	7038.73	3251.28	3111.36
3	25856.58	26714.23	7525.00	12752.69	18331.58	13961.55	4870.92	4031.61
4	38523.03	40083.05	11684.11	18617.99	26838.92	21465.06	5846.79	4628.12

Table 13
Simulation results 2: The ratio of results in % with negative total utility for the ACA process (r_{-}^{ACA}), the ratio of cases, when the CCA process outperformed the ACA ($r_{ACA>CCA}$), and the ratio of allocated and used capacities (r_{AC} and r_{UC}) for the ACA and the CCA process.

NW	r_{-}^{ACA}	$r_{CCA>ACA}$	r_{AC}^{ACA}	r_{AC}^{CCA}	r_{UC}^{ACA}	r_{UC}^{CCA}
1	23.5	55.7	45.5	24.1	23.2	24.1
2	9	79.9	46.9	28.9	27.7	28.9
3	14.4	93.5	49.2	27.4	25.5	27.4
4	8.8	95.8	49.8	27.3	25	27.3

randomly generated networks do not follow this principle. In the case of the simulated random networks it is possible e.g. that one node with a relatively low-capacity inlet edge from a source has high-capacity outlet edges, which cannot be utilized in any case because of the bottleneck in the inflow.

If we analyze the income of the auctioneer, the ACA method results in 25%–31% higher values. This shows that (based on the presented simulation results) the CCA method is not the best choice if one aims to maximize the auction incomes. Let us however emphasize that in the case of ACA, the capacity payments partially correspond to products, which are of no use in the final evaluation for the player. In the case of real world applications, these unused capacities may be possibly traded in secondary markets, where they may generate additional income (and utility) for the participant to whom they have been allocated during the primal auction process. Let us note however, that these unused capacities may exactly represent the possible reason for individual consumers, why they do not participate in the ACA-based allocation, but rather leave the delivery of gas to dominant (outside) entities. Bidding in the ACA framework for individual (especially smaller) consumers is not simple. Even if we use online optimization tools to determine optimal bidding in the ACA process, as the proposed model assumes, we may easily end up with unused capacities. On the other hand, the lower income of the auctioneer (i.e. lower payments for capacities) also contributes to higher net utility values in the case of the CCA. Total UN values are 45%–74% higher in the case of the CCA.

Regarding the maximal difference between the maximal and the minimal (net) utility among players, as a simple measure for fairness, simulation results show that the CCA method results in a more fair allocation. The unfairness measure is higher by 1%–26% in the case of the ACA, showing an increasing tendency as the size of the network is increased.

In the proposed model, the inlet limits of sources have not been considered. This raises a question, which reaches beyond the scope of the current article. If the inlet of the sources may arise as a bottleneck, source capacities have to be allocated as well. To avoid scenarios where a participant may have a route to a source but not enough inlet from the source to fill the pipeline (or vice versa, have a source, but unable to transport it without available network capacity), the allocation of sources may be carried out in the same process as the pipeline capacity allocations. Regarding furthermore the routes of participants to the sources, in the current simulation we used simple assumptions to determine the set of routes in the CCA process, for which the players place their bids. In potential real-life applications of the process, these set of routes may be naturally defined by players (e.g. with a maximal number of routes).

4.2. Possible practical future applicability of the method

Regarding the important question of the possible future availability of the method, several other studies have to be performed as well.

First, as we have seen, the CCA method does not proves to be universally more efficient compared to the ACA method. It is straightforward to ask how the efficiency of the CCA method depends on network topology and parameters, and more importantly, how would these methods perform on realistic models, e.g. in the case of the Eurasian gas network, the structure of which is known. Let us here note that although the topology and transfer capacities of realistic networks are known, and there is also data available about yearly consumption values, the estimation of the inverse demand functions used in this model (which describe the subjective evaluation of natural gas in the context of demand-flexibility) is not straightforward, and requires further research in the case of potential real-world applications.

Although the introduction of the CCA method was the first step in the process of this analysis, several more studies of both theoretical and computational nature will be required to properly characterize the practical applicability of the CCA method. However, we must emphasize that according to the presented analysis, it seems that there may be other capacity allocation methods, which may provide better incentive for the participation of consumers, compared to the current ACA scheme. In particular, the shortcomings of the European capacity allocation methods might stem from other aspects as well in addition to the intrinsic properties of the capacity allocation method used. It is possible that changing the allocation mechanism might not be the easiest and most convenient way for addressing the shortcomings.

Second, the current modeling studies considered only consumers as participants of the auction, which is unrealistic in the current regulation framework. If we consider a setup where consumers may bid for routes, but they are also ready to take the gas at their 'doorstep' as well (for reasonable price), the behavior of producers wishing to market their gas with delivery for maximal profit may be also included in the model. In this case, producers will also be present among participants bidding for routes/capacities and more general results will be obtainable. Let us however point out that while in the case of consumers, under the assumption of the proposed piecewise constant inverse-demand functions and constant source prices, the derivation of optimal biding strategy was quite straightforward. If we wish to include strategic producers in the model, it is plausible to assume that they aim to sell their product at the highest possible price, thus we need to model the price-bargaining related to gas transactions as well. In addition, regarding realistic models, it is not trivial to determine the information the producers have access to about the inverse-demand functions of consumers, which would be required to determine the optimal bidding strategy in the capacity allocation process.

Including producers as bidders in the capacity allocation scheme may be considered as a necessary prerequisite for modeling realistic scenarios, like the European network, where suppliers are also important participants of the transportation, and capacity allocation processes. In fact, it has been pointed out that a dominant supplier is able to acquire most of the transfer capacities without the emergence of real competition (Takácsné Tóth et al., 2017). In our future work we plan to address this issue and also include producers as participants of the capacity allocation process.

The capacity allocation for individual consumers in the network could also fit wider, conceptual policy-related topics. If an efficient,

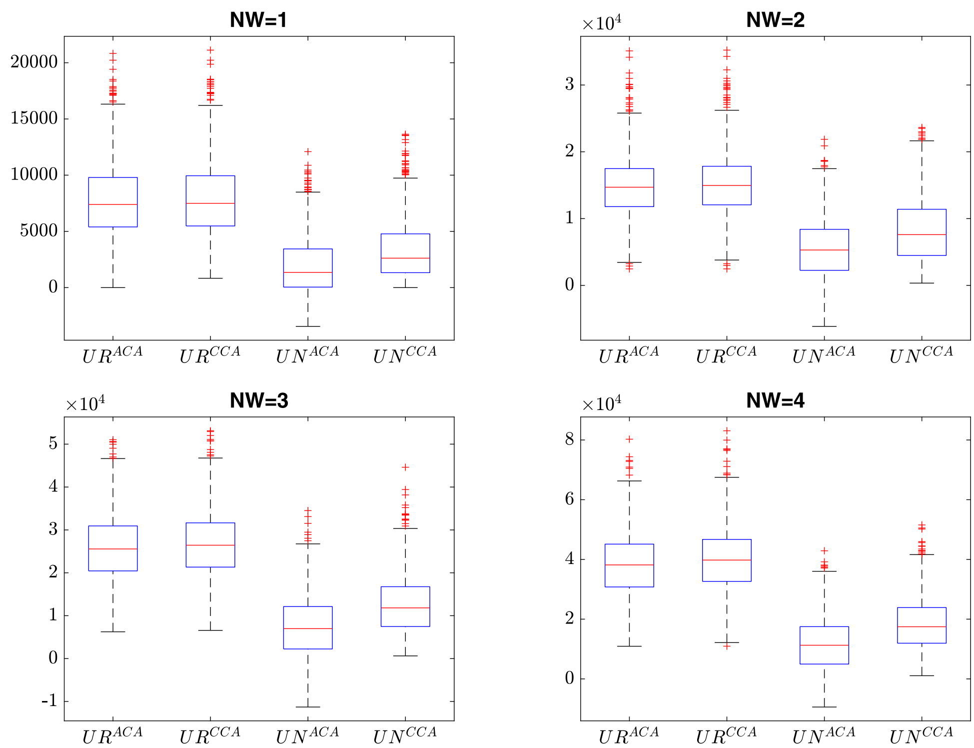

Fig. 4. Distributions of UR and of UN over the 1000 simulations in the case of $NW = 1, 2, 3$ and 4 in the case of ACA and CCA. In the box plots, the central mark is the median, while the edges of the box are the 25th and 75th percentiles respectively. The whiskers extend to the most extreme data points which are considered not to be outliers, and the outliers are plotted individually with red crosses.

generally accepted capacity allocation method could be developed for natural gas networks, in which consumers are motivated to allocate routes for themselves and not to accept gas deliveries on national border points, it may open the potential to reform the trading scheme in the EU. A possible new scheme could be proposed based on the principle of infrastructure independence. In this new scheme, it would not be possible for outside entities to allocate capacity on natural gas network inside the EU. Gas hubs on EU border points could be established, where external producers were able to sell their product on a public, universal price, while internal consumers could allocate paths for themselves in the infrastructure inside the EU to access these hubs. This way, the EU could make a step towards representing itself as a more integrated and independent entity on the global energy market, individually handling and allocating its internal transfer capacities, and acting as a more autonomous global player. Ideas pointing towards such directions resurfaced in the light of the recent invasion and war in Ukraine.

Naturally, the integrated-EU energy policy model has also its pitfalls. Such a framework would possibly be regarded by several member states as curtailing their play-field in establishing individual bargains with gas suppliers (sometimes including other geopolitical elements as well). In the current context however, we only aim to propose a tool which could help to overcome some technical obstacles (the assumed de-motivation of countries to participate in capacity allocation auctions, because of the currently used ascending clock method), in the case an agreement is formed in these points.

Lastly, regarding the actuality of the proposed method, we must note that on the one hand, the congestion problem seems an issue of limited significance at the European gas networks nowadays, and might be even less of an issue as gas consumption might fall due to decarbonization goals, or geopolitical considerations (e.g. cut gas imports from Russia in line with sanctions). On the other hand, the increasing integration of renewable sources in electricity networks increases the proportion of non-controllable sources and thus uncertainty in electricity production. In a more uncertain scenario, the value of controllable power plants with low reaction time may increase, since they are able to provide system level reserves. Modern gas-fueled units are capable of relatively fast start-up and may serve as important reserve capacities in the electricity system, thus it seems that the importance of natural gas will increase in the foreseeable future.

5. Conclusions and future work

In this work we proposed a convex combinatorial auction method (CCA) for the allocation of capacities in capacity-constrained networks, where prior given transfer and source costs apply and the evaluation of routes is subjective by the players. The subjective evaluation originates from the individual flexible demand, described by the assumed piecewise constant inverse demand functions. We compared the proposed method with a 3-round ACA allocation method, which aims to model the current practice of capacity allocation of natural gas networks in the EU.

Let us summarize the key points of our findings.

- In the proposed CCA framework the players place their bid for route–quantity pairs, in contrast to the ACA framework, where quantity bids are placed on multiple individual capacity products simultaneously (considering their actual price in the ACA process). The clearing in the CCA is carried out via the solution of a linear program, which requires low computational effort.
- The ACA framework has usually multiple rounds (in the simulations, motivated by the reality of the practical applications, 3 rounds were considered), and in each round a significant number of steps are present. This means that players do have to recalculate their evaluations in each of these steps. In contrast, in the CCA framework, participants evaluate their respective route–quantity pairs only once at the beginning of the auction, which allocates capacities and determines payments in one single step.

Table 14
Detailed Simulation results 1: Mean (mn), median (md) and standard deviation (σ) values of UR in the case of ACA and CCA.

NW	mn(UR^{ACA})	mn(UR^{CCA})	md(UR^{ACA})	md(UR^{ACA})	$\sigma(UR^{ACA})$	$\sigma(UR^{CCA})$
1	7770.94	7886.32	7398.50	7496.00	3411.43	3415.37
2	14888.16	15267.13	14695.50	14962.00	4800.68	4857.35
3	25856.58	26714.23	25566.50	26432.50	7597.43	7703.33
4	38523.03	40083.05	38168.50	39784.50	10210.44	10338.72

Table 15
Detailed Simulation results 1: Mean (mn), median (md) and standard deviation (σ) values of UN in the case of ACA and CCA.

NW	mn(UN^{ACA})	mn(UN^{CCA})	md(UN^{ACA})	md(UN^{ACA})	$\sigma(UN^{ACA})$	$\sigma(UN^{CCA})$
1	1919.08	3338.94	1355.00	2617.00	2560.72	2624.37
2	5661.71	8228.40	5300.00	7600.00	4508.00	4745.26
3	7525.00	12752.69	6984.00	11823.50	7264.61	7027.09
4	11684.11	18617.99	11268.50	17460.50	8990.49	8863.19

- Optimal bidding is not trivial under the ACA framework (see Section 2.5.2), in other words, we may say that the computational burden is put to the participants in the ACA case, meanwhile the clearing algorithm is simple. In contrast, in the CCA framework bidding is simple, while the clearing algorithm is more complex.
- In contrast to the ACA, the CCA allows simultaneous bidding for multiple alternative routes, without the risk of over-allocation.
- According to the simulation results, when considering the resulting utility values of product bundles (UR) allocated to participants – which is the most commonly used measure for the efficiency of an auction – the CCA is more efficient by 1.5–4.05% in average compared to the 3-round ACA.
- The VCG mechanism applied in the CCA results in significantly lower capacity payments, thus higher resulting net utility (UN) for the participants, and lower income for the auctioneer (IA). The properties of the VCG mechanism furthermore motivate players to bid their true evaluations.
- The proposed CCA method ensures that allocated capacity products always form connected paths between sources and consumers, thus alleviates the need for secondary trading.
- If the useful allocated capacities are considered, the CCA allocates a higher percentage of available network capacities in average, thus contributes to the more efficient utilization of infrastructure.
- The CCA method does not result in explicit individual prices for the capacity products (in contrast to the ACA), which may serve as price signals for investors, however from the dual variables of the underlying linear programming problem, such prices may be easily extracted.

The demonstrated initial results show that the proposed method may be a potential candidate for an alternative capacity-allocation algorithm, but further model extensions and studies are necessary to determine its applicability in the case of realistic scenarios.

5.1. Future work

In addition to the previously discussed potential future improvements of the proposed method (e.g. including gas suppliers in the model as well), the model may be extended to a two sided case as well, when multiple participants on the supply side are offering various capacity products at individual prices. In this case the formalism of CDA will be applied to describe the problem (Xia et al., 2005).

Acknowledgments

The author thanks Borbála Takácsné Tóth and Péter Kotek for their valuable input and for the fruitful discussions. The author also thanks Tim Roughgarden who suggested to use the VCG algorithm for the problem. The author is also thankful for the two anonymous reviewers for pointing out the flaws of the original manuscript. This work has been supported by the Hungarian Academy of Sciences under its Momentum Programme LP2021-2 and by the Fund K 131 545 of the Hungarian National Research, Development and Innovation Office. Dávid Csercsik is a grantee of the Bolyai scholarship program of the Hungarian Academy of Sciences.

Appendix. Further details of simulation results

The box plots of UR and UN in the case of network configurations 1, 2, 4 and 4 ($NW = 1, 2, 3, 4$) may be seen in Fig. 4. (See Tables 14 and 15.)

References

Baranwal, G., Vidyarthi, D.P., 2015. A fair multi-attribute combinatorial double auction model for resource allocation in cloud computing. J. Syst. Softw. 108, 60–76.

Bilgin, M., 2009. Geopolitics of European natural gas demand: Supplies from Russia, Caspian and the Middle East. Energy Policy 37, 4482–4492.

Börgers, T., Krahmer, D., 2015. An Introduction to the Theory of Mechanism Design. Oxford University Press, USA.

Chen, L., Huang, L., Sun, Z., Xu, H., Guo, H., 2015. Spectrum combinatorial double auction for cognitive radio network with ubiquitous network resource providers. IET Commun. 9, 2085–2094.

Csercsik, D., Hubert, F., Sziklai, B.R., Kóczy, L.A., 2019. Modeling transfer profits as externalities in a cooperative game-theoretic model of natural gas networks. Energy Econ. 80, 355–365, URL http://www.sciencedirect.com/science/article/pii/S0140988319300283.

De Vries, S., Vohra, R.V., 2003. Combinatorial auctions: A survey. INFORMS J. Comput. 15, 284–309.

Erdős, P., Rényi, A., 1960. On the evolution of random graphs. Publ. Math. Inst. Hung. Acad. Sci. 5, 17–60.

Ericson, R.E., 2009. Eurasian natural gas pipelines: The political economy of network interdependence. Eurasian Geogr. Econ. 50, 28–57.

EU, 1991. Council Directive 91/296/EEC on the transit of natural gas through grids. Off. J. Eur. Commun. L147.

EU, 1998. Directive 98/30/EC of the European Parliament and of the Council of 22 June 1998 concerning common rules for the internal market in natural gas. Off. J. Eur. Commun. L204.

EU, 2003. Directive 2003/55/EC of the European Parliament and of the Council of 26 June 2003 concerning common rules for the internal market in natural gas and repealing Directive 98/30/EC. Off. J. Eur. Union L176.

EU, 2009a. Directive 2009/73/EC of the European Parliament and of the Council of 13 July 2009 concerning common rules for the internal market in natural gas and repealing Directive 2003/55/EC. Off. J. Eur. Union L211, 94–136.

EU, 2009b. Regulation (EC) No 715/2009 of the European Parliament and of the Council on conditions for access to the natural gas transmission networks and repealing Regulation (EC) No 1775/2005. Off. J. Eur. Union L211.

EU, 2017. Commission Regulation (EU) 2017/459 of 16 March 2017 establishing a network code on capacity allocation mechanisms in gas transmission systems and repealing Regulation (EU) No 984/2013. Off. J. Eur. Union L72.

European Comission, 2005. Council regulation (EU) no 1775/2005 of the European Parliment and of the Council of 28 September 2005 on conditions for access to the natural gas transmission networks. Off. J. Eur. Union L289/1.

Fan, M., Stallaert, J., Whinston, A.B., 1999. A web-based financial trading system. Computer 32, 64–70.
Groves, T., et al., 1973. Incentives in teams. Econometrica 41, 617–631.
Hsieh, F.-S., 2020. A comparative study of several metaheuristic algorithms to optimize monetary incentive in ridesharing systems. ISPRS Int. J. Geo-Inf. 9, 590.
Hsieh, F.-S., 2021. A comparison of three ridesharing cost savings allocation schemes based on the number of acceptable shared rides. Energies 14, 6931.
Hsieh, F.-S., Zhan, F.-M., Guo, Y.-H., 2019. A solution methodology for carpooling systems based on double auctions and cooperative coevolutionary particle swarms. Appl. Intell. 49, 741–763.
Keller, J.T., Kuper, G.H., Mulder, M., 2019. Mergers of Germany's natural gas market areas: Is transmission capacity booked efficiently? Util. Policy 56, 104–119.
Kiedanski, D., Kofman, D., Orda, A., 2021. Design of a combinatorial double auction for local energy markets. In: International Conference on Network Games, Control and Optimization. Springer, pp. 315–320.
Li, L., LIU, Y.-a., LIU, K.-m., Ming, Y.N.G., et al., 2009. Pricing in combinatorial double auction-based grid allocation model. J. China Univ. Posts Telecommun. 16, 59–65.
Madani, M., 2017. Revisiting European Day-Ahead Electricity Market Auctions: MIP Models and Algorithms (Ph.D. thesis). Université catholique de Louvain.
Merino, D., 2016. Implementation Monitoring Report on the Capacity Allocation Mechanisms Network Code. Agency for the Cooperation of Energy Regulators.
Roughgarden, T., Talgam-Cohen, I., 2019. Approximately optimal mechanism design. Annu. Rev. Econ. 11, 355–381.
Samimi, P., Teimouri, Y., Mukhtar, M., 2016. A combinatorial double auction resource allocation model in cloud computing. Inform. Sci. 357, 201–216.
Scherr, B.A., Babb, E.M., 1975. Pricing public goods: An experiment with two proposed pricing systems. Public Choice 35–48.
Takácsné Tóth, B., Kotek, P., Kaderják, P., 2017. Evaluating the 6 March PRISMA Capacity Auctions. Regional Centre for Energy Policy Research, https://rekk.hu/article/145/evaluating-the-6-march-prisma-capacity-auctions.
Vickrey, W., 1961. Counterspeculation, auctions, and competitive sealed tenders. J. Finance 16, 8–37.
Xia, M., Stallaert, J., Whinston, A.B., 2005. Solving the combinatorial double auction problem. European J. Oper. Res. 164, 239–251.
Xu, K., Zhang, Y., Shi, X., Wang, H., Wang, Y., Shen, M., 2014. Online combinatorial double auction for mobile cloud computing markets. In: 2014 IEEE 33rd International Performance Computing and Communications Conference, IPCCC. IEEE, pp. 1–8.