

FUZZY ARITHMETICS IN THE EVALUATION OF QUALITY CHARACTERISTICS OF HIGH-PRESSURE FOOD PRESERVATION PROCEDURES

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Abstract: In recent decades increasing consumer awareness the innovations in the very different fields of technology and changes in societal norms have led to changes in food consumption patterns. The growing demand for minimally processed natural-textured, healthy, and microbiologically safe foods has shifted processing toward high-pressure preservation rather than traditional heat treatment processes. In recent years, fuzzy control has become important in the application of technology, which is able to meet the quality expectations of consumers by dealing with the blurred boundaries of various everyday product characteristics (taste, colour, texture, etc.). The applicability of fuzzy logic has been shown to be effective in a variety of process controls, including preservation procedures. The aim of this work is to summarize the basics of fuzzy arithmetic required in the fuzzy controls used in high-pressure preservation processes.

Keywords: fuzzy logic, fuzzy arithmetic, food safety, food quality, high pressure food processing

1. Introduction

As a consequence of globalization and improved consumer awareness associated with rising living standards, the food industry turned to an innovative stage in recent decades. Consumer trends are constantly changing. It means a daily active challenge for companies in the food preservation sector to satisfy customer needs and maintain their market share. Consumers demand high quality and convenient products, with natural flavour and taste, and very much appreciate the fresh appearance of minimally processed food in addition to microbiological safety [1].

In Hungary, the economic development of the last 10 years is reflected in food consumption as well where international trends are strengthening. The increase in solvent demand has led to a wider range of products, and to an evolution in quality and a higher need for these kinds of products as well [2]. As a result of the strengthening of consumer awareness, low-income consumers are also looking for high-quality products in the appropriate price categories [3].

These shifts in demand have led the international food market suppliers to search for and apply new methods, combine older ones to merge production processes that ensure the high quality with the efficiency of mass-production. Preservation technology plays a key role in quality improvement. Traditional heat treatment-based processes are only partially able to meet requirements such as preserving the appearance and taste of the product due to the harmful effects of heat. In present the usage of modern control processes in food production is fundamental to guarantee efficiency in production both for costs and quality.

Based on international literature for the past 30 years, the appearance of fuzzy control in high-pressure preservation processes is relatively new, but it offers a promising solution to the challenges of modern food quality requirements.

Preservation of food under high pressure dates to the 20th century. Although the technology was available at the beginning of the 19th century [4, 5], wide-range industrial application had not progressed for nearly 100 years. The development of high-pressure equipment started in the early 1990s, particularly with respect to pressure tolerance testing of microorganisms [6]. The technology is known in the industry by several names such as HPP (High Pressure Processing) or HHP (High Hydrostatic Processing), and at the end of the last century it was marked by Knorr [7] in 1993 and by Hoover [8] in 1997 as one of the most promising non-thermal food preservation processes. Its main advantage is the negligible effect on the organoleptic properties and intrinsic values of the treated products.

Nowadays, products that are preserved under high pressure, such as juices, purees, yoghurts, eggs and meat products, account for an increasing share of the market worldwide. In the last 30 years, numerous researches have studied the safety of food processing and the compliance of the customers with the quality requirements. Based on these studies it can be stated that technological developments need to be implemented on regional, company and most of all on the product level. Examination of food ingredients alone is not sufficient to predict their interactions in the products.

2. The role of fuzzy logic

Basic food quality aspects (taste, texture, etc.) are usually described in the words of everyday language: excellent, good, average, acceptable, unsatisfactory. It is easy to see that the bounds of these categories are not sharp enough to handle them with the usage of classic logic. The same is true if we want to characterize how important a particular quality characteristic (such as color) is in the terms of the „image” of the product: very important, fairly important, less important, and so on.

To be able to process these uncertain data (performing operations, make rankings, statistical evaluation), they need to be quantified. Fuzzy numbers are generalizations of crisp numbers that also carry the numerical uncertainty of the qualitative (linguistic) categories.

Fuzzy sets were introduced independently by Zadeh [9] in 1965 and Dieter Klaua [10, 11] in 1965 and 1966 as an extension of the classical notion of set. In the years that followed, fuzzy theory developed slowly, and the first successful technical and industrial adaptation was unpredictable.

Zadeh proposed the concept of linguistic variables in the models describing highly complex systems. Here, instead of an exact numerical value, generalized fuzzy intervals, kernel values and fuzzy membership functions take over the role. The advantage of this approach is that the values in the intermediate regions can be given by convex combinations of membership functions. Although the Zadeh's method resulted reduction in complexity compared to the previous symbolic approach, the number of state variables remained exponential. Computational demands were reduced by Mamdani, and he was the first who also successfully implemented the control to a non-linear model of a steam boiler system.

In our research numerous case studies were reviewed of the industrial implementation of fuzzy controlled high pressure preservation technologies, especially in the term of the connection with food quality characteristics. Víg et al. [12] in 2021 examined several case studies in high pressure preservation, particularly in terms of applicability and process control implementation. In the recent works of Chutia et al. [13] in 2020 and Kausik et al. [14] in 2015 two implementations were presented of fuzzy controlled high-pressure preservation of fruit juices, which technology is proved to be more satisfying in quality and shelf life compared to heat treatment processes. In these works, and generally in the theory of fuzzy control including the process of evaluating data fuzzy arithmetic plays an important role, and understanding this theoretical background is essential to be able to improve methods and algorithms.

Foundation and different types of approaches toward fuzzy set theory has a wide range of literature in mathematics. Gottwald [15] in 2008 classified these approaches in 40-years backward highlighting in conclusion the importance of the classic way presented by Zadeh. The main aim of the present work is to briefly summarize the mathematical background of fuzzy arithmetic.

3. Fuzzy arithmetic

3.1. Fuzzy sets

In this section we summarize those concepts and results of the theory of fuzzy sets which play important role in various applications based on the works of Fodor et al. [16] and Grzegorzewski et al. [17].

The word „fuzzy” has different meanings in suitable contexts. In scientific and technical fields the term „fuzzy” refers to any uncertain, inaccurate object having no clear boundary.

According to the well-known axiom of the classical set theory, for any object x and any set A , it is uniquely decided whether x belongs to A or not. Assuming that A is a subset of a fixed set X , A can be identified with its characteristic function:

$$\chi_A(x) := \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{if } x \notin A. \end{cases}$$

Elements of X belonging to A and those not belonging to A can be distinguished from each other by means of the characteristic function of A .

In case of fuzzy sets, on the other hand, some transitions are allowed. A fuzzy set A can be given by its membership function, a generalization of the characteristic function. A mapping $\mu: X \rightarrow [0,1]$, that is, a function μ defined on X with values between 0 and 1, is called a membership function. For a given $x \in X$, the number $\mu_A(x)$ expresses how much x is compatible with the notion described by the fuzzy set A . The higher this function value is, the more x belongs to the mentioned set.

In addition to numerous classic literary examples of the need and usage of fuzzy sets, food industry applications appear widely in recent research. Korzenszky et al. [18] investigated the connection between the apple (species: Golden Delicious) ripeness and skin coloration based on the CIELAB color space system, which is based on the opponent color model of human vision, where the pairs red and green, and blue and yellow form an opponent pair. In the research ripeness is defined by three categories: unripe, ripe and overripe based on the green-red skin color transition (a^* value in CIELAB). The apples between the a^* values -6 and 10 are regarded as ripe, then the corresponding a^* values belong to the interval $[-6,10]$, the characteristic function of which is shown on Figure 1.

According to the classical set theory, an apple with $a^* = 7$ regarded as unripe suddenly becomes ripe at -6. Instead, it is more reasonable to say that an apple is ripe at $a^* = 2$. It does not mean that any apple with a^* value between -6 and 10 is not ripe, but rather that, the farther the value from 2 is, the less the apple is regarded as ripe. Furthermore, for example any apple is not ripe if $a^* < -6$ or $a^* > 10$.

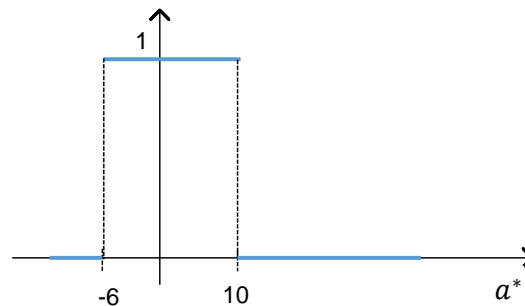


Figure 1. The characteristic function of ripeness (Golden Delicious apple)

Figure 2. represents a possible membership function of the ripe property. The grade (membership value) decreases linearly from 1 to 0 towards the endpoints of the interval $[-6, 10]$.

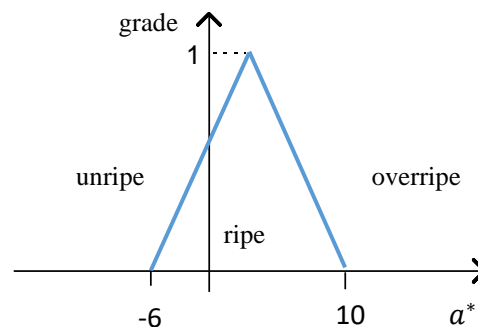


Figure 2. A possible membership function of ripeness (Golden Delicious apple).

Any fuzzy subset of X is identified with its membership function. Since every characteristic function is also a membership function, every traditional subset of X is also a fuzzy subset of X , due to this identification.

The following concepts related to fuzzy sets will be formulated by means of the membership functions so that they will be compatible with the corresponding classical definitions in case of ordinary sets. Thus, two fuzzy sets are equal if and only if their membership functions are identical.

Similarly, the fuzzy set A is a subset of the fuzzy set B , if the membership function of A is smaller than or equal to that of B at any point:

$$A \subset B \text{ if and only if } \mu_A(x) \leq \mu_B(x) \text{ for all } x \in X.$$

Let $\alpha \in]0,1]$, we define the α -cuts of A as

$$A_\alpha := \{x \in X \mid \mu_A(x) \geq \alpha\}.$$

That is, A_α contains all elements of X , which are compatible with A at least at level α . The α -cuts are monotonous in the following sense:

$$0 < \alpha \leq \beta < 1 \text{ implies } A_\alpha \supseteq A_\beta.$$

The *core* of a fuzzy set A consists of those elements in X , which are completely compatible with A :

$$\text{core } A := \{x \in X \mid \mu_A(x) = 1\} = A_1.$$

The *support* of a fuzzy set A consists of those elements in X , for which the membership function values are positive, i.e.,

$$\text{supp } A := \{x \in X \mid \mu_A(x) > 0\}.$$

3.2. Operations on fuzzy sets

Let X be a given set, A be a fuzzy subset of X , whose membership function is $\mu_A: X \rightarrow [0,1]$. The *complement* of A is the fuzzy set \bar{A} , whose membership function is defined as

$$\mu_{\bar{A}}(x) := 1 - \mu_A(x).$$

For example, if A is the fuzzy set of the ripe apples, then \bar{A} denotes that of the non-ripe ones. Using the previous example, the membership function of \bar{A} is the following:

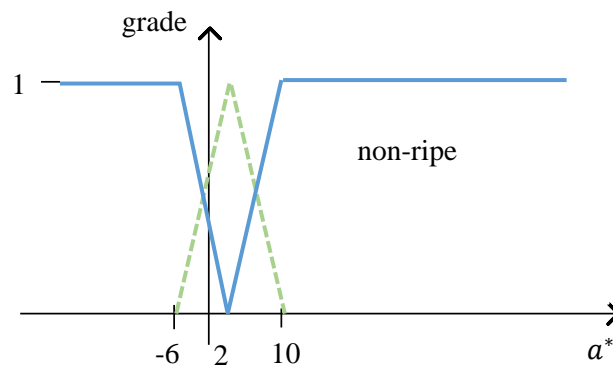


Figure 3. Membership function of non-ripe apples (Golden Delicious apple).

Let A and B be fuzzy subsets of X with membership functions μ_A and μ_B , respectively. The union of A and B is the fuzzy set $A \cup B$, whose membership function is the maximum of those of A and B , that is,

$$\mu_{A \cup B}(x) := \max\{\mu_A(x), \mu_B(x)\} \text{ for all } x \in X.$$

Similarly, the *intersection* of A and B is the fuzzy set $A \cap B$, whose membership function is the minimum of those of A and B , that is,

$$\mu_{A \cap B}(x) := \min\{\mu_A(x), \mu_B(x)\} \text{ for all } x \in X.$$

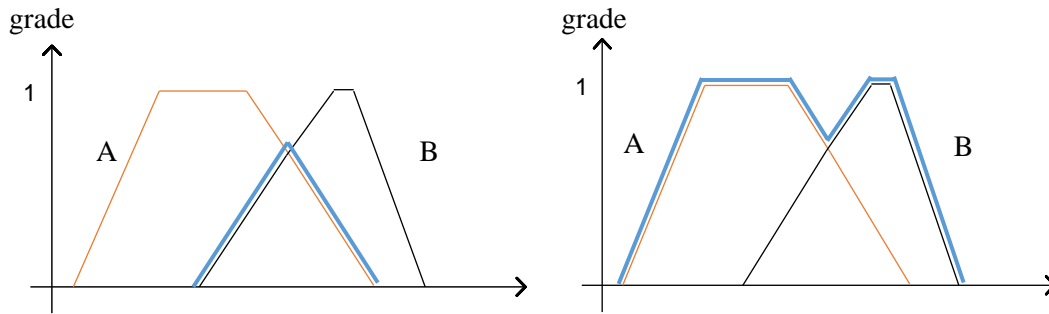


Figure 4. Union and intersection of fuzzy sets.

3.3. Fuzzy numbers

Fuzzy numbers play a central role in the fuzzy modelling of uncertainty.

Let A be a fuzzy subset of \mathbb{R} (the set of real numbers) with membership function μ_A . Then A is called a *fuzzy number* if

1. $\text{supp } A := \{x \in \mathbb{R} \mid \mu_A(x) > 0\}$ is a bounded interval,
2. there exists $x_0 \in \mathbb{R}$, such that $\mu_A(x_0) = 1$,
3. The α -cut $A_\alpha := \{x \in \mathbb{R} \mid \mu_A(x) \geq \alpha\}$ is convex for all $\alpha \in]0,1]$.

Fuzzy numbers defined by these properties can be of various types. Here we present one important special type, the fuzzy numbers of trapezoid shape.

3.4. Fuzzy numbers of trapezoid shape

Let $a \leq b$ be arbitrary real numbers, and α, β be positive numbers. The fuzzy number denoted by $A = (a, b, \alpha, \beta)$ is called of trapezoid shape, if its membership function is given in the following form:

$$\mu_A := \begin{cases} 1 - (a - x)/\alpha, & \text{if } a - \alpha \leq x < a, \\ 1, & \text{if } a \leq x \leq b, \\ 1 - (x - b)/\beta, & \text{if } a < x \leq b + \beta, \\ 0, & \text{if } x < a - \alpha \text{ or } x > b + \beta. \end{cases}$$

That means the interval $[a, b]$ is the core of the fuzzy number A , while the open interval $]a - \alpha, b + \beta[$ is the support of A . The parameters α and β are called the left and right width of the fuzzy number A , respectively. The membership function is linear between the mentioned points, hence its shape in fact is trapezoid (Figure 5.).

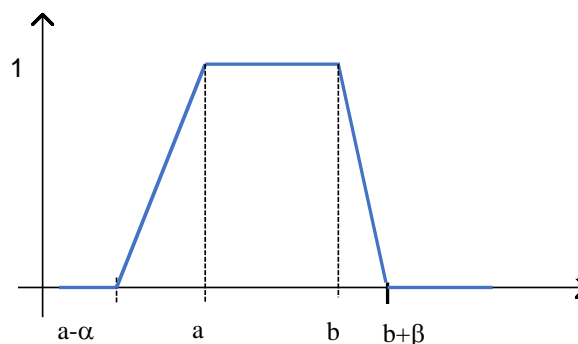


Figure 5. Core and support of a fuzzy number.

Fuzzy numbers of triangular shape are obtained in the special case $a = b$.

In case of some arithmetic operations (e.g., division) it is essential to exclude zero from the possible values of a fuzzy number. Therefore, we introduce the concept of a positive and negative fuzzy number.

A fuzzy number (of trapezoid shape) $A = (a, b, \alpha, \beta)$ is called positive if $a - \alpha > 0$, negative if $b + \beta < 0$.

3.5. Arithmetic operations with fuzzy numbers of trapezoid shape

The main objective when defining arithmetic operations with fuzzy numbers is to ensure their compatibility with the operations with ordinary real numbers. This is called the principle of extension.

Let $A_1 = (a_1, b_1, \alpha_1, \beta_1)$ and $A_2 = (a_2, b_2, \alpha_2, \beta_2)$ be fuzzy numbers of trapezoid shape. Applying the principle of extension, we obtain the following exact form for the sum of A_1 and A_2 :

$$A_1 + A_2 = (a_1 + a_2, b_1 + b_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2).$$

This means that the intervals defining the cores, together with the left and right widths are added. Thus, the uncertainty of the sum, as expected, is at least as much as that of the terms of the sum.

If $A = (a, b, \alpha, \beta)$ and λ is a given real number, based on the principle of extension, we have

$$\lambda \cdot A = \begin{cases} (\lambda a, \lambda b, \lambda \alpha, \lambda \beta), & \text{if } \lambda \geq 0, \\ (\lambda b, \lambda a, |\lambda| \beta, |\lambda| \alpha), & \text{if } \lambda < 0. \end{cases}$$

That means the length of the core and the support is multiplied by λ if λ is nonnegative. If λ is negative, then the λ -multiples of the original endpoints of the core should be interchanged, while the $|\lambda|$ -multiples of the left and right widths are also interchanged with each other.

For $\lambda = -1$ we have

$$-A = (-b, -a, \beta, \alpha).$$

The difference of the fuzzy numbers A_1 and A_2 is defined as $A_1 - A_2 = A_1 + (-A_2)$, that is

$$A_1 - A_2 = (a_1 - b_2, b_1 - a_2, \alpha_1 + \beta_2, \alpha_2 + \beta_1).$$

Our goal is to define the reciprocal of a fuzzy number $A = (a, b, \alpha, \beta)$ of trapezoid shape. As a generalization of the ordinary concept, the reciprocal of a fuzzy number makes sense only if zero is not among the possible values of it. Hence the reciprocal is defined for either positive or negative fuzzy numbers.

The following formula provides an approximative result for the reciprocal of A :

$$\frac{1}{A} = \left(\frac{1}{b}, \frac{1}{a}, \frac{\beta}{b(b + \beta)}, \frac{\alpha}{a(a - \alpha)} \right).$$

If $A_1 = (a_1, b_1, \alpha_1, \beta_1)$ and $A_2 = (a_2, b_2, \alpha_2, \beta_2)$ are fuzzy numbers of trapezoid shape, then their product is not exactly of this type. Hence an approximative formula is applied for the product, where the side functions are approximated linearly, while the endpoints of the core and support are accurate. The „sharper” the fuzzy numbers are, the more accurate approximation is obtained. Let

$$x_m := \min\{(a_1 - \alpha_1)(a_2 - \alpha_2), (a_1 - \alpha_1)(b_2 + \beta_2), (b_1 + \beta_1)(a_2 - \alpha_2), (b_1 + \beta_1)(b_2 + \beta_2)\},$$

$$x_M := \max\{(a_1 - \alpha_1)(a_2 - \alpha_2), (a_1 - \alpha_1)(b_2 + \beta_2), (b_1 + \beta_1)(a_2 - \alpha_2), (b_1 + \beta_1)(b_2 + \beta_2)\},$$

$$y_m := \min\{a_1 a_2, a_1 b_2, a_2 b_1, b_1 b_2\},$$

$$y_M := \max\{a_1 a_2, a_1 b_2, a_2 b_1, b_1 b_2\},$$

then

$$A_1 \cdot A_2 = (y_m, y_M, y_m - x_m, x_M - y_M).$$

Let $A_1 = (a_1, b_1, \alpha_1, \beta_1)$ be arbitrary and $A_2 = (a_2, b_2, \alpha_2, \beta_2)$ be either a positive or negative fuzzy number. Their ratio is obviously defined as

$$\frac{A_1}{A_2} = A_1 \cdot \frac{1}{A_2}$$

3.6. Defuzzification

When carrying out analyses involving fuzzy data, it may be important to represent the resulting fuzzy numbers as classical – so called crisp – real numbers. A procedure of converting fuzzy numbers to crisp ones is called defuzzification. Various methods are known and widely applied, from among which the following are mentioned.

3.6.1. Center of gravity (COG)

If A is a fuzzy number (identified with its membership function), then the crisp number a_0 represented by A is given by

$$a_0 = \frac{\int zA(z)dz}{\int A(z)dz}$$

where the integral is taken over the (bounded) support of A . This is in fact the (x -coordinate of) the center of gravity of the region below the graph of A .

For a fuzzy number $A = (a, b, \alpha, \beta)$ of trapezoid shape the formula gives the following:

$$a_0 = \frac{3b^2 - aa^2 - \alpha^2 + \beta^2 + 3a\alpha + 3b\beta}{6b - 6a + 3\alpha + 3\beta}.$$

In case of a symmetric trapezoid, a_0 is the center of the support of A .

3.6.2. Area (COA)

Let $a_1 \in [a - \alpha, b + \beta]$ be the point, for which the vertical line $x = a_1$ divides the region below A into two parts of equal area. To determine a_1 , we distinguish 3 cases.

- Case 1: $|\alpha - \beta| \leq 2(b - a)$. This is equivalent to $a \leq a_1 \leq b_1$, and then

$$a_1 = \frac{a + b}{2} + \frac{\beta - \alpha}{4}.$$

- Case 2: $\alpha - \beta > 2(b - a)$. This is equivalent to $a - \alpha < a_1 < a$, and then

$$a_1 = a - \alpha + \sqrt{\alpha \left(b - a + \frac{\alpha + \beta}{2} \right)}.$$

- Case 3: $\beta - \alpha > 2(b - a)$. That means $b < a_1 < b + \beta$, in this case

$$a_1 = b + \beta - \sqrt{\beta \left(b - a + \frac{\alpha + \beta}{2} \right)}.$$

3.7. Ordering of fuzzy numbers

For the extension of certain mathematical statistical methods, it is necessary to order the fuzzy numbers (of trapezoid shape). Since there can be considerable overlapping between the supports and cores, that is, they are not disjoint, we introduce the concept of the index of a fuzzy number. For a given $A = (a, b, \alpha, \beta)$ we define

$$I(A) := \frac{a + b}{2} + \frac{\beta - \alpha}{4}.$$

Then, a total order relation can be defined on the fuzzy numbers in the following way: If A and B are fuzzy numbers of trapezoid shape, then A is said to be smaller than or equal to B , if $I(A)$ is smaller than or equal to $I(B)$.

4. Conclusions

The integration of technological innovations that have taken and are just taking place in food industry can be of decisive importance for small and medium-sized companies in Hungary to enlarge their market share. High pressure preservation and fuzzy control of these processes are also known and implemented in some cases in Hungary. However, the region-specific consumption patterns and the microbiological and biochemical speciality of local raw materials demand company and region level adaptation of technologies to improve efficiency of preservation processes in terms of safety, economy and environmental impact. Based on our review we conclude that the importance of high-pressure processing is increasing in quality food preservation, where fuzzy control is particularly useful to apply. The theory of Fuzzy arithmetic supplies a wide spectrum of tools to improve the efficiency the control processes of high-pressure food processing.

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References

- [1] **Hugas, M., Garriga, M., Monfort, J.M.** (2002), New mild technologies in meat processing: high pressure as a model technology, *Meat Science*, 62, Issue 3, 359-371.
- [2] **Huszka, P., Polereczki, Zs.** (2009), A fizetőképes kereslet és az árak hatása a magyar lakosság ételmiszerfogyasztására, *Élelmezési Ipar Szakfolyóirat*, 63 (8), pp. 241-247.
- [3] **Szabó D.** (2017): Determining the target groups of Hungarian short food supply chains based on consumer attitude and socio-demographic factors, *Studies in Agricultural Economics*, 119: 115-122.
- [4] **Hite, B. H.** (1899): The effects of pressure in the preservation of milk, *West Virginia AgricExp. Sta. Bull.*, 58, 15-35.
- [5] **Hite, B. H., Giddings, N. J., Weakley, C. E.** (1914): The effect of pressure on certain microorganisms encountered in the preservation of fruits and vegetables, *West Virginia AgricExp. Sta. Bull.*, 146., 2-46.
- [6] **Patterson, M. F.** (2005): Microbiology of pressure treated foods, *J. Appl. Microbiol.*, 98(6): 1400-1409.
- [7] **Knorr, D.** (1993): Effects of high hydrostatic pressure processes on food safety and quality, *Food Technol.*, 47, 156-161.
- [8] **Hoover, D. G.** (1997): Minimally processed fruits and vegetables: reducing microbial load by non-thermal physical treatments, *Food Technol.*, 51, 66-71 p.
- [9] **Zadeh L. A.** (1964): Fuzzy sets, *Information and Control*, Volume 8, Issue 3, June 1965, Pages 338-353.
- [10] **Klaua, D.** (1965): Über einen Ansatz zur mehrwertigen Mengenlehre, *Monats-berichte der Deutschen Akademie der Wissenschaften Berlin* 7, 859–867.
- [11] **Klaua, D.** (1966): Über einen zweiten Ansatz zur mehrwertigen Mengenlehre, *Monatsberichte der Deutschen Akademie der Wissenschaften Berlin* 8, 161-177.
- [12] **Víg P., Szabó I., Schrempf N., Veres A., Sebestyén Z., Korzenszky P.** (2021): Fuzzy control and its applications, *Mechanical Engineering Letters*, Vol. 21. Issn 2060-3789 MATE, pp. 142-149.
- [13] **Chutia H., Mahanta C. L., Ojah N., Choudhury A. J.** (2020): Fuzzy logic approach for optimization of blended beverage of cold plasma treated TCW and orange juice, *Journal of Food Measurement and Characterization* 14, 1926–1938.
- [14] **Kaushik N., Reddy G. A., Rana R., Rao P. S.** (2015), Application of fuzzy logic technique for sensory evaluation of high pressure processed mango pulp and litchi juice and its comparison to thermal treatment, *Innovative Food Science and Emerging Technologies* 32 (2015) 70–78.

- [15] **Gottwald S.** (2008): Foundations of a theory for fuzzy sets: 40 years of development, *Int. J. Gen. Syst.* 2008. 37:69–82. 17.
- [16] **Fodor J., Roubens M.** (1994): *Fuzzy Preference Modelling and Multicriteria Decision Support*, Kluwer Academic Publishers, Dordrecht, 1994.
- [17] **Grzegorzewski P., Mrówka E.** (2005): Trapezoidal approximations of fuzzy numbers, *Fuzzy Sets and Systems*, 153 (2005), 115-135.
- [18] **Korzenszky P., Kassebi S.** (2021), Changes in physicochemical properties of golden delicious apples during storage, In: 4th International Conference on Biosystems and Food Engineering: Book of Abstracts, Budapest, Paper: E418, 1 p.