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Basics of the atmospheric energetics: A brief summary of global and regional atmospheric energetics reflected upon the university curricula

Réka Ágnes Dávid* and **Péter Tasnádi**

Department of Meteorology,
Eötvös Loránd University,
H-1117 Budapest, Pázmány P. st. 1/A, Hungary

Corresponding author E-mail: davidreka@caesar.elte.hu

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Abstract— The purpose of this study is to provide an overview of the science and development of atmospheric energetics, its so far matured parts to date, and the direction of the researches. However, we restrict ourselves to the discussion of the very basic results of the researches to reveal the parts the introduction of which can be suggested into the compulsory education of the future meteorologist. This became feasible especially due to the rapid development of the personal computer that makes possible the calculation of the atmospheric energies for students by using their own laptops, so this field of meteorology now can be a tactile reality for them. The founder of atmospheric energetics was Lorenz, who formulated for a global, dry atmosphere the concept of available potential energy, which is the difference between the current energy state of the atmosphere and a reference state with minimum energy. His basic results concerning the global description of atmospheric energetics have already become part of the university curriculum. It is important to be able to describe the energy balance of the atmosphere both locally as well as globally, for which the introduction of enthalpy and exergy seemed appropriate. The advantage of examining the dry atmosphere is that significant simplifications can be applied, but the atmosphere is finally moist, so research has also started in this direction, first with a global and then with a local approach. The key is to find the reference state, which is a complex, computationally demanding task. In this paper, we focus on the most important steps of this process and concentrate on the thermodynamic basis of the new concepts.

Key-words: atmospheric energetics, available potential energy, moist available potential energy, enthalpy, exergy, thermodynamics, education

1. Introduction

At the end of the 19th century, Hertz (who is famous for his long-wavelength electromagnetic waves discovery) delivered a lecture, which has been almost forgotten, about the energy budget of the earth and gave quite precise estimations of the energy balance of the Earth (*Mulligan* and *Hertz*, 1997). In atmospheric energetics, the concept of available kinetic energy was introduced by *Margules* (1906). The maximum possible kinetic energy was identified in an isolated part of the air, which can be achieved through adiabatic changes starting from rest. The real development of atmospheric energetics launched with Edward Lorenz's pioneering paper (*Lorenz*, 1955) introducing the concept of available energy.

After Lorenz, many excellent researchers have worked on the field, and the literature on it became very rich. It can be quoted some excellent reviews and books on it *(Dutton*, 1973; *Van Mieghem*, 1973; *Marquet*, 1991[1]; *Wiin-Nielsen* and *Chen*, 1993; *Tailleux,* 2013). However, basic textbooks on dynamic meteorology generally have satisfied with the discussion of Lorenz's work and, except for Dutton, do not step toward more recent results (*Dutton*, 1976; *Panchev*, 1985; *Zdunkowski* and *Bott*, 2003; *Holton*, 2004; *Mak*, 2011; *Hoskins* and *James*, 2014). In this paper, the basics of atmospheric energetics, particularly its thermodynamic, will be reviewed background. Now, atmospheric energetics developed into a field of meteorology which supports highly the complex thinking on the explanation of the global circulation and on the local atmospheric motion. In light of this, dynamic meteorology textbooks devote an undeservedly short chapter to the topic. Maybe, it could be justified on the grounds that the field is still undeveloped and its knowledge is constantly changing, but since Lorenz's work, there have been a number of achievements that have been clarified and can rightly become materials of the textbooks. Therefore, we intentionally focus only on the basic issues and are not dealing with those details that are still in doubt. We emphasize those parts that are worth discussing in the standard educational materials.

The basics of the energetics are also processed in the book of *Dési* and *Rákóczi* (1970) as well as in *Götz* and *Rákóczi* (1981). The first book is based on the work of Margules, while the second uses Lorenz’s energy conversion ideas. In the work of *Práger* (1982), the available potential energy (*APE*) concept of Lorenz is detailed. *Makainé* (1971, 1972, 1974) analyzed in some publications the relevancy of the *APE* in synoptic problems in Hungarian. *Czelnai et al.* (1991) discusses shortly the energy conversions of the atmosphere among the other basic transport processes. *Major et al.* (2002) studied the climatic energetics of Hungary.

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[1] A new (2014) version of the original paper is available on the webpage of the author with valuable new comments. http://www.umr-cnrm.fr/spip.php?article833&lang=en

We overview the question of the availability and its thermodynamic base for atmospheric energetics. The possibility of the use of the concept of available potential energy for global and local processes of the atmosphere will be discussed, and relevant definitions for dry and moist atmospheres will be also investigated. Special attention will be paid to the problem of the reference state. From the literature, some are emphasized in *Table 1* which provides the structure of the topic and which papers, in our opinion, can be regarded as the milestones of the development of atmospheric energetics.

Table 1. Different cases and approaches which are in focus. (The study of *Tailleux* (2013) contains global view as well.)

	Dry case	**Moist case**
Global view	*Lorenz*, 1955 *Dutton*, 1973	*Lorenz*, 1978
Local view	*Marquet*, 1991 *Tailleux*, 2013	*Harris* and *Tailleux*, 2018

2. Conversions of the energies in the atmosphere

There are various forms of energy in the atmosphere, the almost only source of which is the Sun. The study of atmospheric energetics belongs to the field of dynamic meteorology; its general target is to describe the energy generation, transportation, and dissipation (*Wiin-Nielsen* and *Chen,* 1993; *Götz* and *Rákóczi*, 1981).

The energy of the atmosphere consists of the internal, potential, and kinetic energy. The specific energies are the before mentioned forms of the energy per unit mass (in the same order):

$$i = c_v T \,, \tag{1}$$

where c_v is the specific heat of air at constant volume and T is the temperature,

$$\phi = gz, \tag{2}$$

where g denotes the gravitational acceleration, z is the altitude,

$$k = \frac{\mathbf{v}^2}{2}, \tag{3}$$

where $\mathbf{v}$ is the three-dimensional velocity vector relative to the Earth. The previous quantities Eqs. $(1)-(3)$ have to be integrated over the total mass of the atmosphere to get the total amounts of energy of the atmosphere ($E = \int_{V_A} e\rho \mathrm{d}V$, where e can be i, ϕ or k, and V_A is the volume of the atmosphere).

Table 2 below shows the annual mean values of the energy forms for the Northern Hemisphere. It has to be pointed out, that the kinetic energy is a few orders of magnitude smaller, than the internal or the potential energy. Kinetic energy provides the energy for the midlatitude's cyclones. It decreases in summer partly due to the changes in the meridional temperature gradient (*Gertler* and *O'Gorman*, 2019).

Table 2. The annual mean values of the various energy forms for the Norther Hemisphere (1000–75 hPa) expressed in energy per unit area or per unit mass (*Oort*, 1971).

Energy	**[kJ/m^2]**
Internal	1674.8×10^6
Potential	567.5×10^6
Kinetic	1153.4×10^3

The atmosphere is not an equilibrium thermodynamic system, but it is approximately in radiation balance, which means that the incident radiation energy equals the emitted radiation. The processes of the atmosphere, among them the general circulation, are governed by kinetic energy, so to understand these processes, we need to study the generation and dissipation of the kinetic energy and also its conversion from other types of energies.

The posed questions of the atmospheric energetics in the early years of the twentieth century were related to the energetics of the cyclones, whether where the energy of the middle-latitude depressions comes from (*Margules*, 1910). According to the observations, there is not enough energy from the work against the pressure gradient. It turned out, that although the atmosphere contains a large amount of potential and internal energy, only a small part of them can be transformed into kinetic energy. It can be understood using a simple example created by *Wiin-Nielsen* and *Chen* (1993) following *Margules* (1910). Imagine a vessel filled with two fluids of different densities. The two fluids are separated by a vertical wall, so the fluids situate beside each other. When the wall is removed, the denser fluid goes to the bottom of the vessel and the less dense stratifies above it (the fluids do not mix). The potential energy of the fluids is decreasing and reaches its minimum value. Although the potential energy of the fluids might be very high, its available part is only the difference between that of the initial and final state. This available part may be very low due to this process.

The details of the following equations can be found in the work of *Wiin-Nielsen* and *Chen* (1993) and in Hungarian in *Götz* and *Rákóczi* (1981). One of the most general laws of nature, the law of energy conservation also applies to the atmosphere. Applying the law of energy conservation to thermodynamic processes, we get the first law of thermodynamics. If we look for the derivatives of the global energy forms, the derivatives of the internal energy can be obtained from the integral of the internal energy density which can be expressed from the differential form of the first law $c_v dT = \delta q - p d\alpha$, where the continuity equation was used for getting the second term at right-hand side of the above mentioned equation:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \int_{V_A} \dot{q}\rho \mathrm{d}V \; - \int_{V_A} p\nabla \cdot \mathbf{v}\mathrm{d}V \;, \tag{4}$$

Where δq and $\dot{q}$ are the heat absorbed by the unit mass and the heat current to unit mass, respectively, $\rho = \frac{1}{\alpha}$ is the density of the air, α is the specific volume of the air, p is the pressure, and $\mathbf{v}$ is the velocity vector of the wind. In Eq. (4), the first term is the internal energy generation ($G(I)$) and the second one is the conversion from internal energy to kinetic energy ($C(I,K)$). There are two processes, which influence the internal energy: (1) internal energy increases if the atmosphere is heated, where the density is high or it is cooled, where the density is low, (2) internal energy decreases in divergent areas, where the average pressure is high or in convergent areas, where the pressure is low.

The global form of the change of potential energy is the following:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \int_{V_A} g\rho w \mathrm{d}V \;, \tag{5}$$

where w is the vertical wind component. Eq. (5) is the conversion from potential energy to kinetic energy with a minus sign ($-C(P,K)$). And finally, the form of the change of global kinetic energy is:

$$\frac{\mathrm{d}K}{\mathrm{d}t} = \int_{V_A} p\nabla \cdot \mathbf{v}\mathrm{d}V \; - \int_{V_A} g\rho w \mathrm{d}V \; + \int_{V_A} \mathbf{v} \cdot \mathbf{F}\mathrm{d}V \;, \tag{6}$$

where **F** is the frictional force and the other notations have been already defined. The first term of Eq. (6) is known from Eq. (4), which is $C(I,K)$, the second term is the conversion from potential energy to kinetic energy ($C(P,K)$), and the last term is the dissipation of kinetic energy ($D(K)$). So, Eqs. (4)–(6) can be written as:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = G(I) - C(I, K), \tag{7}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -C(P, K), \tag{8}$$

$$\frac{\mathrm{d}K}{\mathrm{d}t} = C(P, K) + C(I, K) - D(K). \tag{9}$$

In the case of long-time average, a steady-state can be considered, which means that there are no changes in the form of energies, which results that the time derivatives of the energy forms are zero, so we get (*Götz* and *Rákóczi*, 1981; *Wiin-Nielsen* and *Chen,* 1993):

$$G(I) = C(I, K) = D(K), \tag{10}$$

$$C(P, K) = 0. \tag{11}$$

Finally, it is worth mentioning, that the gravitational potential and internal energy in an ideal gas of hydrostatic state are proportional to each other, and since the atmosphere can be regarded as an ideal gas, the sum of its specific potential and specific internal energy can be expressed with its specific enthalpy. Lorenz called the sum of the internal and gravitational potential energy simply as potential energy, but enthalpy is a more exact name that helps to avoid misunderstandings originating from the mixing of the terminology. Using this, Lorenz got simpler equations for the cycle of the energy transformations:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = G(H) - C(P, K), \tag{12}$$

$$\frac{\mathrm{d}K}{\mathrm{d}t} = C(H, K) - D(K), \tag{13}$$

where $H = I + P$. However, the numerical value of these conversion factors can only be realistic if the moisture of the air is also taken into account. *Fig. 1* shows the scheme of the energy cycle.

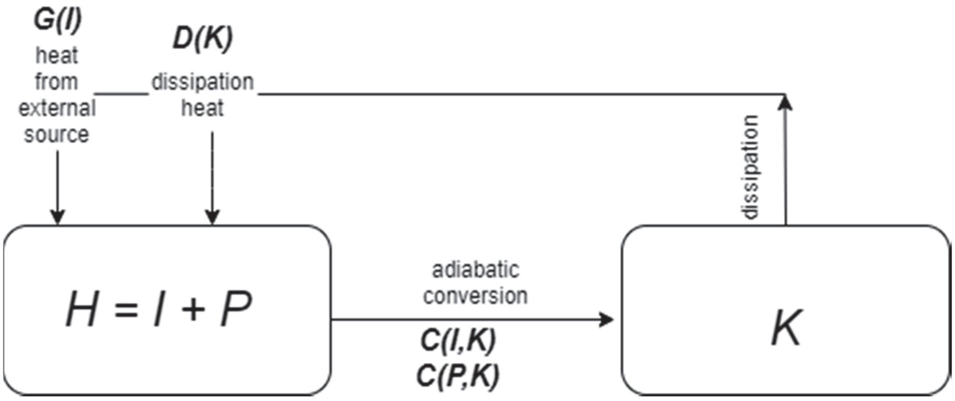


Fig. 1. The scheme of the energy cycle.

3. The equilibrium state of the atmosphere

As we have expounded, the atmosphere is not in a thermodynamic equilibrium state, but it is in approximate dynamic equilibrium, because of the balance of the incoming and outgoing radiation. In contrast to outgoing radiation, incident radiation is not evenly distributed on the surface of the Earth, and due to this, general circulation which continuously transfers the energy from the equator toward the poles is developing.

According to thermodynamics, the equilibrium state of a system can be found in two equivalent ways. If the system is adiabatically (entropically) closed, then its equilibrium state is where its energy is minimum. On the other hand, if the system is energetically closed, then its equilibrium state belongs to its minimum energy one (*Callen*, 1960).

The specific entropy can be obtained by the integration of the differential form of the thermodynamic law $ds = c_v \frac{dT}{T} + \frac{p}{T} d\alpha$ and with the use of the gas law:

$$s = c_v \ln \frac{T}{T_0} + R \ln \frac{\alpha}{\alpha_0} \ , \tag{14}$$

where s is the specific entropy, T_0 and α_0 are an arbitrary reference temperature and specific volume, respectively.

Obviously, the state of a real atmosphere is neither entropically nor energetically closed. However, an equilibrium state can be assigned to either the current state of the atmosphere or some average state if one closes fictitiously the atmosphere and determines the extremum of the energy or the entropy.

Lorenz and Dutton carried out this procedure, Lorenz applied the energy minimum principle, while Dutton used entropy maximum for searching equilibrium state of the dry atmosphere (see *Fig. 2*).

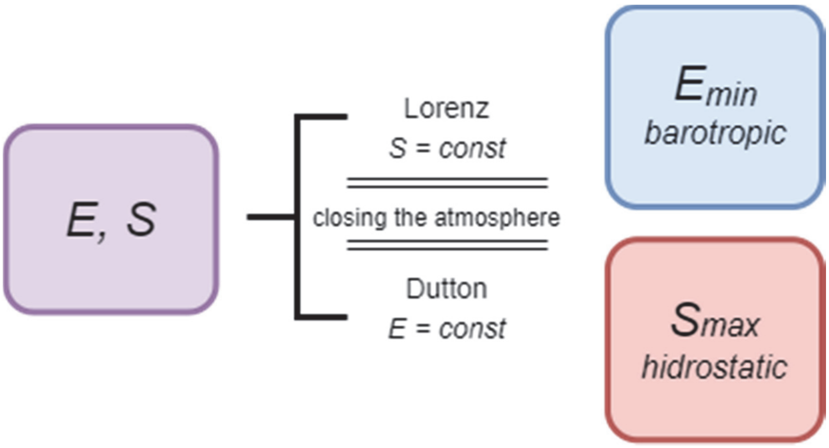


Fig. 2. The state of thermodynamic equilibrium based on the two principles: the approach of Lorenz (up) and Dutton (down). *E* and *S* denotes the global energy and entropy of the atmosphere, respectively.

Despite that the numerical results concerning dry atmosphere do not reflect in every respect the real properties of the atmosphere, they can point to many interesting facts. They can be used in cases when the correlations between processes and not the exact numerical values are in focus. Several atmospheric energy issues are related to the storage and movement of energy that takes place between the components of the climate system (atmosphere, ocean, land, and cryosphere). As an example, we show the correlation between sea ice cover and energies (*Table 2* and *Fig 3*). The details of the calculations can be seen in *Appendix A*.

Table 2. Pearson correlation between the sea ice cover, the total entropy, and the total energy of the atmosphere (1979–2019). ERA5 database was used for the calculations (see *Appendix A*).

	Sea ice cover	**Entropy**
Energy	-0.81	0.99
Entropy	-0.82	

Unsurprisingly, there is a very strong correlation between the energy and the entropy, and negative correlation between the energy and the sea ice cover, and slightly different between the entropy and the sea ice cover, this is illustrated by *Fig. 3*.

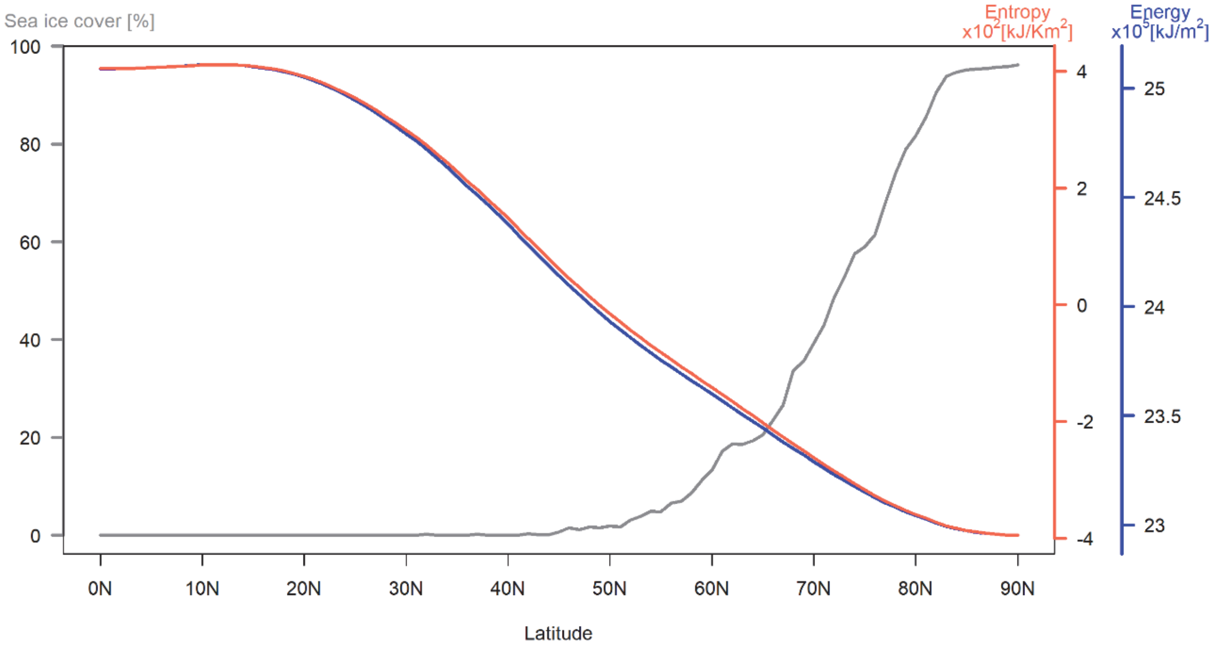
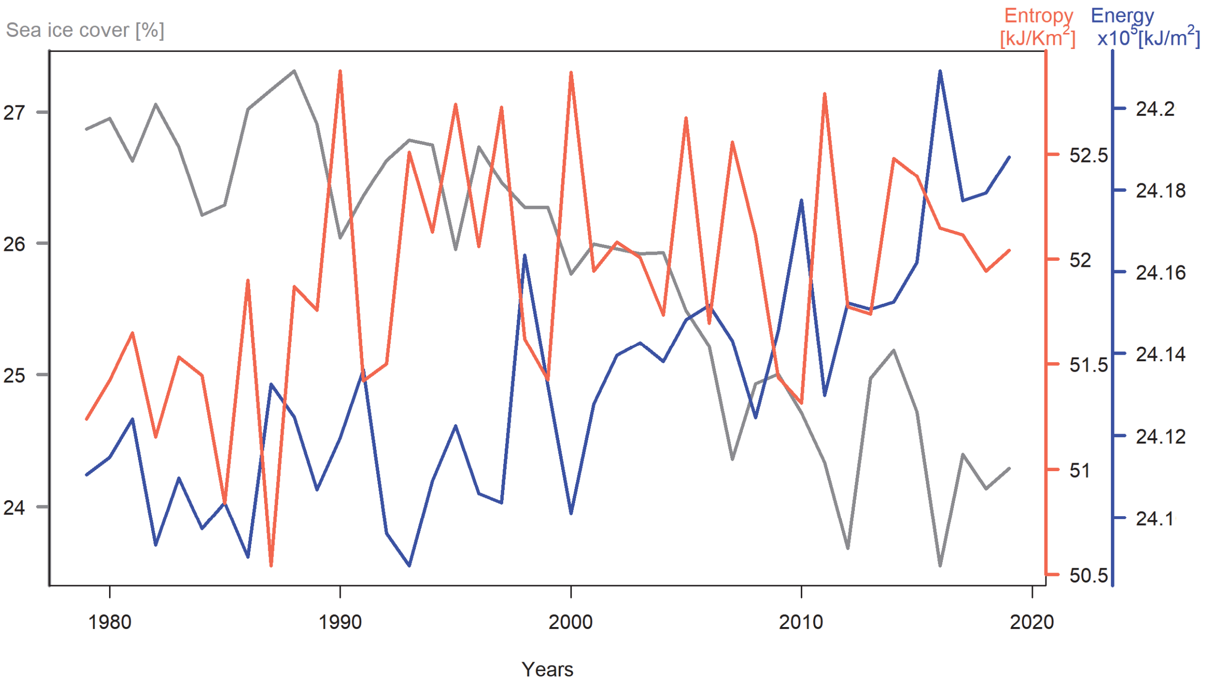


Fig. 3. The average energy and entropy per m^2 cell of a given latitude and the sea ice for the Northern Hemisphere. ERA5 database was used for the calculations (*Appendix A*).

In most cases, the relationship between sea ice and oceanic energy is examined (*Su* and *Ingersoll*, 2016), but researches have also shown that there is also a relationship between the global energy on the top of the atmosphere and the ocean heat content (*Trenberth et al*., 2014) or between atmospheric *APE* and sea ice cover (*Novak* and *Tailleux,* 2018). Particular attention was paid to sea ice around Greenland and the Ross Sea (*Shepherd* and *Wingham*, 2007; *Jacobs et al.,* 2011).

Fig. 4 shows the annual averages of the entropy, energy, and sea ice for the Northern Hemisphere. There is an increasing trend in the ocean heat content from 1993 (*Trenberth et al*., 2014), this can be seen as the decreasing trend in the sea ice cover in *Fig. 4*.



4. Fig. The annual averages of the sea ice, entropy, and energy per m^2 for the Northern Hemisphere. ERA5 database was used for the calculations (see *Appendix A*).

4. The available enthalpy

The question of availability was addressed by Edward Lorenz in 1955 when he introduced the concept of available potential energy (*Lorenz*, 1955). His aim, analyzing the global circulation in the twentieth century, was to find the energy sources and sinks of atmospheric motions, as well as the forms of energy transformations that allow atmospheric motions to persist despite frictional dissipation.

The available potential energy of Lorenz (*APE*) was defined to be the maximum part of the sum of the internal and potential energy (in hydrostatic case, it is the enthalpy) of the atmosphere that can be converted to kinetic energy under ideal conditions during isentropic processes. In the following, the term enthalpy will be used instead of the potential energy used by Lorenz. (The approximations, which Lorenz used, clearly show that Lorenz regarded the actual state of the atmosphere to be hydrostatic.) It follows, that there is some part of enthalpy that cannot be converted into kinetic energy, thus a state of minimum enthalpy in the atmosphere should be defined as a reference state. *APE* was defined by Lorenz as the difference between the enthalpy of the atmosphere's real (actual) and reference states. To find the state of minimal enthalpy, he rearranged adiabatically the actual state of the atmosphere to a statically completely stable and vertically stratified one, and using this state he calculated the APE. Lorenz also determined an approximate formula for the practical calculation of APE. The total enthalpy in a hydrostatic atmosphere is the following:

$$P=\int_{V_A} c_p\rho T \mathrm{d}V \ =\int_{A_G}\int_0^{\infty} c_p\rho T \mathrm{d}z\, \mathrm{d}A\,, \tag{15}$$

where A_G is the surface of the Earth. To execute the integration in a pressure coordinate system (p-system), we use the hydrostatic approximation:

$$\mathrm{d}z=-\frac{1}{\rho \mathrm{g}}\,\mathrm{d}p. \tag{16}$$

Entering Eq. (16) in Eq. (15) we get:

$$P=\frac{c_p}{g}\int_{A_G}\mathrm{d}A\int_0^{p_0} T\mathrm{d}p\,. \tag{17}$$

Applying potential temperature as vertical coordinate:

$$APE=TH_A-TH_R=\frac{c_p}{2gp_0^{\kappa}(\kappa+1)}\int \mathrm{d}S\int_0^{p_0}\left(p^{(\kappa+1)}-\bar{p}^{(\kappa+1)}\right)\mathrm{d}\theta \approx$$
$$\approx \frac{R}{2gp_0^{\kappa}}\int \mathrm{d}S\int_0^{p_0}\bar{p}^{(\kappa+1)}\left(\frac{p^{'}}{\bar{p}}\right)^2 \mathrm{d}\theta, \tag{18}$$

where the total enthalpy (*TH*) is expressed as:

$$TH=I+P=\int \mathrm{d}S\int_0^{\infty}\rho g z \mathrm{d}z+\int \mathrm{d}S\int_0^{\infty}\rho c_v T\mathrm{d}z=\int \mathrm{d}S\int_0^{\infty}\rho c_p T\mathrm{d}z=$$
$$\frac{c_p}{g}\int \mathrm{d}S\int_0^{p_0} T\mathrm{d}p=\frac{c_p}{gp_0^{\kappa}}\,\frac{1}{\kappa+1}\int \mathrm{d}S\int_0^{\infty} p^{\kappa+1}\mathrm{d}\theta\,. \tag{19}$$

In the equations, c_p and c_v are the specific heat under constant pressure and constant volume, respectively, R is the specific gas constant for dry air, κ is the ratio of R to c_p, g is the gravitational constant, p is the pressure, p_0 is the pressure at the surface, which can be regarded to 1 atm, θ is the potential temperature, T is the temperature, ρ is the density, z is the height, $\mathrm{d}S$ is the surface element of the isentropic surfaces, and $p^{'}=p-\bar{p}$.

It shows that the *APE* is proportional to the pressure variance on isentrops to the leading order. Returning to p as vertical coordinate:

$$APE \approx \frac{R}{2gp_0^{\kappa}}\int \mathrm{d}S\int_0^{\propto} \bar{p}^{(\kappa+1)}\left(\frac{p'}{\bar{p}}\right)^2 \mathrm{d}\theta = \frac{1}{2g}\int \mathrm{d}S\int_0^{p_0} \bar{\alpha}\bar{\theta}\left(\frac{\partial\bar{\theta}}{\partial p}\right)^{-1}\left(\frac{\theta'}{\bar{\theta}}\right)^2 \mathrm{d}p. \quad (20)$$

Wiin-Nielsen used the $\frac{\theta'}{\bar{\theta}} \approx \frac{\alpha'}{\bar{\alpha}}$ approximation, while Lorenz applied $\frac{\theta'}{\bar{\theta}} \approx \frac{T'}{\bar{T}}$. The Lorenz approximation leads to a result depending on the temperature variance on isobaric surfaces:

$$APE = \int \mathrm{d}S\int_0^{p_0} \frac{1}{\bar{T}}\frac{T'^2}{\gamma - \Gamma}\mathrm{d}p, \quad (21)$$

where γ and Γ are the actual and the adiabatic lapse rate, respectively. *Wiin-Nielsen* and *Chen* (1993) derived a formula depending on the variance of the specific volume α' and the static stability parameter $\bar{\sigma}$:

$$\bar{\sigma} = \frac{\bar{\alpha}}{\bar{\theta}}\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}p}\ . \quad (22)$$

Eq. (22) is a definition equation, where σ is considered constant, and this constant is taken as the average value of it ($\bar{\sigma}$).

$$APE = \int \mathrm{d}S\int_0^{p_0} \frac{1}{2g}\frac{\alpha'^2}{\bar{\sigma}}\mathrm{d}p. \quad (23)$$

It can be shown that $C(H,K)$ and $G(H)$ equal to $C(APE,K)$ and $G(APE)$.

5. Dutton's entropic energy

Dutton (*Dutton* and *Johnson*, 1967; *Dutton*, 1976) approached the problem of the equilibrium state of the atmosphere through the determination of the maximum entropy. He determined the maximum entropy state of the energetically closed atmosphere with the use of the calculus of variations. According to Dutton's point of view, atmospheric movements are induced by thermal driving forces, so at a given moment he fictitiously insulated the atmosphere from the external energy sources keeping the total energy constant and sought what end state the equalization of internal inhomogeneities leads to.

In a closed atmosphere, due to differences in temperature and pressure, various processes take place that increases entropy. When the processes stop, the entropy will be maximum. This is the state where the atmosphere would converge, if it was cut off from all the energy sources. Dutton called this maximum entropy state an associated state to the actual one. Dutton's calculation based on the calculus of variations led to a final state of the atmosphere, which is isotherm and hydrostatic. This result can be easily predicted also an intuitive way. The maximum entropy state of the atmosphere is a so-called dead state, in which only random thermal motion can occur, and macroscopic processes are perfectly impossible due to the balanced intensive state variables. In this state the temperature is constant, all wind velocity is zero, there is no convection, and consequently, the pressure is hydrostatic.

Let the total energy and the mass at the initial state of the atmosphere be *E* and *M*, respectively, and let the atmosphere be an ideal gas. Then the properties of the associated maximum entropy state are the following. Its temperature is

$$T_0 = \frac{E}{c_p M}, \tag{24}$$

it is motionless and the pressure obeys the barometric height law:

$$p(z) = p_0 \exp(-\frac{gz}{RT_0}), \quad p_0 = \frac{Mg}{4{R_E}^2\pi}, \tag{25}$$

where R_E is the radius of the Earth. It should be mentioned that the energy $(E = E_0)$ in the actual state and in the associated state equal with each other, and while in the actual state the energy consists of internal, potential (gravitational) and kinetic energies $(E_0 = E = I + P + K)$, in the associated state there are only internal and potential energies $(E_0 = I_0 + P_0 = H_0)$. Since the atmosphere is an ideal gas, and in the associated state it is hydrostatic, the energy equals to the enthalpy.

Dutton determined the difference of the entropy of the associated and actual state of the atmosphere and introduced the concept of entropic energy *N* as $N = T_0(S_0 - S)$, where S_0 and S are the entropy of the associated and actual state, respectively. Dutton proved, that

$$N = T_0(S_0 - S) = K + T_0\Sigma, \tag{26}$$

where $T_0\Sigma$ is the static entropic energy. To obtain this equation Dutton expresses $S_0 - S$ as a sum of the leading terms of its Taylor series and a remainder term. The remainder term Σ can be expressed as an integral of the positive definite homogeneous second order form of the temperature and specific volume, which proves that it is positive.

$$\Sigma = \int_V \frac{\rho}{2}\left\{c_v\left[\frac{\frac{T-T_0}{T_0}}{1+\frac{\lambda_1(T-T_0)}{T_0}}\right]^2 + \left[\frac{\frac{\alpha-\alpha_0}{\alpha_0}}{1+\frac{\lambda_2(\alpha-\alpha_0)}{\alpha_0}}\right]^2\right\}\mathrm{dV}', \qquad (27)$$

where λ_1 and λ_2 are constants, which satisfies $0 \le \lambda_1, \lambda_2 \le 1$. λ_1, λ_2 show the position where the remaining term should be calculated in the $(T,\ T_0)$ and $(\alpha,\ \alpha_0)$ interval.

The entropic energy determines a kind of „distance" from the equilibrium state, and in a really closed system, it would decrease monotonously when the system tends to its equilibrium state. It would mean that in the atmosphere the general circulation ought to cease. Clearly, the atmosphere is not an isolated system, and therefore, the net heating provides the entropy destruction that permits N to avoid the monotonic evolution.

6. Comparison of the two theories

Both theories are unrealistic, because besides the arbitrary closing of the atmosphere, they concern a dry atmosphere. The neglecting of the moisture leads by all means unrealistic results. Nevertheless, both theories have significantly contributed to the theoretical clarification and better understanding of the role of changes in the atmospheric energies in the processes of the atmosphere. A further problem is that both theories treat globally the atmospheric energies, so they are not suitable for the description of the local processes which can be important for example in cyclones.

However, contrary to Lorenz's theory, where processes would no longer take place in the atmosphere under the constancy of entropy, this associate end state (under appropriate conditions) could really be reached by the atmosphere. Using thermodynamic argumentation only, the concept of entropy promises a prediction about the direction a thermodynamic process must take. Although this maximum entropy state is also fictitious, it is more realistic than the minimum energy state applied by Lorenz. Despite of this, recently Lorenz' point of view dominates the energy description of the atmosphere. In our opinion, Dutton's approach should be also involved in the standard curriculum, since the entropic approach of the atmospheric processes is at least as important as the energetic one.

7. The local definition by Marquet and Tailleux

Marquet in his excellent paper (*Marquet*, 1991) re-examined the concept of APE and defined a locally practicable version of it. He placed in a historical perspective the development and various definitions of the concept of available energy and, albeit with doubts, identified it with the exergy. However, the main novelty of his paper is the local definition of the specific available potential energy as the generalization of the Lorenz' global available potential energy and the entropic energy of Dutton. As the topic is very diverse and burdened by terminological inconsistencies, we recommend turning for details to the excellent papers of *Keenan* (1951), *Haywood* (1974a,b), and *Marquet* (1991), and to navigate between different technical terms. Especially the application of the various versions of the APE theory was developed by many researchers.

The algebraic expression of the APE density would allow the study of energy conversions in open domains (e.g., cyclones, baroclinic waves), where boundary fluxes are also considered. Furthermore, by focusing on the energies at various distinct pressure levels, the tropospheric-stratospheric energy exchange processes could be interpreted.

The approach of Marquet shows that the hydrodynamic application of available enthalpy allows the generalization of available energy concepts previously introduced by Lorenz and Dutton to global meteorological processes. Marquet defines the specific available enthalpy as:

$$a_h \equiv (h - h_r) - T_r(s - s_r) = (h - T_r s) - (h_r - T_r s_r), \tag{28}$$

which can be expressed as

$$a_h(T,p) = c_p(T - T_r) - c_p T_r \ln\left(\frac{T}{T_r}\right) + R T_r \ln\left(\frac{p}{p_r}\right), \tag{29}$$

where h denotes the enthalpy and T_r and p_r are the reference temperature and reference pressure, respectively. (The exact definitions of T_r and p_r will be given by (43–44), and the physical interpretation of them will be discussed later.) An interesting property of available enthalpy in an ideal gas is that it can be separated into temperature- and pressure-dependent terms:

$$a_h(T,p) \equiv a_T(T) + a_p(p), \tag{30}$$

where the temperature (a_T) and pressure (a_p) dependent terms of the specific enthalpy are:

$$a_T = c_p(T - T_r) - c_p T_r \ln\left(\frac{T}{T_r}\right), \tag{31}$$

$$a_p = RT_r \ln\left(\frac{p}{p_r}\right). \tag{32}$$

Marquet has derived a local energy cycle with the set of energy components: the specific gravitational potential (e_G) and the kinetic energy (e_K), furthermore, with the a_T and a_P ,which are the temperature (T) and the pressure (p) dependent term of the specific enthalpy, respectively. The capital letters with s subscript denote the conversions, generation, and dissipation of the specific quantities. The subscripts distinguish them from those which are in Lorenz's formulas without indices and which concern global quantities (the lower indices denote of the type of the quantities).

$$\begin{aligned}
\frac{\mathrm{d}e_G}{\mathrm{d}t} &= -C_s(G,K) \\
\frac{\mathrm{d}e_K}{\mathrm{d}t} &= +C_s(G,K) + C_s(h,K) - D_s(K) \\
\frac{\mathrm{d}a_T}{\mathrm{d}t} &= +C_s(p,T) - C_s(h,K) + G_s(h) \\
\frac{\mathrm{d}a_p}{\mathrm{d}t} &= -C_s(p,T)\,.
\end{aligned} \tag{33}$$

The $C_s(m,n)$ conversion factors give the increase of type nth energy at the expense of the type mth.

Knowing Marquet's results, Dutton proved them on a much simpler intuitive "ad hoc" way, which is briefly recalled below (*Dutton*, 1992). Multiplying the equation of motion ($\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\alpha\nabla p - g\mathbf{k} + \alpha\mathbf{F} + f(\mathbf{k}\times\mathbf{v})$) by $\mathbf{v}$ we got the equation:

$$\frac{\mathrm{d}e_K}{\mathrm{d}t} = -\alpha\mathbf{v}\cdot\nabla p + C_s(G,K) + D_s(K), \tag{34}$$

where the e_G energy component is the following:

$$\frac{\mathrm{d}e_G}{\mathrm{d}t} = -g\frac{\mathrm{d}z}{\mathrm{d}t} = -C_s(G,K), \tag{35}$$

and D_s is the specific rate of work of all the forces except the gravitation and gradient ones. From the first law of thermodynamics it can be obtained that

$$\frac{\mathrm{d}e_h}{\mathrm{d}t} = \alpha\frac{\mathrm{d}p}{\mathrm{d}t} + \dot{q} = \alpha\frac{\partial p}{\partial t} + \alpha\mathbf{v}\cdot\nabla p + \dot{q}\,, \tag{36}$$

where $\dot{q}$ is the heating rate. Comparing the thermodynamic and kinetic energy equations it can be concluded that

$$\alpha\mathbf{v}\cdot\nabla p = -C_s(h,K). \tag{37}$$

The specific heating rate can be written in the following form:

$$\dot{q} = \frac{T_r}{T}\dot{q} + \left(1 - \frac{T_r}{T}\right)\dot{q}. \tag{38}$$

The first term in the right side of Eq. (38) can be expressed with the specific entropy, Eq. (37):

$$s = s_T + s_p = c_p \ln\frac{T}{T_r} - R\ln\frac{p}{p_r}\ , \tag{39}$$

$$\frac{T_r}{T}q = T_r\frac{\mathrm{d}s}{\mathrm{d}t} = T_r\left(c_p\frac{\mathrm{d}\ln T}{\mathrm{d}t} - R\frac{\mathrm{d}\ln p}{\mathrm{d}t}\right). \tag{40}$$

Introducing

$$C_s(p,T) = -RT_r\frac{\mathrm{d}\ln p}{\mathrm{d}t} = -\frac{R}{p}T_r\omega\ , \tag{41}$$

where $\omega = \frac{\mathrm{d}p}{\mathrm{d}t}$ is the conversion from a_p to a_T, as well as the generation (G_s) of a_T, is $G_s = \alpha\frac{\partial p}{\partial t} + \left(1 - \frac{T_r}{T}\right)\dot{q}$ furthermore using available enthalpy components defined above, we can get with simple algebra the equations of the local cycle Marquet has found. The first term in G_s can be interpreted as the work due to adiabatic expansion, while the second is the heat reduced by the Carnot factor $\left(1 - \frac{T_r}{T}\right)$. The local cycle gives the energy balance of a motion of an atmospheric parcel. Having added the four equations of the cycle, we got the equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}(e_K + e_G + a_h) = G_s - D_s. \tag{42}$$

The right hand side of Eq. (42) vanishes in a frictionless and isentropic steady flow, and then the hydrodynamic derivative equals zero, therefore, $e_K + e_G + a_h = c,\ c =$ constant along any particular streamline, but it may be different on different streamlines. This equation is a form of the Bernoulli equation.

The definition of the reference temperature and reference pressure is an intricate one. Marquet proposed and applied the following formulas:

$$\frac{1}{T_r} = \frac{1}{gM}\int_0^{p_0} \frac{1}{T}\mathrm{d}V_p \ , \tag{43}$$

$$\ln(p_r) = \frac{1}{gM}\int_0^{p_0} \ln(p)\,\mathrm{d}V_p, \tag{44}$$

where $\mathrm{d}V_p$ notes the small domains of the air we sum up as $\mathrm{d}V_p = \mathrm{d}A\mathrm{d}p = {R_F}^2\mathrm{d}\lambda\mathrm{d}\varphi cos\varphi\mathrm{d}p$, where $A = 4R^2\pi$ is the surface of the Earth, the other notations are the same as before. We reproduced the calculations made by Marquet with the ERA5 database (see *Appendix A*). The available enthalpy temperature- and pressure-dependent terms (*Fig. 5*) and their summation (a_h) are presented in this paper. As the latitude increases, the value of a_T decreases, and along the N30° it decreases below 2 kJ/kg. a_T loses its zonal picture around 300 hPa, where it begins to increase again. In fact, the value of a_p decreases with the pressure. *Fig. 6* shows a_h, the sum of a_T and a_p. As expected, a_h is similar to a_p due to it weights heavier on it ($a_p > a_T$). We see a minimal ratio of a curve close to the Equator, which is affected by a_T, where it has the highest values.

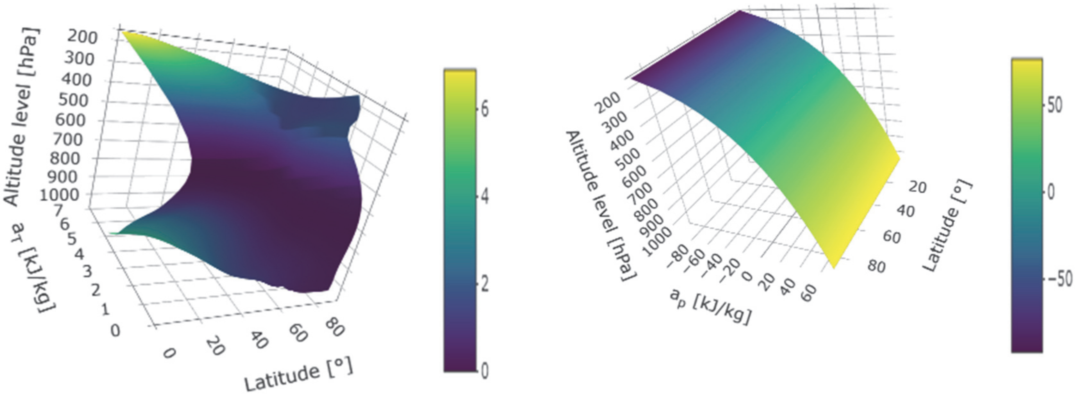


Fig. 5. The available enthalpy temperature- (left) and pressure-dependent (right) terms for the Northern Hemisphere, made by 41 years average.

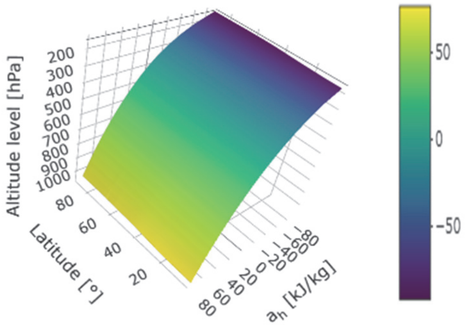


Fig. 6. The available enthalpy for the Northern Hemisphere, made by 41 years average. The calculated enthalpy values are visible on the axis and on the legend.

8. The exergy concept in atmospheric energetics

Marquet's available enthalpy concept can be connected to the concept of the exergy of steady-state open systems. The general description and the embedding of exergy concepts into thermodynamics can be found, e.g., in *Harmatha,* (1982), *Bejan* (2006), and *Dincer* and *Rosen* (2007). However, these studies have summarized the thermodynamic basis focusing on engineering concern of the exergy theory. The atmospheric application of the concept was discussed earlier by *Karlsson* (1990) and *Kucharski* (1997), and the latter author cited a lot of further references concerning the topic. However, we think that it is worth rigorously looking through the basics and origin of this concept and investigating thoroughly its possible connections with atmospheric energetics, particularly with the role of the reference state.

If a system is not in equilibrium with its environment, then spontaneous processes start to equalize the intensive state variables of the system and the environment. Due to these processes, after some time the system will be balanced with the environment. Up till the equilibrium befalls, energy and mass flow through the boundary of the system. The energy transport can be taken place by heat and work. We would like to control the transport processes to make maximum use of its internal energy be maximally exploit as work. The adiabatic process seems to be proper to achieve this goal. However, the processes with finite velocity are always irreversible which means that some part of the work is transformed (for example by friction) into heat and transferred back to internal energy. Because of this, we can conclude that the most efficient processes should be quasistatic, so reversible. It means that the maximum available work belongs to an ideal limiting case. The second law of thermodynamics prevents all internal energy from being transferred from the system as work. Naturally, the available part of the internal energy is the function both the state of the system and the environment.

To determine the maximally extracted work, imagine a system that is insulated from its environment and search the boundary conditions under which the maximum work can be extracted. Let us allow the quasistatic adiabatic processes between the system and the environment. Then the energy can be changed between them by performing work. Achieving the equilibrium state, the internal energy (U_{sys}) of the system becomes a minimum $W_{max} = \Delta U_{sys}$.

Connecting the system to a heat reservoir with constant temperature, the work performed during the setting of the balance between the system and the environment can be done isothermally, so the maximally extracted work equals the change of the free energy $W_{max} = \Delta F_{sys}$. (F_{sys} is the free energy of the system.) Similarly, if the system is connected to a work reservoir, the maximum work is $W_{max} = \Delta H_{sys}$, and in the case of both heat and work reservoirs, it is $W_{max} = \Delta G_{sys}$. (H_{sys} and G_{sys} denote the enthalpy and free enthalpy or Gibbs potential of the system, respectively.)

In the case of a system, which is not in equilibrium with its environment, the balanced state can be achieved by the combination of an isothermal and an adiabatic path. So, first the system is brought adiabatically to the temperature of the environment and after isothermally to the state of the environment. Adding the work performed through the two ways we get the maximum work available. Obviously, the maximum extractable work depends on the structure of the system. According to this, we can obtain two results depending on that the system is closed or it is a stationary open system, where proper reservoirs ensure the constancy of the temperature or the pressure.

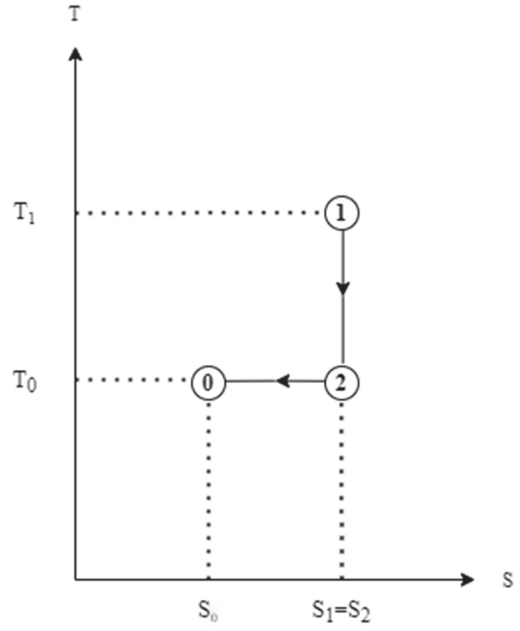


Fig. 7. A sequence of adiabatic and isothermal paths (0 is the state of the environment).

Fig. 7 shows in a T–S diagram the sequence of adiabatic and isothermal paths from state 1 to state 0 (which is the state of the environment). First, the system goes adiabatically from state 1 to state 2, where the temperature equals to that of state 0, after it goes isothermally to state 0. Following the thermodynamic path given in *Fig. 7*, the possible maximum of the extracted work can be determined. In the first adiabatic part of the path, the work performed by the system is $W_{1,2} = U_1 - U_2$, and on the second (isothermal) part it is $W_{2,0} = F_2 - F_0$. Consequently,

$$W_{max} = U_1 - U_2 + F_2 - F_1 = U_1 - U_2 + (U_2 - T_2 S_2) - (U_0 - T_0 S_0)\,. \quad (45)$$

Taking into account that, as *Fig. 6* shows, $T_0 = T_2$ and $S_0 = S_1$:

$$W_{max} = (U_1 - T_0 S_1) - (U_0 - T_0 S_0) = \Delta U - T_0(S_1 - S_0). \quad (46)$$

In a steady state open system, where the constancy of pressure is ensured by a pressure reservoir, the extractable work, during the adiabatic change of state, is $W_{1,2} = H_1 - H_2$, while during the isothermal path, where the system is even connected to a heat reservoir, the work is $W_{2,0} = G_2 - G_0$. Adding these two contributions

$$W_{max} = H_1 - H_2 + G_2 - G_1 = H_1 - H_2 + (H_2 - T_2 S_2) - (H_0 - T_0 S_0)\,. \quad (47)$$

Taking into account again that $T_0 = T_2$ and $S_0 = S_1$:

$$W_{max} = (H_1 - T_0 S_1) - (H_0 - T_0 S_0) = \Delta H - T_0(S_1 - S_0). \quad (48)$$

Strictly speaking, the maximum work extractable from a system is called exergy, and the calculations above show, that it depends on the state variables both the system and the environment. In a broader sense, the maximum work available from a steady state system is called exergy, too. These expressions are very similar to those giving the change of the free energy of the system and the change of the free enthalpy of the system. The only difference is that the free energy and the free enthalpy are state variables of the system and depend on the temperature of it, while the exergy depends on the temperature of the environment too.

The mathematical expressions of the exergy are formally very similar to those that defined for available potential energies (enthalpies) in atmospheric physics and introduced with various names. For example, Dutton's entropic energy:

$$N = T_0(S_0 - S), \quad (49)$$

and Marquet's available enthalpy:

$$a_h \equiv (h - h_r) - T_r(s - s_r) = (h - T_r s) - (h_r - T_r s_r) \quad (50)$$

are exergies. In expressions concerning atmospheric availability, reference states appear everywhere which can be corresponded in exergetics to properties of reservoirs or the surroundings. However, while in technical thermodynamics regarding engines, turbines, solar cells, and so on, there are well defined environment or physical systems which can be identified with reservoirs, in atmospheric physics, there are no such possibilities, the reference states have to be chosen arbitrarily. In the case of atmospheric flows, temperature and pressure reservoirs cannot be found, so the customary definitions cannot be used. It means that the concept of exergy is worth using in atmospheric energetics only as an aid which helps through analogies. Naturally, physical state variables defined using arbitrarily chosen reference state can be expressed well the local atmospheric properties, which are suitable in the description of the local energetics of the atmosphere. This is expressed by Marquet in the following way: „The available enthalpy in this sense is a general thermodynamic local state function which has proved to be significant in order to be described as the part of (thermodynamic) energy that can be transformed into any other form of energy. But this exergy approach was not so easy to introduce in atmospheric energetics, since the temperature of the thermostat (which is the central concern of the exergy theories) must be replaced by the definition of a mere numerical value." (*Marquet*, 1991[2])

9. The available energy of Tailleux

Novak and *Tailleux* (2018) studied the available energy density for dry air, hydrostatic atmosphere, while *Tailleux* (2018) examined the local available energy of compressible stratified multicomponent fluids. Relying on the work of *Andrews* (1981) and *Holliday* and *McIntyre* (1981), they have defined the available potential energy by using simple physical principles, essentially the Archimedes' law. The available potential energy can be identified with the work which is necessary to bring an air parcel with fixed density from the reference state where it is in equilibrium to a state corresponding to the actual state of the atmosphere, against the buoyancy forces. This motion can be described with the equation:

$$\rho_r \frac{\mathrm{d}w}{\mathrm{d}t} = (\rho - \rho_r) g \,, \tag{51}$$

where ρ_r and ρ are the density of the parcel and the density of the environmental atmosphere, respectively. The specific buoyancy force is

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{\rho - \rho_r}{\rho_r} g \,, \tag{52}$$

and the work performed against it is

$$E_a = -\int_{z_r}^{z} \frac{\rho - \rho_r}{\rho \rho_r} \rho g \mathrm{d}z = \int_{p_r}^{p} \left(\frac{1}{\rho_r} - \frac{1}{\rho}\right) \mathrm{d}p = \int_{p_r}^{p} (\alpha_r - \alpha) \mathrm{d}p \,. \tag{53}$$

The crucial point of this definition is the choice of the reference state as well.

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[2] A new (2014) version of the original paper is available on the webpage of the **author** with valuable new comments. http://www.umr-cnrm.fr/spip.php?article833&lang=en

Tailleux (2018) studied the local version of APE which includes diabatic sinks and sources. For simple compressible fluids, the total potential energy of the air portion is the sum of the available elastic energy and the APE. These energies represent the adiabatic shrinkage or expansion work required to move from the reference pressure to the actual pressure. Tailleux writes the APE formula using primitive equations of motion, which is simpler and more clearly visible than the previously published formulas for the relationship between the APE (density given by the Boussinesq approximation) and the basic equation of hydrostatics. The key step was to introduce a hybrid function of β depending on thermodynamic coordinates and altitude, which reminds the non-kinetic energy in the Bernoulli equation, which is actually the sum of the classical potential energy and a $\frac{p_0}{\rho}$ term[3]. According to the definition of Tailleux, β concerns the actual state, while β_r is for the reference state, and it can be expressed in the following forms:

$$\beta(z, S_i, s, p) = \Phi(z) + e_{th}(S_i, s, p) + \frac{p_0(z)}{\rho(S_i, s, p)}, \tag{54}$$

$$\beta_r(z_r, S_i, s, p_r) = \Phi(z_r) + e_{th}(S_i, s, p_r) + \frac{p_0(z_r)}{\rho(S_i, s, p_r)}, \tag{55}$$

where e_{th} is the specific internal (thermal) energy, S_i denotes the components of the fluid, the meaning of the other quantities are usual, and the variables with indices r stand for variables of the reference state. Tailleux proved that $\Pi = \beta - \beta_r$ quantity is positive definite and gives the potential energy density.

In this context, it clarifies the relationship between the Boussinesq approximation and the energetics of stratified fluids and shows the physical background underlying the choice of reference state in the local APE theory.

10. The approach of the moist atmosphere

Cyclones of the midlatitudes strongly influence the climate. In today's climate change, it is important to understand the relationship between the climate and the energies which drive the atmospheric processes. Besides the (vertical and meridional) temperature gradients, the moisture influences the APE. *Gertler* and *O'Gorman* (2019) found that increasing surface temperature causes increasing APE and decreasing meridional surface temperature gradient causes decreasing APE. The extratropical summer cyclones on the Northern Hemisphere are

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[3] Batchelor calls the $\frac{p_0}{\rho}$ term as fictitious potential energy associated with the pressure field in his famous book: An Introduction to Fluid dynamics (2009 p 158).

weakening with the increasing temperature and moisture, however, despite the weakening, the convection intensifies.

The atmosphere at a given moment is determined by its mass and the field of velocity. This means that it depends on the distribution of ρ, T, p, and the phases of water, along with the wind components (*Lorenz*, 1979). Consequently, to use atmospheric energetics in the explanation of the real processes, the role of the moisture must be taken into account. The decisive step toward this was also taken by Lorenz (*Lorenz*, 1978; 1982). First, he elaborated a graphic procedure for determining the available energy of the moist atmosphere, and after it, he gave an algorithm for using this method in numerical calculations. The core problem is the determination of the reference state of the moist atmosphere, that cannot be determined as easily as in the dry case, by an analytical method according to entropy or humidity distribution, considered as state-of-the-art. However, finding the reference state is an analytical assignment problem, but it is computationally intensive. It is based on discretizing the atmosphere with equal mass domains and redistribute them by pressure. An additional difficulty is that with the addition of moisture, several minimal states could be created, consequently, it affects APE.

If the atmosphere is divided into parcels of equal mass, the changes in the atmosphere can be identified if the redistribution of these parcels is detected. The goal is to find the permutation of the parcels of the atmosphere with the least enthalpy (*Harris* and *Tailleux*, 2018). To be able to form parcels with equal mass, an x coordinate (following Lorenz) was created that covers different areas starting from the pole, but the mass belonging to them is equal. Parcels are characterized by the temperature T and by an indicator for the moisture, which can be, e.g., the specific humidity q or relative humidity f_r, these were zonally averaged over pressure levels. The Lorenz coordinate can be obtained:

$$x = -\frac{1}{2{R_E}^2\pi}\int_{\frac{\pi}{2}}^{\varphi}\int_{0}^{\pi} 2R_E cos\varphi \mathrm{d}\lambda R_E \mathrm{d}\varphi = -\frac{1}{2{R_E}^2\pi} 2{R_E}^2\pi[sin\varphi]_{\frac{\pi}{2}}^{\varphi} = \\ = -\left(sin\varphi - sin\frac{\pi}{2}\right), \quad (56)$$

$$x = 1 - sin\varphi\,, \quad (57)$$

where R_E is the radius of the Earth. The sum of the potential and internal energy equals the enthalpy, and it is the function of the thermodynamic variables, while the reference state is the minimum enthalpy one. While in the case of a dry atmosphere the enthalpy of the atmosphere is proportional to the sensitive heat, in the wet atmosphere the latent heat is added to the sensitive heat due to the phase transitions (*Lorenz*, 1979).

The calculation of moist available potential energy ($MAPE$) is based on Lorenz's numerical procedure, which relies on a graphical method elaborated also by Lorenz. The graphical method replaces the equation of a reversible adiabatic process with the plotted state curve. The essence of this procedure is, as it was already mentioned, to divide the atmosphere into parcels by equal mass and to rearrange these parcels. In the wet atmosphere, the dynamics of the parcels are determined by three variables: the pressure, the temperature, and a measure of the humidity, which is, in this case, the relative humidity f_r, but it could be the specific humidity as well (*Lorenz*, 1978).

In the procedure, the atmosphere is divided into N equal masses, each p_n parcel has its p, T, and f_r. All the parcels together completely cover the whole atmosphere between p_A and p_B, it is assumed that the part of the atmosphere above p_B has such a small $MAPE$ that it is negligible. The space between p_A and p_B is divided vertically into M parts, while horizontally into L parts, so: $N = M * L$.

$$n = l * M + m\,, \tag{58}$$

where: $n \in N$, $0 \leq l < L$, and $0 < m \leq M$. The pressure of p_n parcel is chosen to be:

$$p_n = p_0 + n\Delta p\,, \tag{59}$$

where $p_0 = p_A - \frac{\Delta p}{2}$ and $\Delta p = \frac{p_B - p_A}{N}$. The pressure of the reference state is p_1, …, p_n, but the order is different, so the permutation of the actual state is needed (Lorenz, 1978).

The permutation can be obtained by finding the best algorithm, which sorts the parcels fast and accurately enough. This is the question of our time concerning MAPE.

Stansifer et al. (2017) calculated the exact minimum enthalpy of the atmosphere for three cases. Five parcel-sorting algorithms were applied: Munkres algorithm or so-called Hungarian method (*Kuhn*, 1955), divide-and-conquer algorithm, greedy algorithm, Lorenz's algorithm, and the *Randall* and *Wang*'s (1992) algorithm. They lighted upon that the Munkres algorithm is the most accurate, but if the computational speed is preferred, then the divide-and-conquer algorithm should be used.

Harris and *Tailleux* (2018) re-examined the result of *Stansifer et al.* (2017) and extended the list of the analyzed algorithms, these are the top-down-, bottom-up-, and Emanuel algorithm. They tested these algorithms for more than 3700 soundings, and they found that best solution of the calculation of the exact $MAPE$ is the Munkres algorithm, however, it has long runtime. When the approximation of the $MAPE$ is enough *Harris* and *Tailleux* (2018) suggest the use of the divide-and-conquer method.

There are cases, when some algorithms give negative $MAPE$ values, which means that the reference state has higher energy than the actual state. The divide-and-conquer algorithm sometimes gives back negative values, but rarer than the top-down or Lorenz' algorithms (*Harris* and *Tailleux*, 2018).

Peng et al. (2015) have chosen a reference state based on a model atmosphere, so it is independent from both the actual and the average state of the atmosphere. It shows that relying on the exergy theory, an arbitrary state can be defined as reference one.

It can be seen that in this area of atmospheric energetics, numerous open questions can be found, so it is not enough mature to be involved in the university curricula.

11. Summary

The atmospheric energetics was reviewed with particular attention to educational aspects. The decisive step of the development of atmospheric energetics was the introduction of the availability concepts. Having made this by Lorenz, in the development of the field, the global and local description of the energy cycle of the dry atmosphere was made, and recently the global and local description of the moist atmosphere is intensively investigated. The crucial question of these investigations is the definition and finding of the reference state. The deviation of the actual energy from it gives the available energy. Reference state should be chosen on physical considerations, so that it should be an equilibrium state of the atmosphere. However, applying the exergy concept makes it possible to choose a referential state arbitrarily. In our opinion, exergetics is a useful tool for the treatment of nonequilibrium thermodynamics, but for the atmosphere as a whole, it should be used only through analogies. We think that in the current state the basic results of atmospheric energetics are ripe to be involved in the university curricula. This is also supported by the development of computer technology, which enables the reproduction of numerical calculations on desktop computers based on the new databases. Equally important would be to incorporate the foundations of exergetic methods into atmospheric thermodynamics curricula.

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References

Andrews, D., 1981: A note on potential energy density in stratified compressible fluid. *J. Fluid Mech.*, *107*, 227–236. https://doi.org/10.1017/S0022112081001742

Bejan, A. 2006: Advanced Engineering thermodynamics. Wiley, New Yersey. ISBN-13:978-0-471-67763-5

Callen, H.B., 1960: Thermodynamics, Wilkey and sons, London ISBN: 0471862568

Copernicus Climate Change Service (C3S), 2017: ERA5, Fifth generation of ECMWF atmospheric reanalyses of the global climate. Copernicus Climate Change Service Climate Data Store (CDS). https://cds.climate.copernicus.eu/cdsapp#!/home.

Czelnai, R., Götz, G., and *Iványi, Zs.* 1991: Bevezetés a meteorológiába II. A mozgó légkör és óceán. Tankönyvkiadó, Budapest, (Introduction to Meteorology II. The moving atmosphere and ocean.) Book number: J3-1344. (in Hungarian)

Dési, F. and *Rákóczi, F.,* 1970: A légkör dinamikája. Tankönyvkiadó, Budapest. in Hungarian (Dynamics of the atmosphere.) Book number: 42135. (in Hungarian)

Dincer, I. and *Rosen, M.A.,* 2007: Exergy. Elsevier, Amsterdam. ISBN: 978-0-08-044529-8

Dutton, J.A. and *Johnson, D.R.*, 1967: The Theory of Available Potential Energy and a Variational Approach to Atmospheric Energetics. *Adv. Geophys.*, *12,* 333–436. https://doi.org/10.1016/S0065-2687(08)60379-9

Dutton, J.A., 1973: The global thermodynamics of atmospheric motion. *Tellus*, *25*, 89–110. https://doi.org/10.1111/j.2153-3490.1973.tb01599.x

Dutton, J.A., 1976*:* The Ceaseless Wind. McGraw Hill, USA. ISBN 0-07-018407-0

Dutton, J. A. 1992: Energetics with an entropy flavour. *Quart. J. Roy. Meteorol. Soc. 118,* 165–166. https://doi.org/10.1002/qj.49711850309

Gertler, C.G. and *O'Gorman, P.A.*, 2019: Changing available energy for extratropical cyclones and associated convection in Northern Hemisphere summer. *PNAS*, *116*, 4105–4110. https://doi.org/10.1073/pnas.1812312116

Götz, G. and *Rákóczi, F.,* 1981: A dinamikus meteorológia alapjai. Tankönyvkiadó, Budapest. ISBN: 963 17 5580 0 (Foundations of the dynamic meteorology.)(in Hungarian)

Harmatha, A., 1982: Termodinamika műszakiaknak. Budapest, Műszaki Könyvkiadó. ISBN: 963-10-4467-X (Engineering Thermodynamics.) (in Hungarian)

Harris, B.L. and *Tailleux, R.*, 2018: Assessment of algorithms for computing moist available potential energy. *Quart. J. Roy. Meteorol. Soc.*, *144,* 1501–1510. https://doi.org/10.1002/qj.3297

Haywood, R.W., 1974a: A critical review of the theorems of thermodynamic availability, with concise formulations. Part 1. *J. Mech. Eng. Sci., 16,* 161–174. https://doi.org/10.1243/JMES_JOUR_1974_016_046_02

Haywood, R.W., 1974b: A critical review of the theorems of thermodynamic availability, with concise formulations. Part 2.*, J. Mech. Eng. Sci., 16,* 258–267 https://doi.org/10.1243/JMES_JOUR_1974_016_046_02

Holliday, D. and *McIntyre, M. E.,* 1981: On potential energy density in an incompressible stratified fluid. *J. Fluid. Mech., 107,* 221–225. https://doi.org/10.1017/S0022112081001742

Holton, J.R., 2004: An introduction to Dynamic Meteorology. Elsevier, Amsterdam. ISBN: 0-12-354015-1

Hoskins, J.B. and *James, N.,* 2014*:* Fluid dynamics of the midlatitude atmosphere, Wiley Blacwell. ISBN 978-0-470-79519-4 https://doi.org/10.1002/9781118526002

Jacobs, S.S., Jenkins, A., Giulivi, C.F., and *Dutrieux, P.,* 2011: Stronger ocean circulation and increased melting under Pine Island Glacier ice shelf. *Nat. Geosci.*, *4*, 519–523. https://doi.org/10.1038/ngeo1188

Karlsson, S., 1990: Energy, entropy and exergy in the atmosphere (thesis). Chalmers Tekniska Högskola, Bibliotekets Reproservice, Göteborg. ISBN: 91-7032-526-X

Keenan, J., 1951: Availability and irreversibility in thermodynamics. *Br. J. Appl. Phys 2,* 183–192. https://doi.org/10.1088/0508-3443/2/7/302

Kucharski, F., 1997*:* On the concept of exergy and available potential energy. *Quart. J.Roy. Meteorol. Soc. 123,* 2141–2156. https://doi.org/10.1002/qj.49712354317

Kuhn, H.W., 1955: The Hungarian method for the assignment problem. *Nav. Res. Logist. Q., 2,* 83–97. https://doi.org/10.1002/nav.3800020109

Lorenz, E.N., 1955: Available Potential Energy and the Maintenance of the General Circulation. *Tellus*, *7*, 157–167. https://doi.org/10.3402/tellusa.v7i2.8796

Lorenz, E.N., 1978: Available energy and the maintenance of a moist circulation. *Tellus*, 30, 15–31. https://doi.org/10.3402/tellusa.v30i1.10308

Lorenz, E.N., 1979: Numerical evaluation of moist available energy. *Tellus, 31*, 230–235. https://doi.org/10.3402/tellusa.v30i1.10308

Lorenz, E.N., 1982: Low-order models of atmospheric circulations. *J. Meteorol. Soc. Japan 30,* 15–31. https://doi.org/10.2151/jmsj1965.60.1_255

Major, Gy., Nagy, Z., and *Tóth, Z.* 2002: Magyarországi éghajlat-energetikai tanulmányok. A Budapesti Közgazdaságtudományi és Államigazgatási Egyetem Környezettudományi Intézetének tanulmányai. Budapest. ISBN: 963-503-273-0 (Hungarian climate energy studies.) (in Hungarian)

Mak, M., 2011: Atmospheric Dynamics. Cambridge Univ. Press. ISBN: 978-0-521-19573-7

Makainé, Cs., M., 1971: A közepes energetikai szint néhány statisztikai és szinoptikai sajátossága. *Időjárás, 75,* 349–360. (Some statistical and energetical properties of the average energetical level.) (in Hungarian)

Makainé, Cs., M., 1972: Orográfiai hatások elemzése a hasznositható potenciális energia és kinetikus energia változásában. *Időjárás, 76.,* 334–340. (Analisys of orografical influences of the variation of the available potential and kinetic energy.) *(*in Hungarian)

Makainé, Cs., M., 1974: A barotrop légkör energetikai aktivitásáról. *Időjárás, 78.,* 109–115. (On the energetical activity of barotropic atmsosphere.) (in Hungarian)

Margules, M., 1906: The last paper "On the theory of storm" (Zur Sturmtheorie). *Meteorol. Z. 23*, 481–497. translated by *Marquet, P.* in 2017.

Margules, M., 1910: The mechanical equivalent of any given distribution of atmospheric pressure, and the maintenance of a given difference in pressure. *Smithsonian Miscellaneous collection 54,* 501–532.

Marquet, P., 1991: On the concept of exergy and available enthalpy: Application to atmospheric energetics. *Quart. J. Roy. Meteor. Soc. 117,* 449–475. https://doi.org/10.1002/qj.49711749903

Mulligan, J.F. and *Hertz, H.G.,* 1997: An unpublished lecture by Heinrich Hertz: "On the energy balance of the Earth''. *Am. J. Phys, 65,* 36–45. https://doi.org/10.1119/1.18565

Novak, L. and *Tailleux, R.*, 2018: On the local view of atmospheric available potential energy. *J. Atmos. Sci.*, *75*, 1891–1907. https://doi.org/10.1175/JAS-D-17-0330.1

Oort, A., 1971: The observed annual cycle in the meridional transport of atmospheric energy. *J. Atmos. Sci. 28,* 325–339.https://doi.org/10.1175/1520-0469(1971)028<0325:TOACIT>2.0.CO;2

Panchev, S., 1985: Dynamic Meteorology. D. Reidel Publishing Co. Kluwer, ISBN 90-277-1744-1 https://doi.org/10.1007/978-94-009-5221-8_1

Peng, J., Zhang, L., and *Zhang, Y.*, 2015: On the local available energetics in a moist compressible atmosphere. *J. Atmos. Sci.*, *72*, 1551–1561. https://doi.org/10.1175/JAS-D-14-0181.1

Práger, T., 1982: Numerikus Prognosztika I. Tankönyvkiadó, Budapest (Numerical prognostics I.) Book number: J3-1244.(in Hungarian)

R Core Team, 2017: R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/.

Randall, D.A., and Wang, J., 1992: The moist available energy of a conditionally unstable atmosphere. *J. Atmos. Sci. 49,* 240–255. https://doi.org/10.1175/1520-0469(1992)049<0240:TMAEOA>2.0.CO;2

Schulzweida, U., 2019: CDO User Guide. 1–206. https://code.mpimet.mpg.de/projects/cdo/embedded/cdo.pdf

Shepherd, A. and *Wingham, D.*, 2007: Recent sea-level contributions of the of the Antarctic and Greenland ice sheets. *Science 315*, 1529–1533. https://doi.org/10.1126/science.1136776

Stansifer, E.M., O'Gorman, P.A., and *Holt, J.I.*, 2017: Accurate computation of moist available potential energy with the Munkres algorithm. *Quart. J. Roy. Meteor. Soc.*, *143*, 288–292. https://doi.org/10.1002/qj.2921

Su, Z., and Ingersoll, A.P., 2016: On the minimum potential energy state and the eddy size-constrained ape density. *J. Phys. Oceanogr.*, *46*, 2663–2674. https://doi.org/10.1175/JPO-D-16-0074.1

Tailleux, R., 2013: Available potential energy and exergy in stratified fluids. *Annu. Rev. Fluid Mech.*, *45*, 35–58. https://doi.org/10.1146/annurev-fluid-011212-140620

Tailleux, R., 2018: Local available energetics of multicomponent compressible stratified fluids. *J. Fluid Mech.*, *842*, R1. https://doi.org/10.1017/jfm.2018.196

Trenberth, K.E., Fasullo, J.T., and *Balmaseda, M.A.*, 2014: Earth's energy imbalance. *J. Climate*, *27*, 3129–3144. https://doi.org/10.1175/JCLI-D-13-00294.1

Van Mieghem, J., 1973: Atmospheric Energetics. Clarendon Press, Oxford, UK, ISBN 10: 0198516053

Wiin-Nielsen, A. and *Chen, T.C.,* 1993: Fundamentals of Atmospheric Energetics. Oxford and New York and Oxford, ISBN 0-19-507127-1

Zdunkowski, W. and *Bott, A.* 2003: Dynamics of the atmosphere. Cambridge Univ. Press. ISBN-13: 978-0-52100666-8 https://doi.org/10.1017/CBO9780511805462

Appendix A. Entropy and energy calculation

The appendix summarizes the numerical procedures, databases, and software, which were applied in the calculations, the results of which are presented in *Figs. 3–6* and in *Table 2*.

The target area was the Northern Hemisphere on 1°×1° spatial resolution, vertically 19 pressure levels were used (1000–100 hPa by 50 hPa). The examined time period was 1979–2019. The meteorological dataset used in this study was the product of the European Centre for Medium-Range Weather Forecasts (ECMWF), the ERA5. The ERA5 database is the fifth generation of the ECMWF reanalysis, available from 1979 with relatively small delay. ERA5 provides in atmospheric, terrestrial, and oceanic data hourly approximation. It combines large amount of historical observations (satellite and in-situ) with global modeling with advanced modeling and data assimilation systems. ECMWF data is available in NetCDF (network common data form) form, which is advantageous, because such format is suitable for storing array information (*Copernicus Climate Change Service*, 2017).

Monthly averages were calculated by using a free software called CDO (Climate Data Operators) developed by the Max Planck Institute (*Schulzweida*, 2019).

The obtained data were processed with the R program (*R Core Team*, 2017). With the help of this software, calculations and representations were also made.

T_r was calculated numerically from Eq. (43):

$$\frac{1}{T_r} = \frac{1}{\Delta \mathrm{t}} \int_{t_1}^{t_2} \left[\iiint \frac{1}{T} \frac{\mathrm{d}m}{M} \right] \mathrm{d}t = \frac{1}{\Delta \mathrm{t}} \int_{t_1}^{t_2} \frac{1}{T_{m(t)}} dt \,, \tag{A1}$$

where T_r is a characteristic value of the terrestrial atmosphere, which remains close to 250 K under current climatic conditions.

Entropy was calculated by using Eq. (39), where the second (pressure dependent) term diminishes after the integration, because:

$$\int_{V_A} R\ln\left(\frac{p}{p_r}\right) \mathrm{d}V = \frac{R}{g} \int_A \mathrm{d}\lambda \mathrm{d}\varphi \int_0^{p_0} \ln\left(\frac{p}{p_r}\right) {R_F}^2 cos\varphi \mathrm{d}p =$$
$$= \frac{R}{g} \int_0^{p_0} \ln\left(\frac{p}{p_r}\right) 4{R_F}^2 \pi \mathrm{d}p = \frac{R}{g} 4{R_F}^2 \pi \left(-\int_{p_0}^{0} \ln(p) \mathrm{d}p + \int_{p_0}^{0} \ln(p_r) \mathrm{d}p \right) =$$
$$= \frac{R}{g} 4{R_F}^2 \pi \left(-p_0 \ln(p_r) + p_0 \ln(p_r) \right) = 0 \,. \tag{A2}$$

Total energy of the atmosphere was calculated as follows:

$$e_{total} = c_p T + \frac{1}{2}\mathbf{v}^2. \qquad (A3)$$

If we integrate over the atmosphere, we get the energy and the entropy for the atmosphere:

$$E_{total} = \frac{1}{g}\sum e_{total}\, {R_E}^2 cos\varphi \mathrm{d}\lambda \mathrm{d}\varphi \mathrm{d}p, \qquad (A4)$$

$$S = \frac{1}{g}\sum s_T\, {R_E}^2 cos\varphi \mathrm{d}\lambda \mathrm{d}\varphi \mathrm{d}p, \qquad (A5)$$

where g = 9.81$\frac{m^2}{s}$, the gravitational constant, R_E (= 6370 km) is the radius of the Earth, λ is the longitude, and φ is the latitude, so dλdφ = 1°×1° due to the resolution.

a_h is available from Eq. (29), a_p is calculated by Eq. (32), and a_T is reached after simplifying Eq. (31):

$$a_T \approx c_p \frac{(T - T_r)^2}{2T_r}. \qquad (A6)$$