Simplified mechanical model for balancing a motorbike with steering at zero speed

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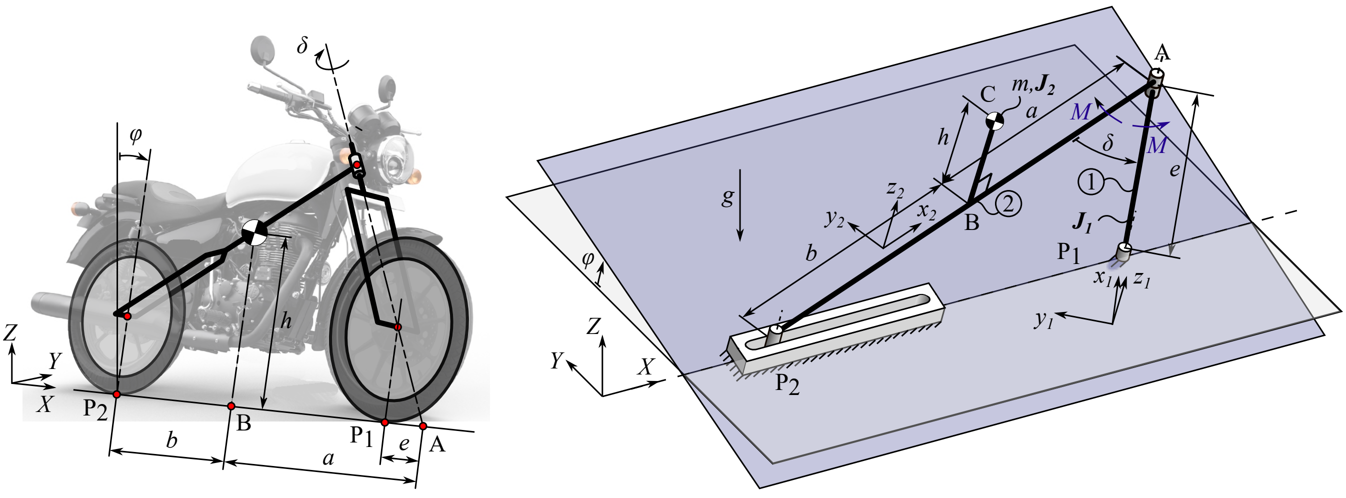
EXTENDED ABSTRACT

1 Introduction

In the development of autonomous vehicles, engineers are facing fascinating but complex tasks. The localization of the vehicle, the detection of the surrounding objects, the decision making, motion planning, and finally, the motion control of the vehicle provide challenges. These challenges are even more complicated when the automated vehicle is a motorcycle. The investigations of bicycles and motorcycles are very complex due to the fact that no in-plane mechanical models can accurately describe the dynamics of these vehicles. However, the governing equation of the required spatial nonholonomic models cannot be handled analytically [1, 2, 3]. To overcome this problem, we present a simplified mechanical model of a motorcycle, by which a linear feedback controller is designed to stabilize the unstable vertical equilibrium at zero longitudinal speed using the steering system.

2 Mechanical model

The simplified mechanical model is shown in Figure 1. When the tilting angle φ and the steering angle δ of the motorcycle are small, the pitch motion is negligible (see [4]). Hence, the crank mechanism in panel (b) is a possible simplified model of the vehicle. The contact point P_1 of the front wheel is modeled as a rotary joint, while the contact point P_2 of the rear wheel can move along the X-axis when the front wheel is steered. The plane of motion of the crank mechanism (marked with blue) can tilt with the tilting angle φ around the X-axis capturing the tilting motion of the motorcycle. The angle between the two arms of the crank mechanism is the steering angle δ. The internal steering torque is marked with M. Using the geometric parameters of the vehicle shown in panel (a), the trail e, the wheelbase $a+b-e$ and the height h of the centre of gravity determine the lengths of the rods of the crank mechanism.



(a) (b)

Figure 1: The geometry of the motorbike (a) and the simplified mechanical model (b).

The equations of motion of this two degree-of-freedom mechanical model can be derived using the Lagrangian equation of the second kind. After linearization around the equilibrium $\varphi \equiv 0$, $\delta \equiv 0$, we obtain:

$$\begin{bmatrix} \dfrac{J_{\mathrm{P}_1}^{z_1}(a+b)^2+(J_{\mathrm{C}}^{z_2}+mb^2)e^2}{(a+b-e)^2} & -\dfrac{mebh}{a+b-e} \\ -\dfrac{mebh}{a+b-e} & J_{\mathrm{P}_1}^{x_1}+J_{\mathrm{C}}^{x_2}+mh^2 \end{bmatrix} \begin{bmatrix} \ddot{\delta} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 & \dfrac{mgeb}{a+b-e} \\ \dfrac{mgeb}{a+b-e} & -mgh \end{bmatrix} \begin{bmatrix} \delta \\ \varphi \end{bmatrix} = \begin{bmatrix} M \\ 0 \end{bmatrix}, \tag{1}$$

where m is the mass of the vehicle, $J_{\mathrm{P}_1}^{x_1}$ and $J_{\mathrm{P}_1}^{z_1}$ refer to the mass moments of inertia of the steered fork-wheel system about x_1 and z_1 axes with respect to the point P_1 (see Figure 1(b)), respectively. $J_{\mathrm{C}}^{x_2}$ and $J_{\mathrm{C}}^{z_2}$ are the mass moments of inertia of the motorcycle body about x_2 and z_2 axes with respect to the center of gravity C.

3 Balancing with steering

The unstable vertical equilibrium position of the motorcycle can be stabilized by using the steering mechanism. This paper investigates the applicability of a PD controller in detail with and without feedback delay τ. The controller actuates trough the steering torque, namely, a lower level control of the steering servo creates the steering torque, while the desired steering angle δ_{des} is calculated by the higher level controller:

$$M = -K_{\mathrm{p}\delta}(\delta - \delta_{\mathrm{des}}) - K_{\mathrm{d}\delta}\dot{\delta}\,, \quad \text{and} \quad \delta_{\mathrm{des}} = -K_{\mathrm{p}\varphi}\varphi(t-\tau) - K_{\mathrm{d}\varphi}\dot{\varphi}(t-\tau)\,, \tag{2}$$

where $K_{\mathrm{p}\delta}$, $K_{\mathrm{d}\delta}$, $K_{\mathrm{p}\varphi}$ and $K_{\mathrm{d}\varphi}$ are the proportional and differential control gains corresponding to the steering angle and the roll angle respectively.

First, the linear stability of the closed-loop system was checked for the zero time delay case, and the stability boundaries were found analytically using the Liénard-Chipart stability criterion [5]. Stability charts were drawn in the $K_{\mathrm{p}\varphi} - K_{\mathrm{d}\varphi}$ plane using realistic, small scale vehicle model parameters, see dashed curves in Figure 2. The linearly stable domain is shaded with light blue color. Closed form formulas can be determined for the upper and lower limits of the lower level control parameters at which vertical position of the motorcycle is stabilizable.

Since the accurate measurement of the tilting angle requires filtering, a non-negligible time delay is introduced in the system. Considering this feedback delay, the stability boundaries were identified using semi-discretization [6]. Even a small value of the delay scales and shifts the stable region significantly in the stability diagram, see the dark blue domain in Figure 2.

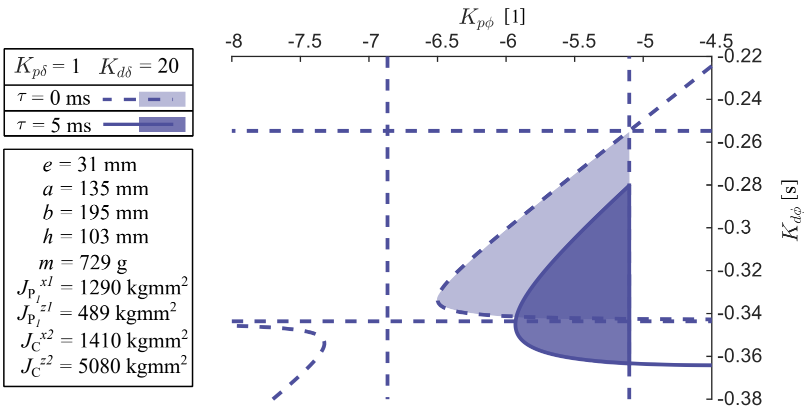


Figure 2: The stability chart.

4 Conclusion

A simplified mechanical model was constructed to design steering controller for balancing a motorcycle at zero longitudinal speed. It was shown that the hierarchical linear feedback controller is capable of stabilizing the vertical position of the motorcycle when the control gains are chosen appropriately and the feedback delay is small enough. The experimental validation of the results is the task of future work, for which a small scale experimental rig is already designed and manufactured.

Acknowledgments

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