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# ABOUT THE GRINDING OF GEAR HOB'S RAKE FACE

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**Abstract**

The most simple and robust construction of the monolithic gear hobs present a common helical rake face for a given line of teeth, whose generatrix is a straight-line segment perpendicular to the hob's axis while its directory is a helix, perpendicular to the pitch helix. As a consequence, constructive rake angles are zero on all edges. Total curvatures of such a surface are negative. Thus, it can be grinded only using the conical surface of a platter type grinding wheel, or a grinding bit. Despite this, some industry practices, possibly for reasons of simplicity and cost lowering, involve the plain grinding surface, supposed to a helical motion. This paper deals with the CAD-simulation of the grinding process using the plain wheel surface, and it shows the differences between the theoretical and rake face and the real obtained helical surface.

**Keywords**: *gear-hob, rake face, grinding, meshing, through-cut.*

**Notations**

$m_n$ – normal module [mm];
$\alpha_0$ – normal rack profile angle [°];
$p_C$ – rake helix parameter [mm];
$R_0$ – pitch radius, [mm];
$R_a$ – addendum cylinder radius, [mm];
$R_f$ – dedendum cylinder radius [mm];
$\lambda_0$ – leading helix declination angle of the gear-hob [°];
$\lambda_x$ – crossing angle between the axis of the gear hob and the plane face of the grinding wheel, [°];
$a_w$ – axis distance [mm];
$\nu$ – surface normal unit vector

## 1. The mathematical model of the rake face

A gear hob's rake face usually is a helical surface of parameter $p_C$ [1, 2], meshed by a straight line that crosses the gear hob's axis (Figure 1). Thus, the parametric equations of the helix surface denoted with Σ are as follows:

$$\begin{cases} x_2(u,\varphi) = p_C\varphi \\ y_2(u,\varphi) = u\cos\varphi \\ z_2(u,\varphi) = -u\sin\varphi \end{cases} \quad (1)$$

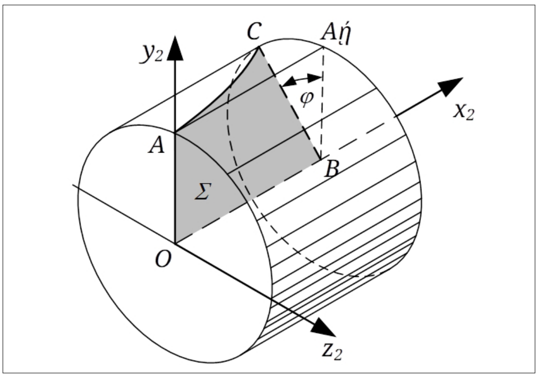


**Figure 1.** *AThe theoretical helical rake face*

## 2. The relative motion between the grinding wheel and the gear hob

Let us consider the geometric elements shown in **Figure 2.** Three frames will be used here: S_0 is the fixed frame, while $S_1$ and $S_2$ are connected to the grinding wheel, and the gear-hob. The relative motion of the grinding wheel related to the gear hob results from a rotation of angle $\varphi$ sof the gear- hob about its own axis while the grinding wheel is translated, corresponding to this, along the same axis with a distance of $p_c\,\varphi$ corresponding to the parameter of the leading helix. Mathematically, this can be formulated using the matrix equation below:

$$\underline{\boldsymbol{r}}_2 = \boldsymbol{M}_{20}\,\boldsymbol{M}_{01}\underline{\boldsymbol{r}}_1 \tag{2}$$

The expressions of the matrices in eq. (2) are the followings:

$$\boldsymbol{M}_{20} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi & 0 \\ 0 & -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{M}_{01} = \begin{pmatrix} \cos\lambda_x & 0 & \sin\lambda_x & p_c\varphi \\ 0 & 1 & 0 & a_w \\ -\sin\lambda_x & 0 & \cos\lambda_x & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

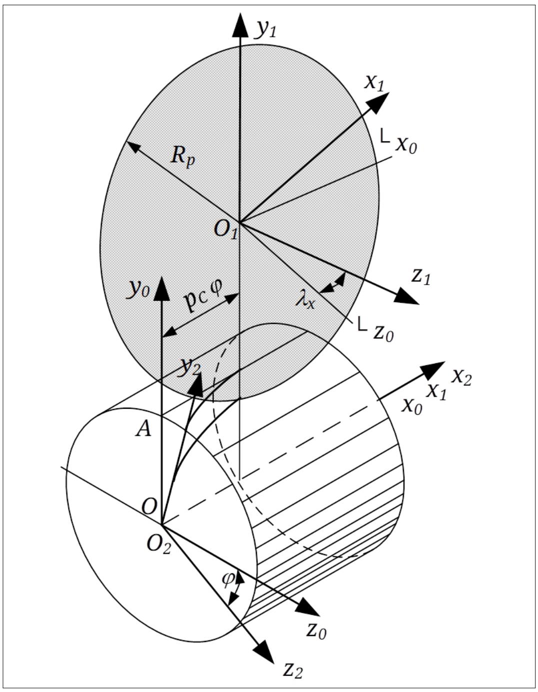


**Figure 2.** *Ahe applied frames*

The axis distance results from the sum of dedendum radius of the gear-hob and the largest radius of the grinding wheel:

$$a_w = R_f + R_p \tag{4}$$

The equation of the plane surface of the grinding wheel is given below in polar coordinates:

$$\begin{cases} x_1(\rho,\theta) = \rho\sin\theta \\ y_1(\rho,\theta) = -\rho\cos\theta \\ z_1 = 0 \end{cases} \tag{5}$$

With the use of eq. (2), it is possible to obtain the motion of any point of the grinding wheel related to the gear hob. Thus, an estimation of the shape of the resulting flute part – the rake face – can be achieved.

First of all, it must be stated here that the grinded rake face results here not from meshing, but from a continuous and infinite series of through-cuts. The through -cut is a specific case of generating, when the tool's imprints are not tangent to the resulting surface, but they intersect it. No common normal vector can be defined here – at least inside the limits of the machined part. A study of the phenomenon related before can be performed using the so called Monge-brick **[3]**.

Let's consider the helix surface given by eq. (1). Omitting the detailed computing **[4]** the invariant quantities of the first and the second fundamental form are as follows:

$$\begin{cases} E = \dfrac{\partial\boldsymbol{r}}{\partial u}\cdot\dfrac{\partial\boldsymbol{r}}{\partial u} = 1 \\ F = \dfrac{\partial\boldsymbol{r}}{\partial u}\cdot\dfrac{\partial\boldsymbol{r}}{\partial\varphi} = 0 \\ G = \dfrac{\partial\boldsymbol{r}}{\partial\varphi}\cdot\dfrac{\partial\boldsymbol{r}}{\partial\varphi} = p_c^2 + u^2 \\ L = -\dfrac{\partial\boldsymbol{r}}{\partial u}\cdot\dfrac{\partial\boldsymbol{v}}{\partial u} = 0 \\ M = -\dfrac{1}{2}\left(\dfrac{\partial\boldsymbol{r}}{\partial u}\cdot\dfrac{\partial\boldsymbol{v}}{\partial\varphi} + \dfrac{\partial\boldsymbol{r}}{\partial\varphi}\cdot\dfrac{\partial\boldsymbol{v}}{\partial u}\right) = \dfrac{p_c}{\sqrt{p_c^2+u^2}} \\ N = -\dfrac{\partial\boldsymbol{r}}{\partial\varphi}\cdot\dfrac{\partial\boldsymbol{v}}{\partial\varphi} = 0 \end{cases} \tag{6}$$

The principal curvature's algebraic equation of second degree $(EG - F^2)k^2 - (EN - 2FM + GL)k + (LN - M^2) = 0$ **[4]** becomes here:

$$(p_c^2 + u^2)k^2 - \frac{p_c^2}{p_c^2 + u^2} = 0 \tag{7}$$

whose roots are:

$$k_{1,2} = \pm\frac{p_c}{p_c^2 + u^2} \tag{8}$$

Either interpreting expression (8) or analyzing the sign of $LN-M^2$ it results that every point of the surface is a hyperbolic point, thus the total or Gauss curvature is negative. From here it results that the tangent plane (i.e., perpendicular to the normal vector in the considered point) intersects the surface in any point along it, due to the fact that the normal curves corresponding to themain curvatures are located on opposite sides of this plane. In conclusion, the surface given by eq. (1) is impossible to grind using a flat grinding wheel. However, practice shows that grinding using a flat surface can lead to a continuous smooth surface, although the modeling of this cannot be realized by applying the meshing theory.

The statement above was verified by a numerical simulation, performed on a gear-hob of $m_n = 5$ mm, $\lambda_0 = 3{,}5°$ and one thread. The point-clouds generated by the flat side of the grinding wheel were realized for the axis cross angle values of $\lambda_x = \lambda_f$, $\lambda_x = \lambda_0$ and $\lambda_x = \lambda_a$. **Figures 3** and **4** show the point-clouds resulting for the smallest and the largest crossing angle value. Here, it must be emphasized that the larger the crossing angle $\lambda_x$ the greater the resulting difference between the theoretical surface and the point-cloud.

In any case the point-cloud covers the ideal surface, namely the ideal surface points are situated inside the border of the generated point-cloud. Consequently, the grinding operation produces continuous through-cuts or interferences.

In the following, real grinded surfaces will be studied using CAD models.

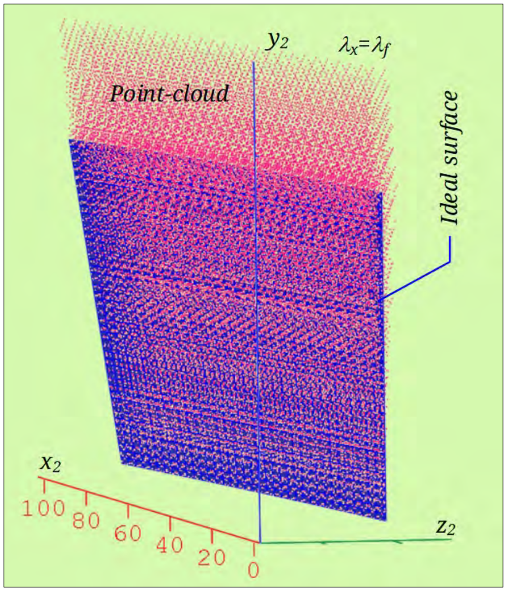


**Figure 3.** *The trace of the grinding wheel (pink) and the ideal surface (blue) for the smallest axis crossing angl* $\lambda_x = \lambda_f$.

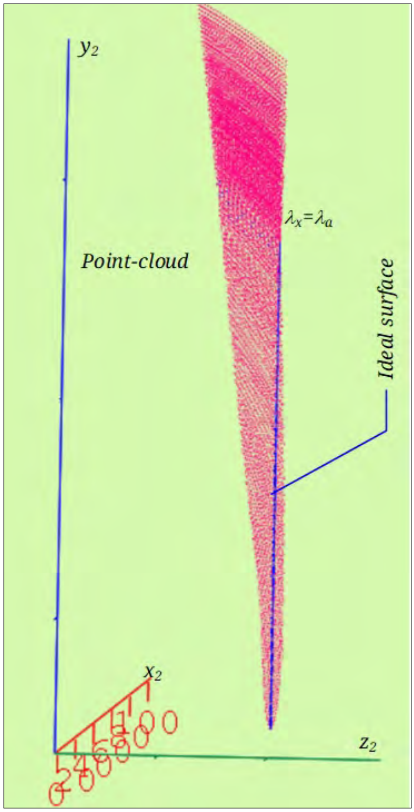


**Figure 4.** *The trace of the grinding wheel (pink) and the ideal surface (blue) for the largest axis crossing angle* $\lambda_x = \lambda_a$.

## 3. The structure of the CAD-model

Construction of an efficient CAD-model requires consistent knowledge of the determining geometric elements of the gear-hob **[5]**. Here must be taken into consideration the technical solutions mentioned in the literature, together with the peculiarities of manufacturing using classical machine-tools and thus the effects of the technological simplifications.

### 3.1. Peculiarities of gear-hob manufacturing

Involute worm gear transmission is that particular case of helical involute gear pairs, where the inclination angle of the tooth of one of the elements becomes so large that the tooth becomes a thread and thus the gear turns into a worm.

The traditional involute gear hob derives from an involute worm. Intersecting this with a finite number of equidistant flutes results in the teeth-

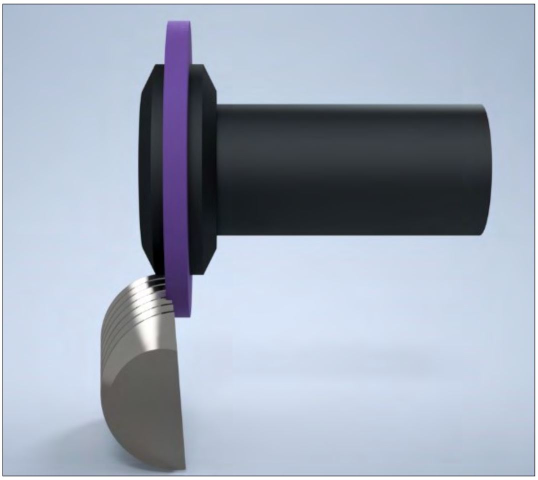

**Figure 5.** *The model of the rake face and the grinding wheel*

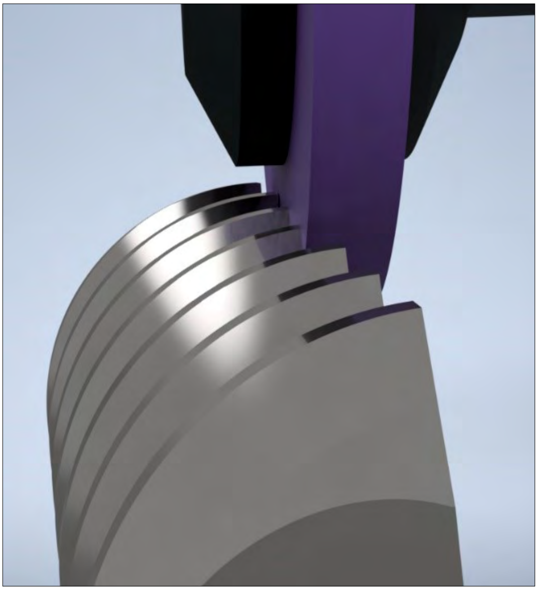

**Figure 6.** *The through-cut of the grinding wheel*

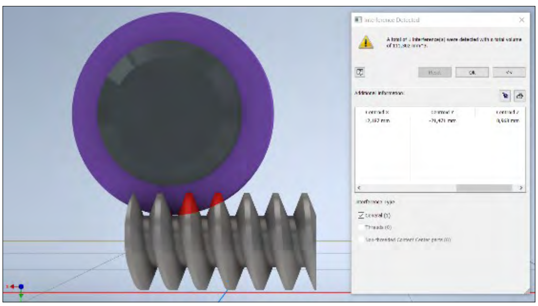

**Figure 7.** *Analysis of interference*

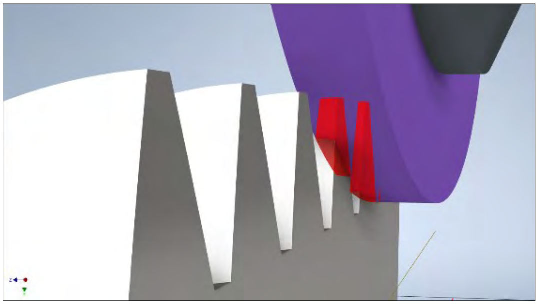

**Figure 8.** *The interference in 3D representation*

lines that later become subject of the relieving operation in order to realize the relief angles on all cutting edges. A remarkable property of the helical rake face consists in its perpendicularity on the leading (pitch) helix of the worm. Therefore, the constructive rake angle result is 0o.

When machining a gear-hob on classical machine-tools, certain simplifications were implemented. Due to the fact that the axial section the flank line of the involute worm is curved, it was in practice replaced often by a straight normal profiled ZN1-type worm **[6]**.

The relevance of the helical rake face consists in producing approximatively equal constructive geometry on the lateral edges, in order to equalize the durability of edges on both tooth flanks.

The CAD-model was created for the sharpening of the rake face with the flat side of the grinding wheel. Different axis crossing angles and wheel diameter values were considered. .

## 4. Numerical evaluation

As a consequence of the interference phenomenon presented above the resulting rake face is truncated. The error value depends on the grinding wheel diameter and the axis crossing angle.

### 4.1. The influence of the axis crossing angle

The influence of the axis crossing angle (**Figure 9**) was investigated for three distinct values that equal respectively the dedendum, the pitch and the addendum helix angle.

The results are shown in **Table 1.** Numerical values indicate the maximum of interference, considered in the normal direction.

### 4.2. The influence of wheel diameter

The diameter of the grinding wheel also influences significantly the value of the interference. The obtained results are also included in **Table 1.** The considered diameters were respectively 112.5 mm, 160 mm and 200 mm.

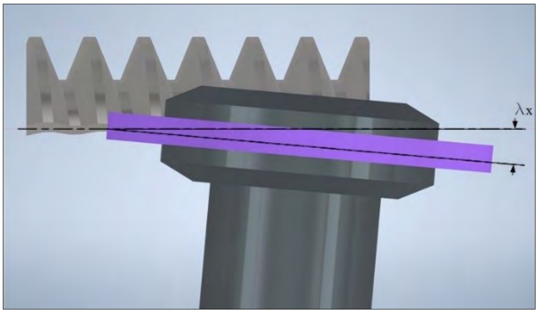


**Figure 9.** *The representation of the axis crossing angle (lx)*

**Table 1.** *Interference values for different crossing angle and diameter values.*

| Maximal normal interference [mm] | | | | |
|---|---|---|---|---|
| **No.** | **Wheel diameter [mm]** | **la** | **lo** | **lf** |
| 1 | 112.5 | 1.136 | 0.965 | 0.792 |
| 2 | 160 | 1.268 | 1.082 | 0.893 |
| 3 | 200 | 1.511 | 1.291 | 1.064 |

## 5. Conclusions

Gear-hob sharpening with a disk type grinding wheel is a common manufacturing technology. There exist a lot of demonstrative videos on the subject. The setup of a flat disk is simpler than those of a profiled grinding wheel. The disadvantage of the procedure arises from the interference that is inevitable. In the real process, errors of interference cumulate with the grinding machine setup errors and grinding wheel errors.

All these errors will result in inappropriate functional edge geometry and profile errors. As a result, a decrease in durability and precision of the manufactured gear will occur.

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