Political inequality in participation index - a gini-based measure of inequalities in political participation

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Abstract
Measuring inequalities in political participation across social groups is a challenging task as participation is typically coded in dummy variables. For instance, social scientists record whether their respondents have voted in the previous elections (1) or not (0). In this paper, we identify a list of desirable criteria that an inequality index used for empirical comparative studies should meet. Existing inequality indices fail to satisfy one or more of these criteria. Building on our list, we define a new Gini-type index, the Political Inequality in Participation Index (PIPI), suitable for cross-country comparisons. We show that inequalities measured by the PIPI are correlated to, but are qualitatively different from the best-known measurements. In particular, using data simulation techniques, we demonstrate that this correlation is decreasing in the complexity of societies’ structure. Moreover,
by replicating an existing study, we further demonstrate that when working with real-world data, the PIPI provides new empirical results.

**Keywords**
Political inequality, political inequality in participation index, gini index, data simulation

It would be hard to find a topic more popular in social sciences than inequalities, since social equality is morally desired in modern, democratic social and political systems. Based on the theories about democracy and according to contemporary political philosophers, modern democracy requires equality in political participation as “a fundamental premise” (Dahl, 2006: ix).

We adhere to the well-known reasoning that inequalities in education or political participation are the consequences of individual decisions (Holm and Breen, 2016; Breen and Goldthorpe, 1997), even though inequalities are measured at the macro-, societal level.

However gauging political inequality within and across societies is hard because researchers measure participation with binary variables. For example, political sociologists want to know whether people voted in the last national elections, or took part in demonstrations in the last 12 months. Responses to these questions about political participation are discrete variables by nature, therefore they are coded as dummies (voted/not voted, demonstrated/not demonstrated, etc.).

In this paper, we develop a new index that measures political inequalities in participation across social groups. We call it the Political Inequality in Participation Index (PIPI). We demonstrate that the PIPI satisfies a list of three criteria that we believe are important for comparative studies. First, an inequality index should be independent of the number of participants in a sample (e.g., number of protesters in a given country). Second, it should also be independent of the size of social categories (e.g., proportion of highly educated citizens in the country). Third, the index should be normalized, so that it takes values from 0 to 1.

To the best of our knowledge, none of the existing inequality indices meet all three criteria. In particular, we demonstrate that the most-widely used measures fail to satisfy one or more criteria, however, the PIPI meets all of them and thus provides less biased empirical results. Finally, through data simulations we show that the PIPI highly correlates with traditional measures (GAP - Bühlmann et al. (2012); EGINI – Hu (2015)) when comparing similar societies. However they differ qualitatively when comparing more different societies, which is typically the case in real-world applications.
The rest of the paper is organized as follows: After the literature review about inequalities in participation, we define a set of criteria that indices used in comparative studies should meet. In the third section, the most popular inequality indices are described. In the fourth section, we demonstrate that these widespread indices fail to meet one or more of the criteria. To remedy this issue, we develop the Political Inequality in Participation Index (PIPI). In the fifth section, using complex data simulation techniques we test how the PIPI and other indices are different. In the sixth section, using real data we show that the PIPI might bring essentially and significantly different results than the most frequently applied inequality measurements. The last section concludes and presents the limitations of the PIPI.

**Inequality in Political Participation**

For modern democratic political systems, political equality seems to be a fundamental premise Dahl (2006). Political equality requires meeting the 'one man one vote principle’ referring to the universal right to vote, as well as the 'equality before the law principle’. Moreover, the idea of democracy contains the concept that public decisions that politicians make should represent the interests not only of elite groups but the demands of all social strata. If in a society there are groups which do not participate in politics and do not raise their voice, politicians are unable to follow these hidden or latent preferences. As Marion Young argues, all members of a society should "have the opportunity to try to influence public policy to serve or protect their interests." (Young, 2000: 17) This argumentation builds upon the traditional concept of political participation developed in the 1970s (Barnes et al., 1979; Parry et al., 1992; Verba and Nie, 1972; Verba et al., 1995). Verba and his colleagues (1995) define political action as an instrument in the hands of ordinary people to shape their political and social environment. Unequal distribution of political participants across social groups may have serious consequences for the work of democracy (Bühlmann and Kriesi, 2013; Verba et al., 1995; Schlozman et al., 2013; Bühlmann et al., 2012). Large inequality in political participation signals an exclusive polity and political culture Schlozman et al. (2013). Thus participation as a means of shaping politics is strongly related to the model of 'responsive democracy’, which is described as a system where politics is responsive to citizens’ needs and preferences Teorell (2006).

Social and political inequalities, on the other hand, may also serve as crucial motivation to participate in political collective actions and social movements (Kriesi et al., 2020; Opp, 2012; Somma, 2021). Moreover, social and political structures shape individuals’ motivations to act collectively (Brady et al., 1995; Dalton et al., 2010; Zhou and Wang, 2018).
Although the topic of inequalities in political participation is as old as political sociology itself, there is no consensus among researchers on measurement and methodology. Sociologists want to know whether people voted in the last national elections, or took part in demonstrations in the last 12 months. Thus in social surveys, respondents are asked about their political activities in the last few years. As responses to these questions about political participation are discrete variables by nature, they are coded as dummies (voted/not voted, demonstrated/ not demonstrated etc.). These surveys are the main data sources for political sociology, but there is no unified approach in analytical strategies. However, speaking of severe or mild inequalities demands a well-defined measurement.

Jan Teorell and his colleagues define two approaches to study political inequalities: a process-oriented and an outcome-oriented interpretation of inequality (Teorell, 2006; Teorell et al., 2007). The process-oriented approach focuses on the causes behind inequalities and uses association-based measures like the odds ratio. On the other hand, the outcome-oriented approach deals with the unequal distribution of participants and applies distributional measures like the Gini coefficient. This differentiation is very similar to how the methodologist Ottar Hellevik explains inequalities in education:

Association or effect - how class affects the probability of obtaining higher education - must be captured by statistical measures such as [...] regression coefficients, or [...] odds ratio [...]. Inequality or unrepresentativity - how the distribution of higher educational positions between classes deviates from their share of the population - must be captured by measures like the Gini coefficient or the ratio of advantage. (Hellevik, 1997: 377-378)

Hu (2015) provides a detailed review of the stormy debate generated by Hellevik’s claims. Some scholars prefer associations, whereas others prefer the distributional measure of inequality. The former school of thinking prefers the concept inequality of opportunity measured by odds ratio, while the latter prefers the concept of inequality of condition and uses distributions instead (Hu, 2015: 281). Teorell and his colleagues do not refer to Hellevik’s suggestion; notwithstanding that they apply the same methodological strategy when scrutinizing process and outcome-oriented inequalities in political participation.

The academic literature is abundant in analyses that operationalize inequality using odds ratio or other association based measures (Verba et al., 1995; Gallego, 2007, 2010; Marien et al., 2010). Also there are other acknowledged scholars who have studied the unequal distribution of participants instead of investigating odds ratios (Rosenstone and Hansen, 1993; Teorell et al., 2007; Bühlmann et al., 2012; Dubrow, 2010; Somma and
Since political participation is defined as an instrument to shape the social and political environment, sociologists want to know to what extent this instrument is distributed across society. Empirical studies usually aim to study gender (e.g., Coffé and Bolzendahl (2010); Quaranta and Dotti Sani (2018)), education (e.g., Bovens and Wille (2010); Dassonneville and Hooghe (2017)) or age gaps (e.g., Rubenson et al. (2004); Teorell et al. (2007)) in political participation. The distribution of participants in demonstrations is unequal between social categories such as age groups, gender or education, because youths, men and well-educated citizens tend to participate more than older people, women and less educated citizens. However, these differences may vary across countries and may also depend on forms of participation, for example, senior citizens may be more active in voting (Dalton, 2017). Differences between social groups regarding political activity are outcomes of a complex system of social mechanisms. We will not attempt to clarify all possible mechanisms revealed in the literature that may encourage political participation. In this article we do not deal with the processes that bring about inequality; instead, we intend to describe how unequally distributed participants are across social groups (e.g., level of education, gender).

Political sociologists are also highly interested in comparative country research (Teorell et al., 2007; Dubrow, 2010; Somma and Bargsted, 2018). Let us take two examples. Dubrow (2010) studies the European regions and claims that Post-communist countries were not much more unequal in protest participation than their Western-European counterparts. On the other hand, Dassonneville and Hooghe (2017) examine the growing educational gap between voters and non-voters in the United States and Western-European countries. These two landmark studies, and most empirical papers, however, do not share a standardized methodology and measurement.

Overall, in empirical works three dimensions should be taken into consideration: (1) location (e.g., country, region), (2) social categories (e.g., gender, education), and (3) form of political action (e.g., voting, demonstrating, contact with politicians). Comparative studies intend to compare countries (e.g., Germany to Poland, or the USA to Japan) and find out which society is burdened more with inequalities in political participation. Another research question would scourge the comparison of different participation forms within a given country. e.g., "What form of political participation is the most unequally distributed in Hungary?" or "Is inequality in demonstration more severe than inequality in voting?". The third dimension regards social categories like gender or level of education since scholars often want to know which social groups are most deprived.

In the next sections, we focus on the concept of unequal distribution, investigating the validity of distributional measures. We introduce the
Political Inequality in Participation Index, and demonstrate how it facilitates the comparative analysis of inequalities in political participation.

**List of Criteria**

Inequality studies intend to reveal differences between countries as well as social groups. Scholars are inclined to say that political inequalities are higher in country A than in B; or in a given country voting inequalities are more moderate than inequalities in demonstration; or political inequality across educational groups is higher than across gender.

A desirable inequality index contains purely the information resulting from social mechanisms. When doing a comparative analysis, we should be certain that the differences described between two countries are due to social processes rather than outcomes of measurement bias. In order to meet these aims, an inequality measure should consider the following criteria:

1. Responsiveness to the number of participants
2. Responsiveness to the size of social categories
3. Range of the index is $[0,1]$  

The first criterion means that it should be possible to compare groups (e.g., countries) with different levels of participation. Let us assume that our database contains 2,000 respondents representative of the Hungarian and another 2,000 of the German population. In Hungary the culture of political activism is less developed than in Germany, thus the Hungarian sample contains only 60 demonstrators, while the German sample comprises 200. A measure of inequality is valid if it is insensitive to the number of participants (i.e. demonstrators), or in other words, the size of inequality does not hinge on the number of participants. In the case of comparing two countries differing in the number of participants, a sensitive measure is biased since differences between the countries’ inequalities may simply be due to the different number of participants rather than the underlying social mechanisms that bring about the unequal distribution of protesters. A desirable index should factor in the increased variance of inequality arising due to a lower number of demonstrators. Consequently, at the special case of equal participation rates between social groups of the same size, the index should return zero inequality.

Thanks to the second criterion, the inequality index includes the population structure which we argue is a desirable property. In other words, the measure takes into consideration the relative size of social groups. Indeed, this is one of the main differences between the odds ratio and the Gini coefficient. Namely, the former disregards social composition, whereas Gini can take it into consideration. Therefore, the second criterion ensures that the
index belongs to the Gini inequality measures (see Hellevik, 1997, 2000). In practice, following the example above, the index should yield the same inequality (in comparison of Germany and Hungary) regardless of whether the share of highly educated in Germany is smaller or larger than in Hungary.

Finally, according to the third criterion it is ideal that the range of the index run from zero to one. Otherwise we would be unable to compare two cases with different social categories. Clearly, the value of 0.6 on the 0 to 1 scale does not equal the value of 0.6 on a scale where the theoretical minimum is 0.2 and the theoretical maximum is 0.7.

Inequality Measures in Political Sociology

Measures of inequalities in political participation examine the distribution of participants across social groups or social classes. In this section, we describe the most frequently used indices. As we will see, all of them follow the logic of gap measures. Thus, they calculate differences between the expected and the observed share of a group.

There are, however, other disciplines in social sciences struggling with similar – but not identical – methodological difficulties. Scholars study territorial distribution of votes using party nationalization measures (e.g., Jones and Mainwaring, 2003; Bochsler, 2010; Morgenstern et al., 2014). One other direction within sociology pursues revealing inequalities in educational attainment (Hellevik, 1997, 2000; Hu, 2015). Given the similarities in technical and methodological issues for measuring inequalities in political participation, and homogeneity of votes across electoral districts and for educational inequalities, we decided to interweave these strands of methodological developments.

Jan Teorell and his co-authors made excellent contributions to the theory of political inequalities when they distinguished outcome oriented and process inequalities in participation (Teorell, 2006; Teorell et al., 2007). It seems, however, that the operationalization of outcome oriented (unequal distribution) inequalities is highly biased. Teorell et al. (2007) define their Normed Distance to the Equality Point (NDQP) index as:

\[
NDQP = \frac{k}{k-1} \sqrt{\sum \left( P_i - \frac{1}{k} \right)^2}
\]

with

- \( P_i \): ith group’s share of participants.
- \( k \): number of groups.

NDQP is defined solely for cases where the social categories analyzed are equal in size, but real-world societies hardly ever meet this assumption. Even
the gender groups’ distribution in the population is somewhat different from the 50:50 percent proportions. Clearly, using this measure is far from ideal for the type of situations we aim to describe. Therefore, we do not use this index in the following analysis.

Dubrow (2010) describes inequalities across groups with different levels of political resources (e.g., information, knowledge, free-time). Dubrow (2010) defines his Index of Dissimilarity (ID) as:

$$\text{ID} = \frac{1}{2} \sum |p_{ij} - e_{ij}|$$

where,
- $p_{ij}$: observed proportion of participants in the $i$th quintile in the $j$th country.
- $e_{ij}$: expected proportion of participants in the $i$th quintile in the $j$th country.

The third inequality measure, used by the outstanding researchers of the Democracy Barometer project, uses education gap to detect inequalities in political participation across educational levels (Bühlmann et al., 2012).

$$\text{GAP} = \frac{\sum |A_i - p_i|}{k}$$

where,
- $A_i$: $i$th group proportion in the population.
- $p_i$: $i$th group’s share of participants
- $k$: Number of levels of education, the number of groups

Finally, the fourth index in the literature is the Educational GINI (EGINI), developed by Hu (2015) for measuring inequalities in the field of education. As Hu (2015: 283) emphasizes, “The computation of the Gini coefficient is based on the Lorenz Curve, which, for the educational inequality research, plots the cumulative percentage of the population against the cumulative percentage of educational opportunities.” The EGINI index according to Hu (2015) only works for a two-level setup with upper and lower social classes:

$$\text{EGINI} = 1 - p_1k_1 + p_2k_2 - 2k_2$$

where
- $k_i$: $i$th group’s proportion in the population, and
- $p_i$ denotes the $i$th group’s share of participants.

Next, we adapt the logic behind the EGINI to the research field of political participation. In our case, the Lorenz Curve shows the distribution of participants across pre-defined social classes (e.g., level of education). For this, we first need to generalize the EGINI to work with an arbitrary number of $n \geq 2$ social classes. Without loss of generality, let
\[ \frac{p_1}{k_1} \leq \frac{p_2}{k_2} \leq \ldots \leq \frac{p_n}{k_n} \]

This assumption is simply a relabeling of groups in a way that we order them according to their relative share of participants, \( p_i/k_i \).

We generalize the original EGINI index to the case of \( n \) groups as follows:

\[
\text{GEGINI} = 1 - \sum_{i=1}^{n} k_i p_i - 2 \sum_{i=1}^{n} k_i \left( 1 - \sum_{j=i}^{n} p_j \right)
\]

where GEGINI stands for “Generalized EGINI”. To illustrate the definition, let \( T \) denote the area below the Lorenz Curve. It can be calculated as

\[
T = \sum_{i=1}^{n} \frac{k_i p_i}{2} + \sum_{i=1}^{n} k_i \left( 1 - \sum_{j=i}^{n} p_j \right)
\]

It is conventional for Gini-type measures to subtract this from 0.5 (perfect equality) and multiply it by \( 2 \) to get a normalized measure of inequality. Indeed, \( 2(0.5 - T) \) equals the definition of GEGINI. To see that GEGINI is indeed a generalization of EGINI, note that for the special case of \( n = 2 \) we have

\[
\text{GEGINI}_{(n=2)} = 1 - k_1 p_1 - k_2 p_2 - 2k_1 (1 - p_1 - p_2) \quad -2k_2 (1 - p_2) = 1 - k_1 p_1 - k_2 p_2 - 0 + 2 k_2 p_2 - 2k_2 = \text{EGINI}
\]

Clearly, GEGINI is a function of not only the distribution of participants (\( p \)), but also of the composition of society, since in the formula \( k \) refers to the distribution of the population over the social grouping scheme.

In the next section we will show that none of these measures meet all three desirable criteria described in the previous section, and introduce a new measure that does.

**Testing Inequality Indices**

**Bias caused by the number of participants**

In this session, we will demonstrate that the GEGINI is sensitive to the number of participants, and we will argue that a good measure of inequality should be insensitive to it. In particular, we will show a simulation where the participation probabilities of two groups is perfectly equal by construction, however, the GEGINI will measure positive inequality. Moreover, we will show that this measured inequality decreases in the number of participants.
By flipping a fair coin, it lands on ’head’ or on ’tail’ with equal chance. Toss this coin once, and check whether it is head or tail. What we notice now is that the distribution of heads and tails is fairly unequal, since all the tosses are, say, head. Of course, this is because we flipped the coin once only. In the next step, flip the coin ten times. The result should be a much more equal distribution, say seven heads and three tails. However, it is far from the supposed outcome, where 50 percent of the cases would be heads. As a third step, throw the coin 10,000 times. In this case, the proportion of heads and tails is very close to 50:50 percent, thus inequality of tosses across heads and tails is low. This correlation between the number of tosses and the size of inequality is based on the well-known “law of large numbers”.5

We can simulate data to test whether the inequality measures are sensitive to the number of participants. In our simulation, we generated 191 countries with different numbers of participants (for instance demonstrators). At the lowest level there are 100 participants, while at the highest the country has 2,000. Between the two extremes the number of participants (observations per simulation run) rises by ten (100, 110, 120…2,000). Moreover, there are two categories for gender: female and male. We label our participants as male or female by equal (50:50 percent) chances, which results in a random distribution. Differences between the distributions are caused purely by the different number of participants.

After we gauge the inequality measure (GEGINI), in the next step in the simulation we calculate the Spearman-correlation between the number of participants and inequality. Thus, we get 191 values of GEGINI and one correlation coefficient between inequalities and number of demonstrators. As a final step, we repeat the simulation ten thousand times.

Figure 1. The level of inequality (GEGINI) changing with the number of participants.
In Figure 1 we show the results of this data simulation: There is a strong negative correlation between the level of inequality and the number of participants measured by GEGINI. In the simulation, the participants are labelled randomly as ‘male’ or ‘female’, thus there is no social mechanism causing inequality. These results lead us to the conclusion that correlation exists purely due to mechanical factors rather than the underlying social mechanism.

Another explanation of the negative correlation is that the smaller the sample, the larger is the standard deviation of the mean value of the random variable gender (or any other variable describing the social structure) across multiple draws of the sample. The simulation demonstrates that differences in the number of participants lead to variation in the inequality measure because indeed between multiple draws of a smaller sample there is more inequality than between multiple draws of a larger sample.

Based on this, we argue that a good inequality index takes into account the number of participants and is insensitive to it, because the law of large number says that more participants in a society brings about lower inequality per se, in a purely mechanical way. For instance, the average number of protesters is about 20 times larger in Spain than in Finland each year (Borbáth and Gessler, 2020). We do not want an inequality index to return a lower inequality in participation between genders in Spain purely as a result of the number of participants. In other words, if a standard inequality measure finds lower inequality in Spain, we cannot decide if this is caused by real-world gender differences or it is an artifact of the inequality measure used.

Our proposed index, the PIPI, will be constructed to return a zero measure of inequality in the simulation above. The correction of the GEGINI we propose factors in a larger expected deviation from the true population mean for smaller countries. In fact, we will use the GEGINI values calculated as above as the minimum values of GEGINI and use it for normalizing our proposed index, the PIPI. Thanks to this property, the PIPI will allow for a better comparison of countries like Spain and Finland.

Correcting the bias of different numbers of participants: The minimum of GEGINI

Our aim is to compare political inequalities between countries with different numbers of activists. However, as show in the section above, using GEGINI we are unable to decide whether the degree of inequality in a country is engendered by chance or brought about by a social mechanism.

The simulations above that we used to check whether GEGINI meets our first criterion show the minimum level of inequality for a given number of participants. This is the empirical minimum level of inequality, where we
take into consideration a given social situation where the number of participants and the structure of the given society are observed. The empirical minimum value is very useful for our analysis since it displays whether the measured inequality is significantly different from the values we would obtain by chance. Moreover, it will be used for the normalization of our proposed index in order to meet the third criterion, the zero to one range.

Correcting the bias of different number of participants: The maximum of GEGINI

Next, we show that the theoretical maximum value of the GEGINI does not equal 1, as this maximum depends on the number of participants, the number of social categories, and also on the relative size of these groups. Determining the maximum value of GEGINI is necessary for normalizing the index to fall between 0 and 1. In turn, the normalization is necessary to pursue a meaningful comparative study between samples differing in the number of participants, the number of social categories, and/or the relative size of these categories. Calculating the maximum value of GEGINI is laborious, however. We first theoretically prove a Lemma, which we then use in constructing a mixed integer programming algorithm that computes its exact value.

\textbf{Lemma.} When reaching its maximal value, the GEGINI index has at most one social category \( i \) for which \( 0 < \frac{p_i}{k_i} < 1 \), i.e. whose relative share of participants is neither zero nor one.

\textbf{Proof.} We prove the Lemma by contradiction. Assume that when reaching the maximal GEGINI value, there are exactly two categories, \( i \) and \( j \), with \( 0 < \frac{p_i}{k_i}, \frac{p_j}{k_j} < 1 \). We will show that by rearranging participants it is possible to achieve a higher GEGINI value.

Clearly, GEGINI is maximal if and only if \( T \), the area below the Lorenz curve is minimal, thus we will subsequently argue in terms of minimizing \( T \). Without loss of generality, let \( i < j \) which implies \( \frac{p_i}{k_i} \leq \frac{p_j}{k_j} \). The relative share of participants must equal 1 for all social categories with labels higher than \( j \), and it must equal 0 for all social categories with labels smaller than \( i \). Therefore, the area below the Lorenz Curve can be calculated as

\[
T = 0 + \frac{k_ip_i}{2} + \frac{k_jp_j}{2} + k_jp_i + \frac{m}{2} (1 - 1 - m) \quad \text{where} \quad m = \sum_{l=j+1}^{n} k_l.
\]

As none of the shares of social categories is either 0 or 1 by assumption, there exists an \( \varepsilon > 0 \) such that moving \( \varepsilon \) participants from category \( i \) to category \( j \) is possible, and we still have \( 0 \leq \frac{p_i - \varepsilon}{k_i} < \frac{p_j + \varepsilon}{k_j} \leq 1 \). The area below this new Lorenz curve can be written as
\[ T' = 0 + \frac{k_i(p_i - \varepsilon)}{2} + \frac{k_j(p_j + \varepsilon)}{2} + k_j(p_i - \varepsilon) + \frac{m}{2} \left(1 + 1 - m\right) \]

Therefore, we have

\[ T' < T \iff \varepsilon \left(-\frac{k_i}{2} + \frac{k_j}{2} - k_j\right) < 0 \iff -\frac{k_i}{2} - \frac{k_j}{2} < 0 \]

Which is always satisfied. Therefore, such a transformation reduces the area below the Lorenz curve and thus increases the value of the GEGINI. This contradiction proves that there cannot be two social categories whose relative share is neither 0 nor 1. Clearly, the same argument can be used for three or more categories as well.

**Defining PIPI**

In the previous sections, we have mathematically proven that the level of inequality depends on the number of participants, the number and size/distribution of social categories. These parameters determine both the minimum and the maximum values of participants’ unequal distribution across social categories. Using data simulation techniques, we have gained the minimum level of inequality which we denote by \( GEGINI_{MIN} \). Moreover, our algorithm computes the maximum value of inequality, \( GEGINI_{MAX} \), (for a given situation where the number of participants, and the number and proportion of social groups are fixed). Applying these pieces of information, the Political Inequality in Participation Index corrects and normalizes GEGINI:

\[
\text{Definition.}
\]

\[ PIPI = \frac{GEGINI - GEGINI_{MIN}}{GEGINI_{MAX} - GEGINI_{MIN}} \]

By construction, the PIPI satisfies all three criteria described above.

**Comparing Inequality Indices**

In the previous section, we defined the Political Inequality in Participation Index (PIPI). The index provides an unbiased measurement of inequality in political participation, since it takes into consideration the population’s and the participants’ distribution across social categories, as well as the number of participants. As we have shown, these parameters shape the level of inequality in any given society. Using the PIPI, researchers and practitioners
can conduct a valid cross-country comparison because, thanks to the normalization, the range of the index spans from 0 to 1.

It is not clear, however, to what extent the PIPI is different from traditional indices. In particular, to what extent do former indices meet the set of criteria we have provided above? In this section, we demonstrate that the PIPI may bring us new empirical results in comparative research. To learn how the PIPI works in practice, we run three complex data simulations bearing in mind the logic of comparative political analyses.

As an example, imagine a study where researchers want to analyse inequalities of protest participation in several countries. These fictitious researchers are focusing on educational inequalities, and they classify educational levels using four categories: no formal schooling, low, middle and high level of education. In this example, they have survey samples from 54 countries, and the sample size is 5,000 respondents in each country.

We are looking for similarities and differences between our newly defined index, the PIPI, and the most commonly used index, the GAP, as defined above (based on Bühlmann et al. (2012)). In particular, we are calculating correlations between the values of the GAP and the PIPI to reveal whether the indices work similarly. A high correlation coefficient means that the PIPI is high when the GAP is also high, and vice versa; the PIPI tends to be lower when the GAP is lower. In the simulations we will show how the following three parameters shape correlations between the two inequality indices:

1. Distribution of protesters in the sample across educational categories
2. Number of protesters in the sample
3. Distribution of the sample across educational categories

We set up three types of simulations to study correlations. The three simulations differ in their complexity, i.e. in the number of parameters that vary. In the first simulation, the distribution of protesters may vary across

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<th>Parameters</th>
<th>Distribution of protesters</th>
<th>Number of protesters</th>
<th>Distribution of educational categories</th>
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<td>1st simulation</td>
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<td>2nd simulation</td>
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<td>3rd simulation</td>
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countries, but the number of protesters and the distribution of educational categories are held constant. In the second simulation, in addition to the distribution of protesters, the number of protesters may also vary. Finally, in the third and most complex simulation we let every parameter vary. Table 1 summarizes the differences between the three simulations.

**First simulation**

In the first simulation, the only parameter that varies is the distribution of protesters across the educational categories. Thus, we first have to fix the other two parameters. Firstly, we fix the number of protesters at 500 (10%) in each of the 54 countries. Secondly, let the distribution of the whole sample over the four educational categories be 10%, 20%, 30% and 40%, respectively. The distribution of the population does not vary between countries, thus every country has the same educational social structure. Namely, the numbers of respondents in the social categories are: 500 respondents in the no formal schooling category, 1,000 respondents in the low, 1,500 respondents in the middle, and 2,000 respondents in the high level education categories.

The result of this first simulation is that the GAP and the PIPI are perfectly correlated ($\rho = 1$) in this very simplistic case. This is obviously true. GAP’s formula contains only the parameters of protesters’ distribution and the distribution of the sample over educational categories. However, the PIPI hinges on the number of participants, through the minimum and the maximum of the index. Since we held constant of the number of protesters PIPI - just as the GAP - is insensitive to that parameter. Thus, this simulation mainly serves as a benchmark for the following, more complex situations.

**Second simulation**

In the second simulation, in addition to the distribution of protesters, we also vary the number of protesters across the countries. We set the minimum number of participants to 100 (2% of the sample), and the maximum to 2,000 (40% of the sample). Between the two extremes the number of participants always rises by ten (100, 110, 120…2,000), which indicates 191 different levels of protest participation. The distribution of participants is the same as in the first simulation, and the distribution of education is still held constant. However, we randomly assign different numbers of protesters to each country, thus the number of participants and their distribution could vary across countries. We replicate the random assignment of participation level 10,000 times. Now we calculate PIPI values for every country. Table 2 contains GAP values for the 54 countries and comprises 10,000 other
columns, where each column shows the PIPI values for the possible realization of a sample of countries with different participation levels.

Investigating similarities and differences between the two inequality indices, we use the correlations between GAP and PIPI values. Correlations between column B, on the one hand, and C, D, E .. F on the other hand in Table 2 show whether the indices work similarly. According to the results of the 10,000 correlations the GAP highly correlates with the PIPI (0.90), however the range of these correlations is between 0.81 and 0.95 (Table 3). We can draw the conclusion here that the GAP and PIPI work similarly, but they are not identical in our simulated world. It is necessary to emphasize that the social structure parameter (the distribution of citizens over the categories of education) was held constant. There were no variations between countries regarding their social structure.

The cases simulated (columns C to F in Table 2) have their uniqueness in the number of participants. Thus, if we look at Figure 2, it is easy to see that higher variation in the number of participants across countries within a random set of countries (e.g., column D) leads to lower correlations between

<table>
<thead>
<tr>
<th>Country ID</th>
<th>GAP</th>
<th>PIPI_1</th>
<th>PIPI_2</th>
<th>PIPI_3</th>
<th>…</th>
<th>PIPI_10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.22</td>
<td>0.67</td>
<td>0.56</td>
<td>0.56</td>
<td>…</td>
<td>0.84</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>0.80</td>
<td>0.60</td>
<td>0.57</td>
<td>…</td>
<td>0.64</td>
</tr>
<tr>
<td>3</td>
<td>0.22</td>
<td>0.53</td>
<td>0.60</td>
<td>0.50</td>
<td>…</td>
<td>0.49</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>54</td>
<td>0.07</td>
<td>0.21</td>
<td>0.22</td>
<td>0.26</td>
<td>…</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 3. Correlation between GAP and PIPI indices.

<table>
<thead>
<tr>
<th></th>
<th>Mean (sd)</th>
<th>Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Simulation - Moderately skewed distribution of social structure</td>
<td>0.8959 (0.0190)</td>
<td>0.8573 - 0.9306</td>
</tr>
<tr>
<td>II. Simulation - Uniform distribution of social structure</td>
<td>0.9965 (0.0006)</td>
<td>0.9952 - 0.9975</td>
</tr>
<tr>
<td>III. Simulation - Highly skewed distribution of social structure</td>
<td>0.7632 (0.0612)</td>
<td>0.6386 - 0.8769</td>
</tr>
</tbody>
</table>
the GAP and PIPI. As the graph suggests, the higher the standard deviation is in the number of participants across countries, the lower the correlation between the GAP and PIPI. This is because the PIPI takes into consideration the number of participants, whereas the GAP does not. In a situation where the countries studied tend to differ regarding the level of participation, the two indices should diverge more significantly. In reality, such a scenario would be wondrous.

Third simulation

Now we move one step further and turn to our third simulation. The analysis above shows how the PIPI correlates with the GAP if the distribution of educational categories is held constant (10%, 20%, 30% and 40%). The question that arises is how these correlations between the inequality indices change with different distributions of social categories (levels of education in our example).

Thus, we run the same simulation as before, but modify the distribution of education. In the next two simulations, we choose extreme distributions: 1) uniform distribution, 2) highly skewed distribution. In the first case, all four educational levels share the same number of respondents. A quarter of citizens belong to each of the four educational groups (no formal schooling, low, middle and high levels of education). However, at the second extreme, one educational group (e.g., low educated people) has an extremely high share of citizens, which produces a highly skewed distribution of educational groups (80%, 10%, 5%, and 5%).

The results of these two further simulations are summarized in Table 3. If the distribution of social groups is uniform, the PIPI works the same way as
the GAP. The means of the correlations is very high (0.997), and much higher than in the first simulation; moreover, confidence intervals do not overlap (Figure 3). According to the outcomes of the third simulation, correlations between the PIPI and GAP are much lower if the distribution of social categories is highly skewed.

The three simulations demonstrate that the Political Inequality in Participation Index reveals a significantly different level of inequality if there is a high variation in the number of participants, and if the distribution of social groups is not uniform across the countries studied.

Datasets containing observed data from numerous different countries (e.g. ESS, EVS, WVS) show that variation in participation levels and a skewed distribution of social groups seem to be the rule rather than the exception. This highlights the benefits of using the PIPI for measuring inequalities in political participation.

**Comparing Inequality Indices: An Empirical Analysis**

Data simulations show that the PIPI is statistically different from the GAP index developed by the Democracy Barometer project (Bühlmann et al., 2012). In this chapter, we take this reasoning further. Using European Social Survey (ESS) data, we demonstrate that the more sophisticated PIPI index detects substantially different results.

Dassonneville and Hooghe (2017) stress that the correlation between voting and the level of education has become stronger as the voter turnout has decreased in the last decades. These trends seem to be important for analyses on voter representation in democracies and on the populist turns in European countries Huber and Ruth (2017). As we find these questions very
interesting, we have chosen the case of educational inequalities as an illustration.

In the ESS project voting is measured by the question "Did you vote in the last national election?". Level of education was recoded into three categories: primary, secondary, tertiary education.

We calculated both the GAP and PIPI index for each country. In Figure 4 two countries are highlighted in red: Germany and Italy. If we measure educational inequalities in voting with the GAP index, we might conclude that there are hardly any differences between these two countries, as the GAP level is 0.025 and 0.027 respectively, while the PIPI yields that in Germany inequalities in voting across groups of education is essentially higher than in Italy. In Germany the level of educational inequality in voting measured by the PIPI is as high as 0.40, however in Italy it is only 0.24. This comparison suggests not only meaningful differences between the two countries, but the 95% confidence intervals buttress that the difference is statistically significant.

While this example is illustrative, we want to present that the differences are more general. Therefore, we used a pooled ESS dataset containing data from 33 countries between 2010-2018 (rounds 5-9), which adds up to 125 country-year combinations. After the calculation of both the GAP and PIPI indices the rank-correlation (Spearman’s $\rho$) between the measures shows a moderate similarity ($\rho = 0.54$).

Empirical analyses often focus on the relationship between inequalities in political participation and different features of the political system (Gallego, 2010; Dubrow, 2010; Huber and Ruth, 2017). For example, Dubrow (2010) analysed the correlation between political inequalities and the quality of democracy measured by the Economist Intelligence Unit’s (EIU) Democracy

![Figure 4. Educational inequality in voting measured by GAP or PIPI.](image-url)
He found a negative association between inequality in protest participation and the quality of democracy. In the following, in order to further test our proposed inequality index, we replicate the analysis of Dubrow (2010). Firstly, using the same ESS dataset (round 4) as Dubrow (2010), we calculate both the social status (SES) gap in protesting and the Political Inequality in Participation Index. Secondly, we calculate Spearman’s rank-correlations between the inequality measures and the EIU Democracy Index. We find a negative correlation between the Gap and the quality of democracy ($\rho = -0.76$, $p < 0.01$), while a weak and statistically non-significant positive correlation between PIPI and EIU. ($\rho = 0.20$, $p = 0.40$). Thus we were able to show that the Gap and the PIPI indices are different not only in an “artificial” or “simulated” world but also analyses on real-data yield qualitatively different results.

While there may potentially be many reasons for the differences, one can at least be sure that the PIPI is not biased by the social structure or the number of participants of different countries. As we show above, the results of earlier studies in general, and Dubrow (2010) in particular, are expected to be biased for several reasons. First, the range of the inequality indices used is not normalized, making the interpretation of cross-country comparisons difficult. Second, their inequality indices do not correct for the different variances induced by the cross-county differences in the number of participants and the social structure. Overall, our empirical analysis confirms the findings of Simulation 3: when countries differ both in the number of participants and their social structure, PIPI provides qualitatively different and novel results.

Conclusions

This article focuses on measurement issues of distributional inequalities regarding political participation, one of the most intensively studied topics in political sociology. This strong interest is confirmed, among others, by the well-known research project led by Sydney Verba and his colleagues (Verba et al., 1995; Schlozman et al., 2013) or the Democracy Barometer Project (Bühlmann et al., 2012). We set up a list of criteria an inequality index used for comparative analyses should meet.

Since societies often differ in their main characteristics, a desired measure of unequal distribution should be responsive to the number of participants, distribution of social groups and categories. It is also necessary that the range of the index should run from zero to one. Results of the analysis show that neither GAP-like indices (e.g., Bühlmann et al., 2012) nor EGINI (Hellevik, 1997; Hu, 2015) meet the criteria defined, they are sensitive at least to the number of participants. Studies applying these measures to compare inequalities in voting or demonstrating lead to vague interpretations, since the differences in the level of inequalities contain different levels of political
activity. To remedy this bias, we suggest using the Political Inequality in Participation Index (PIPI), a normalized Gini-based index.

As a first step towards defining the PIPI, we generalized the EGINI index to be able to treat more than two social groups. The newly defined GEGINI index still does not satisfy the desired criteria. Our next contribution is calculating the minimum and maximum values of the GEGINI. Both its minimum and maximum values hinge on the number of participants and the size of the social groups analysed. Using simulation techniques, we find the minimal level of inequality taking into consideration the given participant number and social constellation. The index reaches its minimum when participants are sorted across social categories by chance and not through social mechanisms. On the other hand, gauging the maximal value is presented as an optimization problem, solved with a non-convex mixed-integer programming algorithm.

We have run complex data simulations to investigate how social contexts shape both the PIPI and GAP indices. Our findings underscore the differences between these measures. The more different countries are regarding their level of political activity, the distribution of their social structures and the distribution of participants across these social groups, the more the PIPI diverges from the GAP index.

Finally, we used a real data case study to illustrate, that the PIPI brings essentially and significantly different results than other, more biased inequality indices.

Due to its flexibility, the PIPI enables more nuanced comparisons between different forms of political participation, like institutionalized (e.g. voting) and non-institutionalized (e.g. demonstrating, boycotting) types of political actions. Since inequality indices of political participation are building blocks of complex indicators, like the democracy index (e.g. Bühlmann et al., 2012) they can contribute to empirical investigations in political science. In addition, the grievances evoked by inequalities can be drivers of political protests, and other forms of political collective actions (Dalton et al., 2010; Opp, 2012; Somma, 2021).

We are convinced that the PIPI is a useful measurement tool for any social research aiming to describe unequal distribution where the dependent variable is a dummy. For example, it would be applicable in education sociology (attend or not attend), or for inequalities in access to the Internet (accessed or not accessed).

**Limitations**

One apparent disadvantage of using the PIPI instead of more standard measures of inequalities is that it is harder to compute. To alleviate this problem, we prepared a software with a customer-friendly interface that
calculates PIPI and made it publicly available at https://gitlab.com/kogentum/pipi. Another limitation is that, in some countries, high ‘nominal’ political participation hides very low ‘real’ participation. For instance, compulsory voting in Belgium naturally engenders higher participation levels than in other countries. Also, social surveys always use self-reported items, and ‘real’ political participation is not directly observed. In our view, these are mainly data issues, which thus can bias standard inequality analyses as well. However, we caution the reader to keep this in mind when applying our index.

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**Notes**

1. “By political participation we refer simply to activity that has the intent or effect of influencing government action – either directly by affecting the making or implementation of public policy or indirectly by influencing the selection of people who make those policies.” (Verba et al., 1995: 38).

2. In such cases, gap or distortion are synonyms for inequality.

3. Providing only one example, we cite the excellent review of Persson (2015) of the link between educational level and political participation. As Persson (2015) demonstrates, education may cause participation through a set of various mediator mechanisms. Formal education enhances students’ political knowledge, civic skills, and other politically relevant attitudes. As a result of these mechanisms, more educated citizens are politically more active than less educated people. Inequality studies try to grab the outcome of these mechanisms and distil
information about how (un)equally distributed participants are across social groups/countries.

4. Somma and Bargsted (2018) suggest a different inequality concept, namely unequal distribution of political voice among citizens instead of inequality across social groups. Their Political Gini index is very promising, however, it is based on a composite measure of political participation. The main advantage of the PIPI index compared to Political Gini is that it makes it possible to analyse different forms of political participation separately. For instance, it allows us to compare inequalities in voting to inequalities in demonstrations, or signing petitions.


6. If we measure inequality by the GAP index, the results of the simulations are very similar: Inequalities negatively correlate with the number of participants.

7. For the details of the algorithm, see: https://gitlab.com/kogentum/pipi.

8. We calculated correlations between the PIPI and GEGINI using the same simulations. We found that PIPI/GEGINI correlations are very similar to PIPI/GAP correlations.

9. Respondents not eligible to vote in the last elections were left out of the analysis.

10. Austria, Belgium, Bulgaria, Switzerland, Cyprus, Czechia, Germany, Denmark, Estonia, Spain, Finland, France, United Kingdom, Greece, Croatia, Hungary, Ireland, Israel, Iceland, Italy, Latvia, Lithuania, Montenegro, Netherlands, Norway, Poland, Portugal, Serbia, Russian Federation, Sweden, Slovakia, Slovenia, Ukraine.


12. Dubrow (2010) combined years in education and household income into one SES variable. Since we could not reconstruct the exact same SES index, we followed Ganzeboom et al. (1992) cited by Dubrow (2010): both variables were z-scored then summed up. Finally, we calculated the SES quintiles.

13. Here Dubrow (2010) used a complex protest index, combining the following participation forms: 1) contacted a politician, 2) worked in a political party, 3) worked in another organization, 4) worn a campaign badge, 5) signed a petition, 6) taken part in a lawful demonstration 7) boycotted certain products.

References


