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THE TRANSFER AND STORAGE OF INFORMATION WITH THE HELP OF DOMINATING SETS IN GRAPH, WITH PARTICULAR REGARD TO THE ORGANIZATION OF TRANSPORT

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Abstract
In this work, we present some proposals for the transfer and storage of information with the help of dominating sets in graph. In traffic science, the right flow of information is very important: the right person must receive the information at the right time, with the help of which the necessary decision can be made. It is important to note that many disasters could have been prevented or avoided with the appropriate flow of information. If we also add to this that the necessary theoretical results are available and the flow of information can be ensured with minimal financial expenditure, then it becomes clear why publications of this nature are so important. Of course, there are several methods and approaches for theoretical descriptions, but all of them can be used and none of them can be said to be the best: they all have their advantages. Thus we select the method, which our needs and we use the appropriate IT background.

Keywords: *transfer and storage of information for optimal decisions, application of graphs, dominating set.*

1. Introduction

In the transport science very important that one can get the correct information quickly. For this reason, researchers are looking for methods that enable a quick and fair transfer of information between the partners. What do partners mean? The partners or participants are divided into two groups. The first group includes those who have certain information (e.g. what to do in a certain situation, for example, in train traffic, when an unexpected obstacle appears on an open track etc.). The other group includes those to whom the information must be passed on in certain situations. Those who belong to the first group can be called the dominant group (see Figure 1. where we denoted the dominant person in red colour). The second group receives the information from the first group in certain cases (see Figure 1. where we denoted the member of the second group in green colour). Using the traffic science and graph theory results, and we will give a proposal for the optimal transfer and storage of information.

A number of valuable publications have been recently published in the area of transport. Añez, Barra and Pérez examine (Añez et al., 1996) the dual graph representation of transport networks. Arumugam and Velammal (Arumugam and Velammal, 1998) show the edge domination in graphs. Caro, West and Yuster (Caro et al., 2000) examine the connected domination and spanning trees with many leaves. The three authors provide a thorough overview of the topic. Cockayne, Dreyer, Hedetniemi and Hedetniemi in (Cockayne et al., 2004) give a further topic to consider about the dominating sets. Cockayne and Hedetniemi (Cockayne and Hedetniemi, 1977) examine the domination in graphs. Das and Bharghavan in (Das and Bharghavan, 1997) examine the routing in ad-hoc networks using minimum connected dominating sets.

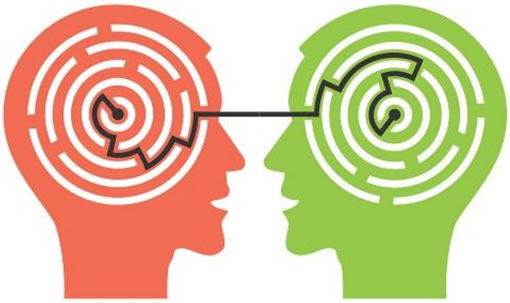

Figure 1. *The dominant person is on the left.*

Deijfen and Lindholm (Deijfen and Lindholm, 2009) show the growing networks with preferential deletion and addition. Derrible and Kennedy in (Derrible and Kennedy, 2011) give an application of graph theory and network science to transit network design. Duchet and Meyniel in (Duchet and Meyniel, 1982) provide a theoretical approach that can form the basis of further investigations. Duckworth and Mans in (Duckworth and Mans, 2009) examine the connected domination of regular graphs. Durrett in (Durrett, 2006) deals with the dynamics of the random graph, which is very useful in some cases. Dvořák in (Dvořák, 2013) shows the constant-factor approximation of the domination number in sparse graphs. Scholz-Reiter, Makuschewitz and Frazzon in (Ehm et al., 2015) examine the graph-based integrated production and intermodal transport scheduling with capacity restrictions. From Feller's book (Feller, 1957) we can study the general aspect of probability. This is necessary, or possible, if we remember that things only exist with a certain - usually high, close to one - probability. Flaxman, Frieze and Vera in (Flaxman et al., 2007) examine the adversarial deletion in a scale-free random graph process. Here, too, there is information about random graphs. Gilbert in (Gilbert, 1959), and Glebov, Liebenau and Szabó in (Glebov et al., 2015) also provide a background for a possible accidental implementation. Grenander (Grenander, 2008) contains usefuul generalizations, but his book is basically theoretical and not practice-oriented. Guha and Khuller in (Guha and Khuller, 1995) show the approximation algorithms for connected dominating sets. Haynes in (Haynes, 2017) examines the domination in graphs. Haynes, Hedetniemi and Slater in (Haynes et al., 1998a; Haynes et al., 1998b) examine dominating sets in graphs. These are very thorough and good summary works on the subject. Klieštik in (Klieštik, 2013) shows the optimization of transport routes. His work is based on graph theory. It derives the connection points to intelligent transport systems from this aspect. Li, Wu and Yang in (Li et al., 2018) make a dominating set of a graph connected. Liang, Weng, Zhou, Baez, Ma and Rong model in (Liang et al., 2018) the individual travel behavior of a public transport passenger based on graph construction. Liu, Wang and Guo in (Liu et al., 2010) examine the survey on connected dominating set construction algorithm for wireless sensor networks. Lovasz, in (Lovasz, 2012) shows the large networks and graph limits methods. The study is very interesting, although it has a rather theoretical approach. Métivier, Allain, Brossier, Mérigot, Oudet and Virieux in (Métivier et al., 2018) show the optimal transport for mitigating cycle skipping in full-waveform inversion. In (Morgado and Orgado, 2011) Morgado and Orgado show the COSA graph-based model to transport networks analysis through GIS. Niepel and Knor in (Niepel and Knor, 2009) show the domination in a digraph and in its reverse. Odor and Thiran in (Odor and Thiran, 2021) give a good overview of the sequential metric dimension for random graphs, which can play a significant role in the further thinking of the paper. O'Malley, Karra, Viswanathan and Srinivasan, in (O'Malley et al., 2018) show the efficient Monte Carlo method with graph-based subsurface flow and transport models. Ore in (Ore, 1962) provides a good overview of graph theory basics. Scrocca, Comerio, Carenini and Celino in (Scrocca et al., 2020) deal with the transformation of traffic data to meet EU standards while enabling a multimodal traffic knowledge graph. What we have described can also be applied to what is described in the article. Sun, Zhang, Mao, Mensah and Liu in (Sun et al., 2020) examine the relation extraction through a convolutional network over learnable syntax transport graph. Our study can also be applied to this approach. Viswanathan, Hyman, Karra, O'Malley, Srinivasan, Hagberg and Srinivasan in (Viswanathan et al., 2018) show the advancing graph-based algorithms for predicting flow and transport in fractured rock. Their study can be well connected to our article. Wieland and Godbole in (Wieland and Godbole, 2001) provide a theoretical overview on the domination number of a random graph. This may be important for the further development of this article. Xu, Kang, Shan and Zhao in (Xu et al., 2006) show the power domination in block graphs.

We note that what is described and suggested in this paper can be further developed. We also note that appropriate programs must be prepared for the application, which, however, can be easily done after studying this article. Of course, in the case of implementation, if we only have a small number of vertex of graphs (i.e. the company has a small number of employees), then a small or at least a medium-performance computer is sufficient. Otherwise, the graph has a large number of vertices (i.e. the company has many employees), then of course the computer must be powerful.

If the information had been passed on at the right time, serious tragedies such as the Szajol tragedy in 1994 (see Figure 2.) or the Taiwan tragedy in 2018 (see Figure 3.), both train accidents, could have been prevented. In none of the tragedies did the right person get the right information in time.

We note that the methods described in this article are mainly theoretical, so they are independent of the actual software or hardware. Consequently, the method can be applied even if the computing environment changes.

Of course, the method described here can also be improved and thought about further. We consider this paper as a basis and we are already thinking about its further development. Further thinking can take place in several directions. For example, the definition of the dominant set can be modified: we also accept it if n (n is a given natural number) can be reached by a long way to any vertex. This is suitable if it is not important that the flow of information be fast.

If during the search we get several dominant sets with the same number, we can choose (obviously only one) among the several sets by randomization.

2. Proposal for the introduction of dominating sets for the transmission of information

In this work, we propose the introduction of a certain set, with the help of which the efficient flow of information would be perfectly feasible (this set is called the dominating set and will be defined shortly). Note that this set is usually ambiguous. Although this is not a problem, but one must strive to choose a path where the set in question has the smallest possible number. For example, the whole company can be imagined as a graph: the vertices or peaks of the graph are the company's employees. Consider the graph in Figure 4: if there is a connection between two workers, this is indicated by the drawn edge. If we take a closer look at Figure 4, it can be read from the figure that, for example, there is a connection between points 1 and 3, because there is an edge between them. Or the same can be said about points 1 and 2, but it is also true for peaks 1 and 4. However, for example, there is no connection between points

2 and 4, because there is no edge between them. But the same can be said for 5 and 3, or 3 and 4 to name a few. Diagram models can, for example, model the relationship between employees of a company: of course, we also mean here that if two employees are related to each other, this is indicated by connecting the corresponding vertices of the graph representing the two employees. Of course, the modeling is usually done with a computer, for example in our case it is worth drawing the connections with a graph drawing program first. Later, of course, we will also need to use other software for the appropriate computer implementation.

Figure 2.

Figure 3.

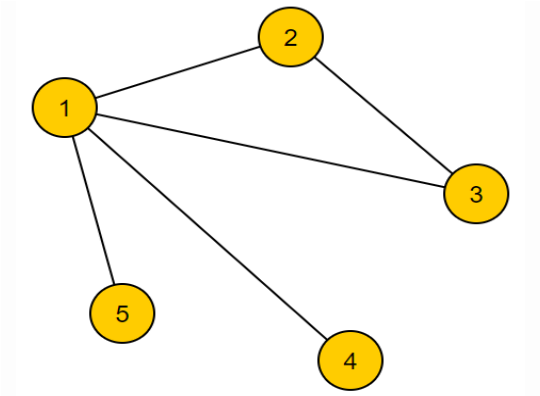

Figure 4.

The idea is to select a subset of the set of vertices where the selected vertices represent the people who can pass the information on to the others. However, it is important that every element of the selected set can reach the element of the other set. Thus, we have arrived at the definition of the dominating set.

Definition. Given a graph G = (V, E) with vertex set V and edge set E, a dominating set is a subset D ⊆ V such that every vertex in V \ D is adjacent with at least one vertex in D.

The task is thus given: we have a graph in which we should find a set D corresponding to the definition. Of course, the vertices of the graph represent the workers, while the edges indicate whether there is a connection between the workers. We note that in the case of a graph with a small number of vertices, it is relatively easy to find the dominant set. It is important to keep in mind that we always try to find the dominating sets having the smallest possible number of vertices. Therefore, if we look at the graph shown in Figure 7, which will be presented later, we should not choose the eight-element dominant set, but the one-element one. If our graph contains isolated points, then of course these points form the elements of the dominant set. It is important to note that it can be proven that the search for dominating sets in any graph is an NP-complete problem, but in our case, that is, in a graph with vertices that correspond to the employees of a company, the search problem for dominating sets can be solved with the help of a computer. Of course, we don't even need a computer for some high-end graphs (at a small company). The problem is rather that if we have found several dominating sets with the same number of elements, which one should we choose. In this case, usually any of the found sets is suitable. If it is a larger graph and the structure is less clear, then it is sufficient to examine a part of it. For example, in the graph examined earlier and shown in Figure 4, it is easy to see that if point 1 is selected, the set consisting of this point will be dominant. We note that a slightly modified version of the definition of dominating sets described in this article exists in the literature. But we will not deal with that definition now within the framework of this article; however, the above-mentioned definition may be the subject of an investigation in the future.

Below (see Figure 5.) we consider the Petersen graph and the dominating set of the graph (in red colour).

Figure 6 shows that the dominating set in star graphs can be chosen in such a way that it has only one point. This means exactly that only one person has the information to be transferred and he must transfer it in a certain case. This corresponds to the classical management method, where there is a leader (for example, a director) and the others receive his information transfer. In this sense, the method proposed in our article includes the classical case as a special case.

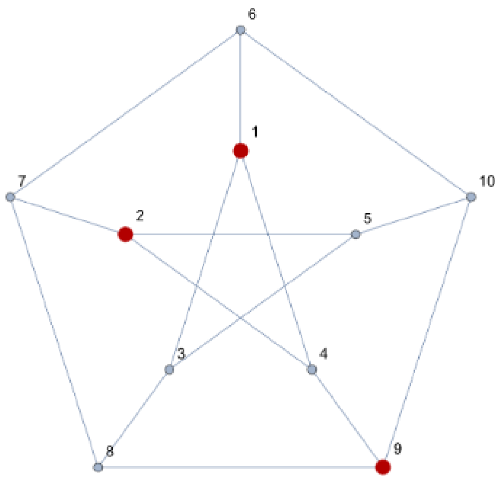

Figure 5.

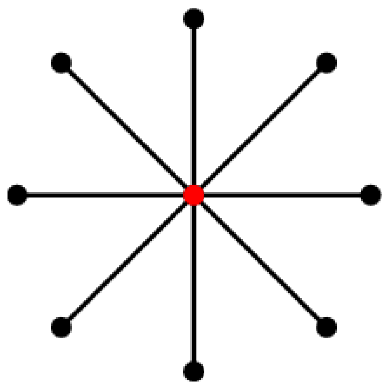

Figure 6.

In Figure 7 we show two graphs with different dominating sets (marked in red). In the first case, the dominating set consists of 1 point, while in the second case, it consists of 8 points. It can therefore be seen that the dominating sets can be very different even within a graph.

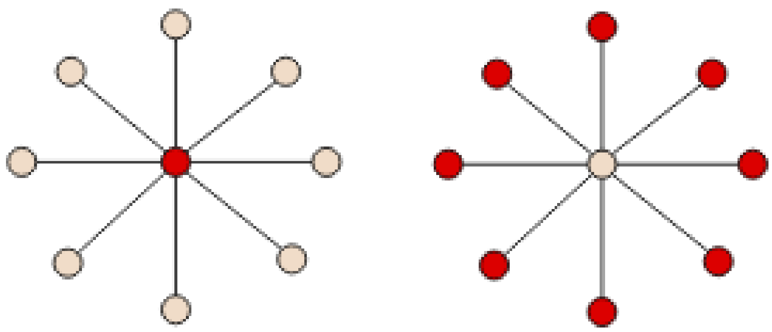

Figure 7.

Recommended construction. Let's do the following steps.

(i) Workers are denoted by the vertices of the graph,

(ii) Find one of the smallest dominating sets,

(iii) Let's provide the people identified with elements of the dominating set with the appropriate information to be handed over as the case may be.

The construction described above, which is actually an algorithm, provides an opportunity for efficient work organization. Note that this is of course not only about efficiency, but also about the fact that certain things can be prevented - for example, the tragedies in Shayoli and Taiwan. Note that, whenever possible, the elements of the dominating set should be selected so that the elements are in senior positions, such as traffic controllers and officers on the railway. Of course, if we have no other option, a subordinate can also be included among the dominant elements. It is important that the basic graph itself may change over time (new colleagues may arrive or leave the company), so the graph and its dominating set must be prepared again from time to time. We did not write this into the algorithm, but it is important to do so.

3. Transfer of information to the appropriate person

The transfer of information can be done with the appropriate IT systems, which are usually given, so it is not necessary to create a new IT system. For example, in railway traffic, the locomotive deck (see Figure 8. and Figure 9.) equipment can also be used to store information.

Of course, data transfer can take place from the simplest method (e.g. from telephone) to the most complex method. However, it is important that the information should not be distorted, or if it is, then to the smallest extent. If possible, we should have several information transmission systems at our disposal, as they can fail.

Figure 8.

Figure 8 above shows the so-called locomotive on-board equipment, which informs the locomotive driver about many things in real time, so this equipment is suitable for us to transmit information. Of course, it is even better to have another device available as a backup, such as a phone.

Figure 9.

4. Summary

In this paper we deal with the efficient transfer and storage of information with the help of dominating sets in graph, with particular regard to the organization of transport. The introduction of the dominating set proposed in the article enables the rapid flow of information e.g. in a railway system, so that in some cases the catastrophe caused by the lack of information can be prevented. Of course, in addition to finding the dominant group, it is also important to have the appropriate information tools. Furthermore, it is extremely important that the information is not damaged during transmission or as little as possible.

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