

# Atomic Physics Constraints on the $X$ Boson

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Recently, a peak in the light fermion pair spectrum at invariant  $q^2 \approx (16.7\text{MeV})^2$  has been observed in the bombardment of  ${}^7\text{Li}$  by protons. This peak has been interpreted in terms of a protophobic interaction of fermions with a gauge boson ( $X$  boson) of invariant mass  $\approx 16.7\text{MeV}$  which couples mainly to neutrons. High-precision atomic physics experiments aimed at observing the protophobic interaction need to separate the  $X$  boson effect from the nuclear-size effect, which is a problem because of the short range of the interaction (11.8 fm), which is commensurate with a “nuclear halo”. Here, we analyze the  $X$  boson in terms of its consequences for both electronic atoms as well as muonic hydrogen and deuterium. We find that the most promising atomic systems where the  $X$  boson has an appreciable effect, distinguishable from a finite-nuclear-size effect, are muonic atoms of low and intermediate nuclear charge numbers.

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## I. INTRODUCTION

Recently, the reaction

$${}^7\text{Li} + p \rightarrow {}^8\text{Be}^* \rightarrow {}^8\text{Be} + \gamma \rightarrow {}^8\text{Be} + e^+e^- \quad (1)$$

has been observed at the MTA ATOMKI (Institute for Nuclear Research of the Hungarian Academy of Sciences) in Debrecen, and deviations from Standard Model predictions have been recorded [1–3]. While the primary aim of the study had been the hunt for a massive (“dark”) photon, the experimental data are described satisfactorily in terms of a new (“fifth-force”)  $X$  boson (vector boson) which couples to fermions according to [4, 5]

$$\mathcal{L} = -e \sum_f \varepsilon_f \bar{\psi}_f \gamma_\mu X^\mu \psi_f, \quad (2)$$

where  $X^\mu$  is the spin-1  $X$  boson field,  $f$  sums over the fermions (fermion flavors), and the  $\varepsilon_f$  coefficients describe the flavor-dependent couplings to the  $X$  boson. A family-dependence is disfavored by the authors of Refs. [4, 5]. Rather, the  $X$  boson is advocated as a possible partial explanation for the observed  $3.6\sigma$  discrepancy of the observed muon  $g$  factor [6], while assuming a family independence (electron versus muon) of the couplings  $\varepsilon_f$  (i.e., in particular,  $\varepsilon_e \approx \varepsilon_\mu$  for electron and muon).

If, accidentally, the following combination of couplings to the up and down quarks add to a value close zero,

$$2\varepsilon_u + \varepsilon_d \approx 0, \quad (3)$$

then the interaction with the  $X$  boson becomes protophobic, i.e., protons are effectively decoupled. By contrast, a numerical value of

$$|\varepsilon_n| = |\varepsilon_u + 2\varepsilon_d| \approx \left| \frac{3}{2} \varepsilon_d \right| \approx \frac{1}{100} \quad (4)$$

explains the observed  $6.1\sigma$  peak seen in the experiments [1–5] [see Eq. (10) of Ref. [4]]. The proposed vector boson has a mass of  $m_X = 16.7\text{MeV}/c^2$ . Light particles similar to dark photons in this mass range have been considered a possible solution to problems related to the understanding of certain isotope abundances in the Universe [7], and other experiments have been designed to cover the conjectured parameter range of the  $X$  boson [8] (for a more detailed discussion of the particle physics aspects of the proposed boson, see the Appendix).

From below, the parameter  $\varepsilon_e$  for the electrons is further constrained by electron beam dump experiments, which search for dark photons [4, 5], while a high bound on  $\varepsilon_e$  is set by electron  $g - 2$  experiments. Numerically, one finds that [4, 5],

$$2 \times 10^{-4} < \varepsilon_e < 1.4 \times 10^{-3}. \quad (5)$$

Traditionally, atomic high-precision experiments have been used with good effect to constrain any conjectured additions to the low-energy sector of the Standard Model (see, e.g., Refs. [9, 10]). Moreover, it has been one of the goals of high-precision atomic spectroscopy to explore the low-energy sector of the Standard Model, and to possibly discover a “hidden” sector of fundamental interactions at low energy [11]. Several recent papers explore the consequences of the proposed  $X$  boson for atomic spectroscopy, notably, isotope shifts [12–14]. The purpose of the current paper is twofold. First, we briefly discuss possible implications of the  $X$  boson for the proton and deuteron radius puzzle, which still has not been completely solved [12, 15] (see Sec. III). Second, we attempt to find a simple atomic system, in which the effect of the  $X$  boson could be discerned, based on a straightforward theoretical analysis, without resorting to numerical many-body calculations of isotope shifts [12–14] (see the discussion in Sec. V).

Also, we shall attempt to develop an intuitive understanding for the observation [14] that it is rather difficult to obtain a signal from the  $X$  boson in electronic bound systems (as discussed in Sec. II). A promising alternative appears to involve muonic systems with medium and high nuclear charge numbers, for reasons to be discussed in the following.

## II. ENERGY SCALES

In order to obtain a somewhat intuitive understanding of the  $X$  boson in terms of atomic physics, it is instructive to explore the energy scales involved in the problem. Indeed, the proposed vector boson mass of  $m_X = 16.7 \text{ MeV}/c^2$  is much larger than both the effective mass  $\alpha m_e$  of bound electronic systems, as well as the momentum scale  $\langle p \rangle = Z \alpha m_e c \approx 0.343 \text{ MeV}/c$  of hydrogenlike Uranium ( $Z = 92$ ), and also larger than the bound-state momentum  $\langle p \rangle = \alpha m_\mu c \approx 0.772 \text{ MeV}/c$  of muonic hydrogen [16], but not necessarily larger than the momentum scale  $\langle p \rangle = Z \alpha m_\mu c$  of a one-muon ion with medium charge number  $Z$ . E.g., for muonic carbon, one has a momentum scale  $\langle p \rangle = 6 \alpha m_\mu c \approx 4.63 \text{ MeV}/c$  which is commensurate with the  $X$  boson mass. For muonic magnesium, one has  $\langle p \rangle = 12 \alpha m_\mu c \approx 9.25 \text{ MeV}/c$ . These considerations are relevant because the  $X$  boson mass determines the range of the interaction mediated by the new particle, which is  $\langle r \rangle = \hbar/\langle p \rangle$ .

For electronic systems, the energy scale of the  $X$  boson is “detached” from both electronic bound systems as well as low- $Z$  muonic bound systems. The range of the  $X$  boson interaction is equal to its reduced Compton wavelength,

$$\lambda_X = \frac{\hbar}{m_X c} = 11.8 \text{ fm}, \quad (6)$$

which has to be compared to the generalized Bohr radius for muonic hydrogen,

$$\lambda_{\mu\text{H}} = \frac{\hbar}{\alpha m_\mu c} = 256 \text{ fm}, \quad (7)$$

and the (ordinary) hydrogen atom,

$$\lambda_H = \frac{\hbar}{\alpha m_e c} = a_0 = 52917.7 \text{ fm}, \quad (8)$$

where  $a_0$  is the (ordinary) Bohr radius. As already indicated, the Bohr radius for a one-muon carbon ion,

$$\lambda_{\mu^{12}\text{C}} = \frac{\hbar}{6 \alpha m_\mu c} = 42.6 \text{ fm}, \quad (9)$$

is closer to the range of the  $X$  boson interaction. In the following, we refer to the bound system with a single, negatively charged muon circling around a carbon nucleus, as “muonic carbon”. For muonic magnesium,

as defined analogously, we have  $\lambda_{\mu^{24}\text{Mg}} = 21.3 \text{ fm}$  (with nuclear charge number  $Z = 12$ ).

From now on, we shall use natural units with  $\hbar = c = \varepsilon_0 = 1$ . By matching the scattering amplitude generated by the Lagrangian (2) to an effective Hamiltonian in the no-retardation approximation (zero energy of the virtual boson), we obtain the following interaction Hamiltonian  $H_X$  for electronic bound systems (in the low-energy limit),

$$H_X^{(e)} = \varepsilon_e \varepsilon_n (A - Z) (4\pi\alpha) \frac{\delta^{(3)}(\vec{r})}{m_X^2}. \quad (10a)$$

Here,  $A$  is the mass number of the nucleus, while  $Z$  is the charge number, so that  $A - Z$  counts the number of neutrons in the nucleus. If the orbiting particle is a muon, then we need to replace  $\varepsilon_e \rightarrow \varepsilon_\mu$  and obtain

$$H_X^{(\mu)} = \varepsilon_\mu \varepsilon_n (A - Z) (4\pi\alpha) \frac{\delta^{(3)}(\vec{r})}{m_X^2}. \quad (10b)$$

The finite-nuclear-size (FNS) Hamiltonian is [17]

$$H_{\text{FNS}} = \frac{2\pi}{3} Z \alpha r_n^2 \delta^{(3)}(\vec{r}), \quad (11)$$

where  $r_n = \sqrt{\langle r_n^2 \rangle}$  is the root-mean-square charge radius of the nucleus. The two Hamiltonians (10) and (11) are both proportional to a Dirac- $\delta$  function.

## III. X BOSON AND DEUTERON RADIUS

Let us explore a possible role of the  $X$  boson in the proton and deuteron radius puzzle [16, 18, 19], and take into account a possible family dependence of the interaction, i.e., ask the question of whether a coupling constant dependence  $\varepsilon_e \neq \varepsilon_\mu$  could contribute to an explanation of the puzzle. The current status of this puzzle can be summarized as follows: For the proton, a recent measurement [20] of the  $2S-4P$  transition has indicated a possible reconciliation, by analyzing a cross-damping term (“nonresonant shift”) of the transition due to neighboring fine-structure states [21]. The revised value of the proton radius [20], derived from hydrogen spectroscopy, is  $r_p = 0.8335(95) \text{ fm}$  and in better agreement with the muonic hydrogen value  $r_p = 0.84087(39) \text{ fm}$  than the previous CODATA value of  $r_p = 0.8775(51) \text{ fm}$ , which is primarily derived from an analysis of the most accurately measured hydrogen transitions (see Table XXXVIII of Ref. [22]). One notes that the “larger” proton radius of  $r_p \approx 0.88 \text{ fm}$  is mainly derived in combining very accurate  $1S-2S$  measurements [23] with  $2S-nD$  measurements [24, 25] and  $1S-3S$  atomic hydrogen measurements [15] of the Paris group. One might speculate about an incomplete analysis of the systematic effects in the measurements of the Paris group; however, a very recent work [15] reaffirms the correctness of the analysis performed for the  $2S-8D$  and  $2S-12D$  transitions, and

1S–3S transitions [15]. One can thus, at present, not conclusively confirm that the proton radius puzzle has been solved. In any case, for the proton, it turns out that the  $X$  boson cannot contribute to an explanation of the puzzle, because of the protophobic character of the proposed interaction [see Eq. (3)].

For the deuteron, the CODATA value of  $r_d = 2.1424(21)$  fm is primarily derived from (ordinary) deuterium spectroscopy [19]. It has to be compared to the value  $r_d = 2.12562(78)$  fm derived from muonic deuterium spectroscopy [19]. The relative difference of these values is

$$\frac{\delta r_d^2}{r_d^2} = 0.016(2). \quad (12)$$

Let us assume, for the moment, that this difference is due to a lepton family non-universality of the  $X$  boson interaction. To this end, we evaluate the ratio of the energy shift due to the  $X$  boson, to the finite-size energy shift. This ratio is equal to the ratio of the change  $\delta r_d^2$  in the root-mean-square radii to the root-mean-square charge radius of the deuteron itself,

$$\frac{\langle H_X^{(e)} - H_X^{(\mu)} \rangle}{\langle H_{\text{FNS}} \rangle} = \frac{6(\varepsilon_e - \varepsilon_\mu)\varepsilon_n}{m_X^2 r_n^2} \frac{A - Z}{Z} = \frac{\delta r_d^2}{r_d^2}, \quad (13)$$

where  $A = 2$ ,  $Z = 1$ ,  $\varepsilon_n \approx 1/100$  [see Eq. (32) of Ref. [5]]. Plugging in values, one obtains

$$(\varepsilon_e - \varepsilon_\mu) \approx 0.012. \quad (14)$$

The sign can be understood from the fact that the conceivable existence of the  $X$  boson, for electronic systems, would enhance the finite-size Dirac- $\delta$  potential, for  $\varepsilon_e > 0$ , and thus lead to a larger value of the deuteron radius, if determined from electronic bound systems. The result (14) is incompatible with the bound (5) for the coupling parameter of the electron, assuming an approximate family independence  $\varepsilon_e \approx \varepsilon_\mu$  of the couplings. Furthermore, assuming  $\varepsilon_e \approx 0$ , the value  $\varepsilon_\mu = -0.012$  leads to a severe discrepancy with the muon  $g - 2$  experiment, inducing a contribution to the muon anomaly  $(g - 2)/2$  of about  $1.58 \times 10^{-7}$  [see Eq. (4) of Ref. [6]]. The  $X$  boson can thus be excluded as an explanation for the deuteron radius puzzle.

However, the conceivable existence of the  $X$  boson would (slightly) affect the determination of the deuteron radius from experiments. Namely, one normally defines the deuteron radius as the slope of the charge form factor  $G_C$  of the deuteron at zero momentum transfer, after all QED effects and effects of “external” interactions (virtual gauge bosons, etc.) have been subtracted [see Eq. (13) of Ref. [26]]. The slope of the charge form factor  $G_C$  leads to the deuteron radius (see [27] and Sec. 4.2 of Ref. [28])

$$r_d^2 = 6 \left. \frac{dG_C(q^2)}{dq^2} \right|_{q^2=0} = -6 \left. \frac{dG_C(Q^2)}{dQ^2} \right|_{Q^2=0}, \quad (15)$$

where  $Q^2 = -q^2$  is the squared four-momentum transfer. Taking the  $X$  boson into account, the deuteron radius would shift according to the replacements

$$r_d^2 \rightarrow r_d^2 - \frac{6\varepsilon_\mu\varepsilon_n}{m_X^2} \quad (16)$$

for the determination from muonic deuterium, and according to

$$r_d^2 \rightarrow r_d^2 - \frac{6\varepsilon_e\varepsilon_n}{m_X^2} \quad (17)$$

for determinations involving ordinary deuterium atoms. Taking into account the bound (5) and assuming that  $\varepsilon_e \approx \varepsilon_\mu$ , the shifts (16) and (17) are seen not to exceed 0.003 fm when expressed in terms of the root-mean-square radius  $r_d$ .

Finally, let us note that the  $X$  boson does not affect the determination of the Rydberg constant from hydrogen and deuterium spectroscopy [29]. We recall that the Rydberg constant is one of the most accurately known physical constants, with a relative accuracy on the level of  $10^{-12}$  [20, 22]. However, one notes that the inclusion of the  $X$  boson Hamiltonian (10) in the theoretical model for the determination of the Rydberg constant from hydrogen and deuterium spectroscopy would not affect the Rydberg constant, because the additional term is of the same functional form as the finite-size Hamiltonian (11) and thus reabsorbed in the nuclear radius.

#### IV. X BOSON AND MUONIC IONS

In principle, one might hope to determine the coupling parameter  $\varepsilon_e$  from isotope shifts of atomic transitions. The essential idea is to write the isotope shift as a linear combination of the mass shift of a transition (due to the change in the reduced mass of the system), of the field shift (due to the isotopic change in the nuclear radius), and due to the  $X$  boson [see Eq. (2) of Ref. [14]]. We note that in principle, the mass shift could be obtained by very accurate Penning trap measurements and thus subtracted. However, the observation of a single isotope shift does not determine the  $X$  boson coupling because of the unknown field shift, i.e., the unknown radius difference. One might think that the radius could be determined independently by a scattering experiments and subtracted. However, in scattering experiments, the  $X$  boson term (10) modifies the scattering cross section just like the finite-nuclear-size term (11) and thus could not be subtracted separately.

Measurements of isotope shifts between the same isotopes but more and different atomic transition also do not help because in the leading-order approximation, both the  $X$  boson Hamiltonian (10) as well as the finite-size Hamiltonian (11) are proportional to a Dirac- $\delta$ . One might observe isotope shifts involving more than two isotopes, considering that the prefactor of the  $X$  boson term

depends on the isotope (via the change in the neutron number, which enters the nuclear mass number  $A$ ). Even so, within the Dirac- $\delta$  approximation, one still cannot accurately determine the  $X$  boson coupling because each addition of an isotope also implies the addition of a field shift term, i.e., an additional radius difference which cannot be determined independently.

For electronic bound systems, the reduced Compton wavelength  $\lambda_H/(1+n_e)$  [see Eq. (8)], where  $n_e$  is the charge number of the ion, is much larger than the reduced Compton wavelength  $\lambda_X$  of the  $X$  boson, as given in Eq. (6). Thus, for electronic bound systems, the  $X$  boson potential remains a Dirac- $\delta$  to good approximation. If at all, then the  $X$  boson coupling could be determined based on higher-order terms beyond the Dirac- $\delta$  approximation used in Eqs. (10) and (11) (see Refs. [13, 14] for a comprehensive discussion, especially in the context of “King linearity violation” as envisaged originally in Ref. [30]). In the end, even under the optimistic assumption of an increase in the precision of isotope spectroscopy to better than 1 Hz, the range of coupling parameters and masses for the conjectured  $X$  boson [4, 5] remains out of the observable range of high-precision isotope shift measurements (specifically, see the black bar in the right panel Fig. 3.2 of Ref. [14]). A more optimistic point is taken by Ref. [13], where in Fig. 3, it is claimed that a measurement of isotope shifts in  $\text{Yb}^+$ , involving nuclei with  $A = 168, 170, 172, 174, 176$ , could potentially resolve the  $X$  boson if an experimental accuracy of 1 Hz is reached. This would correspond to an increase in the current level of experimental accuracy by four to five orders of magnitude. Additionally, the drastic difference between the resolving power of  $\text{Sr}^+$  and  $\text{Yb}^+$  reported in Fig. 3 of Ref. [13] might be considered as a little surprising because both ions have  $n_e = 1$ , and so the reduced Compton wavelength (effective length scale of the atomic binding, effective nuclear charge number) is the same for the outer electrons in both systems. It would be somewhat awkward if the electron density in  $\text{Yb}^+$ , which has a nuclear charge radius of about 5.3 fm [31] for the isotopes in question, remains essentially constant over the nuclear volume, while displaying a drastic deviation from the value inside the nucleus on a distance scale of 11.8 fm, which is the range of the  $X$  boson interaction. Such a behavior would be required in order to substantially invalidate the Dirac- $\delta$  approximation used in Eqs. (10) and (11), thus explaining the resolving power of isotope shifts in  $\text{Yb}^+$  as compared to  $\text{Sr}^+$ , reported in Fig. 3 of Ref. [13]. In any case, the precise understanding of the expansion coefficients used in Ref. [13] may depend on the details of the many-body atomic structure code used in Ref. [13].

Here, we pursue a different route and attempt to find a simple atomic system where the  $X$  boson contribution could naturally be extracted based on a straightforward analytic model. We need to find an atomic system where the Dirac- $\delta$  approximation to the  $X$  boson term (10) is

insufficient, and the  $X$  boson Hamiltonian changes into

$$H_X^{(e,Y)} = \varepsilon_e \varepsilon_n (A - Z) \alpha \frac{e^{-m_X r}}{r}, \quad (18)$$

$$H_X^{(\mu,Y)} = \varepsilon_\mu \varepsilon_n (A - Z) \alpha \frac{e^{-m_X r}}{r}. \quad (19)$$

Here, the superscript  $Y$  reminds us of the Yukawa character of the potential. If the functional form of the  $X$  boson term (10) and the finite-size term (11) are different for a particular atomic system, then we can distinguish the two effects. For muonic carbon, according to Eq. (9), we have  $\lambda_{\mu^{12}\text{C}} = 42.6$  fm, which is commensurate with the reduced Compton wavelength of the  $X$  boson given in Eq. (6), but much larger than the  $^{12}\text{C}$  radius of about 2.4 fm. Hence, we have

$$r_{12\text{C}} \ll \lambda_{\mu^{12}\text{C}}, \quad \lambda_X \lesssim \lambda_{\mu^{12}\text{C}}. \quad (20)$$

This implies that in  $^{12}\text{C}$ , the finite-nuclear-size Hamiltonian can still be approximated by a Dirac- $\delta$  potential, while the  $X$  boson Hamiltonian changes into the form given in Eq. (19).

We note that at nuclear charge number  $Z = 6$ , one can still use nonrelativistic (Schrödinger) wave functions to good approximation. In the relevant spectroscopic experiments on muonic carbon [32, 33] (for scattering data, see Ref. [34]), one observes the  $1S-2P$  transition, where the main nuclear-size effect is generated by the expectation value of the finite-nuclear-size potential (11) in the ground state. The ratio of the expectation values of the exact  $X$  boson potential to the Dirac- $\delta$  approximation in the ground state is

$$\xi_{nS} = \frac{\langle nS | H_X^{(\mu,Y)} | nS \rangle}{\langle nS | H_X^{(\mu)} | nS \rangle}, \quad (21a)$$

$$\xi_{1S} = \frac{\chi^2}{(\chi + 2)^2}, \quad (21b)$$

$$\xi_{2S} = \frac{\chi^2 (1 + 2\chi^2)}{2(1 + \chi)^2}, \quad (21c)$$

$$\xi_{3S} = \frac{3\chi^2 [16 + 27\chi^2 (8 + 9\chi^2)]}{(2 + 3\chi)^6}, \quad (21d)$$

where  $\chi$  is the ratio of the generalized Bohr radius to the reduced Compton wavelength of the  $X$  boson,

$$\chi = \frac{\lambda_{\mu^{12}\text{C}}}{\lambda_X} = \frac{m_X}{6\alpha m_\mu} \approx 3.610, \quad (22)$$

and so we have  $\xi_{1S} = 0.4140$ ,  $\xi_{2S} = 0.3904$ , and  $\xi_{3S} = 0.3865$ . Of course, we have  $\xi_{nS} \rightarrow 1$  for  $\chi \rightarrow \infty$  ( $m_X \rightarrow \infty$ ). In momentum space, the suppression of the correction for muonic carbon can be traced to the importance of spatial exchange momenta in excess of  $m_X$ , which are important in the Coulomb exchange in the discussed atomic system.

In the 1970s, there has been some discussion regarding a possible discrepancy in the determination of the charge

radius of the  $^{12}\text{C}$  nucleus, with values from electron scattering (without dispersion corrections) converging to a root-mean-square value of  $r_{12\text{C}} = 2.471(6)$  fm [34, 35], while muonic spectroscopy led to a value of  $r_{12\text{C}} = 2.4829(19)$  fm [33, 34]. Under the assumption that this discrepancy is due to  $X$  boson, an analysis similar to the one carried out in Eqs. (13) and (14) leads to a value of

$$(\varepsilon_\mu - \varepsilon_e) = 0.0070(46), \quad (23)$$

which deviates from zero by more than one standard deviation. However, the large absolute magnitude of the required coupling coefficients excludes the  $X$  boson as a viable explanation for the carbon charge radius discrepancy. After the (somewhat ad hoc) application of dispersion corrections to the scattering data, the value as determined from scattering has been shifted to  $r_{12\text{C}} = 2.478(9)$  fm [34], corresponding to

$$(\varepsilon_\mu - \varepsilon_e) = 0.0029(64), \quad (24)$$

which is fully compatible with zero.

Obviously, in order to access physically sensible values of the coupling constant [see Eq. (5)],

$$2 \times 10^{-4} < \varepsilon_e \approx \varepsilon_\mu < 1.4 \times 10^{-3}, \quad (25)$$

one needs to increase the experimental precision. In view of the inequality (20), muonic carbon appears to be well suited for an extraction of the  $X$  boson contribution, based on spectroscopic data alone. The idea is to use the state dependence of the  $\xi$  parameter, in order to be able to write a non-singular system of the equations which can be solved for the nuclear radius and the coupling parameters of the  $X$  boson. Let us denote by  $\nu_{1S2P}$  and  $\nu_{2S2P}$  the remainder frequencies obtained after subtracting all known relativistic and quantum electrodynamic (QED) contributions to the transition frequencies. Because the finite-size effect and the  $X$  boson Hamiltonian primarily shift  $S$  states, one may write for the  $nS$ - $2P$  transition,

$$\begin{aligned} \nu_{nS2P} &= \xi_{1S} \left\langle 1S \left| H_X^{(\mu)} \right| 1S \right\rangle + \langle 1S | H_{\text{NFS}} | 1S \rangle \\ &= r_{12\text{C}}^2 \frac{2}{3} \frac{(Z\alpha)^4 m_\mu^3}{n^3} \\ &\quad + \varepsilon_\mu \xi_{nS} \frac{4(A-Z)(Z\alpha)^3 \alpha \varepsilon_n}{m_X^2 n^3}. \end{aligned} \quad (26)$$

We here ignore reduced-mass corrections. The system of equations

$$\nu_{1S2P} = \xi_{1S} \left\langle 1S \left| H_X^{(\mu)} \right| 1S \right\rangle + \langle 1S | H_{\text{NFS}} | 1S \rangle, \quad (27a)$$

$$\nu_{2S2P} = \xi_{2S} \left\langle 2S \left| H_X^{(\mu)} \right| 2S \right\rangle + \langle 2S | H_{\text{NFS}} | 2S \rangle, \quad (27b)$$

can be solved for  $\varepsilon_\mu$  and  $r_{12\text{C}}$ , because of  $\xi_{1S} \neq \xi_{2S} \neq 1$ .

The solution is

$$\varepsilon_\mu = \frac{(\nu_{1S2P} - 8\nu_{2S2P}) m_X^2}{2(A-Z)(Z\alpha m_\mu)^3 \alpha \varepsilon_n} f(\chi), \quad (28a)$$

$$f(\chi) = \frac{(1+\chi)^4 (2+\chi)^2}{\chi^2 [\chi(4+3\chi) - 2]}, \quad (28b)$$

$$r_{12\text{C}}^2 = \frac{3\nu_{1S2P}}{2(Z\alpha)^4 m_\mu^3} + \frac{3(\nu_{1S2P} - 8\nu_{2S2P})}{(Z\alpha)^4 m_\mu^3} g(\chi), \quad (28c)$$

$$g(\chi) = \frac{(1+\chi)^4}{2 - \chi(4+3\chi)}, \quad (28d)$$

where  $\chi$  has been defined in Eq. (22). Plugging in the parameters for  $^{12}\text{C}$  (see Ref. [31]), one obtains for the sensitivity

$$\delta\varepsilon_\mu \approx 31.234 \frac{\delta(\nu_{1S2P} - 8\nu_{2S2P})}{\nu_{1S2P}} \approx 31.234 \frac{\delta r_{12\text{C}}^2}{r_{12\text{C}}^2}, \quad (29)$$

where  $\delta(\nu_{1S2P} - 8\nu_{2S2P})$  is the uncertainty with which  $\nu_{1S2P} - 8\nu_{2S2P}$  could be determined experimentally. Also, we should clarify that  $\delta r_{12\text{C}}^2$  is the difference in the nuclear radii, determined from the two transitions separately, assuming that one ignores the possible presence of the  $X$  boson. A comparison to recent determinations of nuclear radii for simple atomic systems [16, 18, 19] reveals that an increase in the current experimental accuracy by about two orders of magnitude will be sufficient to discern the  $X$  boson from atomic spectroscopy. For muonic magnesium, the sensitivity coefficient in Eq. (29) changes according to the replacement  $31.234 \rightarrow 58.515$ .

Various generalizations of the system of equations (27) are possible. One obvious generalization would concern additional carbon isotopes such as  $^{13}\text{C}$ , for which the expansion coefficients are a little different. In this case, if one obtains a consistent result for  $\varepsilon_\mu$  from two different isotopes, this will serve as an independent confirmation of the result. Other generalizations would include combinations of transitions in muonic systems ( $\xi_{nS} \neq 1$ ) with electronic bound systems, where  $\xi_{nS}$  is nearly equal to unity, in view of the relation  $\lambda_X \ll \lambda_H$  [see Eqs. (6) and (8)]. Also, generalizations to transitions involving the  $3S$  state are straightforward [see Eq. (21d)].

## V. CONCLUSIONS

In this article, we have studied the  $X$  boson [4, 5] from the point of view of atomic physics, both in terms of possible connections to the proton and deuteron charge puzzles [16, 18, 19] (see Sec. III) as well as muonic bound systems (see Sec. IV). As outlined in Sec. II, the parameter range of the  $X$  boson is energetically somewhat outside of the range of atomic physics and therefore, the particle is hard to detect by pure atomic physics techniques. This fact, in particular, explains why it has not been seen in atomic experiments, despite heroic efforts of experimentalists to increase the precision of measurements in simple atomic systems (see, e.g., Ref. [23]). In

fact, the range of the  $X$  boson interaction somewhat overlaps with the atomic nucleus; it can be characterized as an interaction present in some extended “nuclear halo” with a range of about 11.8 fm [see Eq. (6)].

For interactions involving bound muons, one has to use the Yukawa potential (18) instead of the Dirac- $\delta$  approximation (10b). This, however, does not imply an electron-muon nonuniversality; it simply means that the  $X$  boson effect has to be evaluated differently for bound electrons as opposed to muons. The same phenomenon is observed (for electronic systems) with vacuum polarization, where a good approximation is formed by a Dirac- $\delta$  potential for ordinary hydrogen, but one has to carry out a detailed integration for muonic systems (see Ref. [36]), because the length scale of the bound muonic system is commensurate with the electron Compton wavelength, which in turn defines the extent of the vacuum-polarization mediated modification of the Coulomb interaction.

This latter observation leads to a possible pathway toward the observation of the  $X$  boson in atomic systems, as described in Sec. IV. A model calculation involving muonic carbon illustrates that a nontrivial dependence of the  $X$  boson effect on the principal quantum number is introduced for  $S$  states in muonic systems, which leads to a separation of the effect from the nuclear-size contribution, rendering the  $X$  boson effect observable [see Eq. (28)].

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## APPENDIX: COUPLINGS IN THE NEUTRINO SECTOR

This brief appendix is devoted to the discussion of the  $X$  boson model in a particle physics context, with a particular emphasis on the neutrino sector. We recall that in Eq. (2), the couplings to the fermion fields are left as free parameters in the  $X$  boson coupling Lagrangian. In Sec. I, we have discussed constraints on these parameters for electrons, protons and neutrons, the latter being determined according to their quark content [4, 5].

Important constraints on the coupling parameters for

neutrinos have been discussed in Sec. VI.C of Ref. [5]. Namely, according to Sec. VI.C.1 of Ref. [5], some of the most stringent constraints come from the TEXONO experiment, where electron (anti-)neutrinos scatter off electrons. Because of a relatively small length of the interaction region (of about 28 meters), the electrons (of energy 1–2 MeV) remain in pure electronic flavor eigenstates.

Depending on the sign of the coupling parameters of electrons and neutrinos, the interference of the  $X$  boson term can lead to constructive or destructive interference with the Standard Model prediction. According to Sec. VI.C.1 of Ref. [5], for the electron coupling parameter range given in Eq. (5), one finds bounds for  $|\varepsilon_\nu|$  in the range from  $10^{-6}$  to  $10^{-4}$  for constructive and destructive interference alike. Here,  $\varepsilon_\nu$  is the electron (anti-)neutrino coupling parameter.

Neutrino-nucleus scattering has not yet been observed, but it is the target of a number of upcoming experiments that use reactors as sources. According to Sec. VI.C.2 of Ref. [5], from SuperCDMS, CDMSlite, and LUX, one obtains bounds for  $|\varepsilon_\nu|$  in the range from  $10^{-5}$  to  $10^{-4}$  for the electron neutrino coupling parameter  $\varepsilon_\nu$ , assuming that  $|\varepsilon_n| = 1/100$ . These constraints are not in disagreement with any other experimental observations.

Interesting connections to the neutrino sector have also been pointed out in Ref. [37], where a dark matter particle  $D$  with mass 8.4 MeV is being proposed, which would give rise to the reaction  $D + D \rightarrow X$ , where the  $X$  particle has a predicted mass of 16.8 MeV, just twice the  $D$  mass, almost perfectly matching the proposed  $X$  boson mass [4, 5]. The  $D$  particle is required for the interpretation of the Mont Blanc neutrino burst [38], as proposed in Ref. [37].

In Ref. [39] (see also Ref. [40]), the authors identify the  $X$  boson as the massive vector boson of a new  $U(1)$  gauge group, which, by virtue of the interaction Lagrangian [see Eq. (2) of Ref. [39]], is called a baryon minus lepton ( $B - L$ ) symmetry. In addition to explaining the ATOMKI anomaly [1–3], the  $U(1)_{B-L}$  also provides a possible explanation for the lightness of the neutrinos, by proposing a radiative seesaw model in which neutrinos acquire their tiny masses only by a one-loop diagram whose value is proportional to the vacuum expectation value  $v_s$  of a scalar field  $S$  which takes the role of an added Higgs-like particle [see Eq. (5) of Ref. [39]]. Likewise, the mass of the  $X$  boson is proportional to  $v_s$  [see Eq. (12b) of Ref. [39]]. In the context of the  $U(1)_{B-L}$  models, the authors of Ref. [41] point out that it could be quite natural to assume a protophobic interaction ( $\varepsilon_p \simeq \varepsilon_e \ll 1$ ), but then, it would be more natural to assume that the couplings to neutrinos are not as suppressed as indicated in Sec. VI.C of Ref. [5], but rather, that  $\varepsilon_n \simeq -\varepsilon_\nu$ . Finally, according to Ref. [42], the new  $X$  boson could also help in resolving a 2–3  $\sigma$  discrepancy between theory [43] and experiment [44] for the rare decay  $\pi^0 \rightarrow e^+ e^-$ .

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