Skyrmion robustness in non-centrosymmetric magnets with axial symmetry: The role of anisotropy and tilted magnetic fields

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We investigate the stability of Néel skyrmions against tilted magnetic fields, in polar magnets with uniaxial anisotropy ranging from easy-plane to easy-axis type. We construct the corresponding phase diagrams and investigate the internal structure of skewed skyrmions with displaced cores. We find that moderate easy-plane anisotropy increases the stability range of Néel skyrmions for fields along the symmetry axis, while moderate easy-axis anisotropy enhances their robustness against tilted magnetic fields. We stress that the direction, along which the skyrmion cores are shifted, depends on the symmetry of the underlying crystal lattice. The cores of Néel skyrmions, realized in polar magnets with C_{nv} symmetry, are displaced either along or opposite to the off-axis (in-plane) component of the magnetic field depending on the rotation sense of the magnetization, dictated by the sign of the Dzyaloshinskii constant. The core shift of antiskyrmions, present in chiral magnets with D_{2d} symmetry, depends on the in-plane orientation of the magnetic field and can be parallel, anti-parallel, or perpendicular to it. We argue that the role of anisotropy in magnets with axially symmetric crystal structure is different from that in cubic helimagnets. Our results can be applied to address recent experiments on polar magnets with C_{3v} symmetry, GaV₄S₈ and GaV₄S₈.

I. THE INTRODUCTION

In magnetic compounds lacking inversion symmetry, the underlying crystal structure induces a specific asymmetric exchange coupling, the so-called Dzyaloshinskii-Moriya interaction (DMI), which can stabilize longperiod spatial modulations of the magnetization with a fixed rotation sense^{1,2}. Within a continuum approximation for magnetic properties, the DMI is expressed by inhomogeneous invariants involving first derivatives of the magnetization **M** with respect to the spatial coordinates:

$$\mathcal{L}_{i,j}^{(k)} = M_i \frac{\partial M_j}{\partial x_k} - M_j \frac{\partial M_i}{\partial x_k}.$$
 (1)

Depending on the crystal symmetry, certain combinations of these Lifshitz invariants (LI) can contribute to the magnetic energy of the material³.

In the last few years, a renewed interest in noncentrosymmetric magnets has been inspired by the discovery of two-dimensional localized modulations, commonly called magnetic skyrmions⁴⁻⁶. Recently, skyrmion lattice states (SkL)^{5,7} and isolated skyrmions (IS)^{8,9} were discovered in bulk crystals of non-centrosymmetric magnets near the magnetic ordering temperatures^{10–12} and in nanostructures with confined geometries over larger temperature regions^{5,7,13,14}. The small size, topological protection and easy manipulation of skyrmions by electric fields and currents^{15–17} generated enormous interest in their applications in information storage and processing^{18,19}.

The DMI provides not only a unique stabilization mechanism⁴, protecting magnetic skyrmions from radial

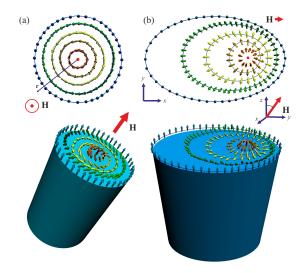
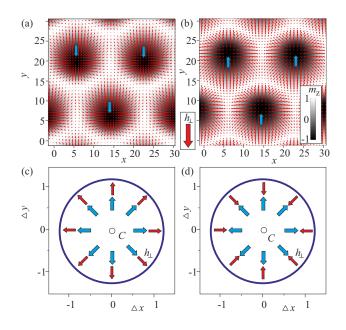


FIG. 1. (Color online) The spin texture of Bloch (a) and Néel (b) skyrmions in a tilted magnetic field. While the Bloch skyrmions in cubic helimagnets co-align their axes with the field, the Néel skyrmions displace their cores and lock their axes to the the high-symmetry (polar) axis of the host crystal.

instability, but also governs their internal structure. In Ref. 3 by Bogdanov and Yablonskii, three distinct types of skyrmions and two types of antiskyrmions (having opposite topological charge as compared with skyrmions) were predicted to occur in non-centrosymmetric magnets with different crystallographic symmetries.

The first observed type of skyrmions is called *Bloch* skyrmions, whose internal structure is schematically

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(Color online) Néel SkL in an oblique magnetic FIG. 2. field in magnets with C_{nv} symmetry, h = 0.2, $\alpha = 0.7$. (a), (b) the color plots of the m_z -component of the magnetization plotted for two types of Néel skyrmions realized with opposite signs of the Dzyaloshinskii constant D in Eq. (2). In-plane components of the magnetization are shown with thin red arrows. The in-plane projection of the magnetic field h_{\perp} is shown by the thick red arrow and is the same for both cases. h_{\perp} leads either to the parallel (a) or antiparallel (b) shift of the skyrmion cores for the "in-ward" and "out-ward" rotation sense of the magnetization, respectively. (c), (d) Displacement of skyrmion centers plotted with respect to the skyrmion centers in a "stright" magnetic field with $\alpha = 0$, point C. The radius of the curve shows the magnitude of the core shift. Blue and red arrows show corresponding directions of the field and core shift in each point of the curve.

shown in Fig. 1 (a). It is the most ubiquitous skyrmionic archetype observed in materials with B20 structure like MnSi¹⁰, FeGe¹¹ or Cu₂OSeO₃²⁰. The magnetization in Bloch skyrmions is perpendicular to the radial direction. Since the magnetic energy term corresponding to the DMI is reduced to the isotropic form $w_{DMI} = \mathbf{M} \cdot \operatorname{rot} \mathbf{M}$, the axes of Bloch skyrmions in cubic helimagnets always co-align with the applied magnetic field, irrespective of the direction of the field.

Antiskyrmions were recently observed in Mn-Pt-Sn Heusler materials with acentric D_{2d} crystal symmetry²¹. They were shown to exist over a wide temperature interval both as IS and in ordered SkL. Although the antiskyrmions break the cylindrical symmetry and carry a quadrupolar moment of the magnetostatic charges, they are invariant under all symmetry operations of D_{2d} , including the $\overline{4}$ symmetry (4-fold rotation followed by inversion).

Polar magnets with C_{nv} symmetry, such as $GaV_4S_8^{12}$ and $GaV_4Se_8^{22,23}$, host *Néel* skyrmions in which the magnetization rotates radially from the skyrmion center as shown in Fig. 1 (b). The helicity angle $\chi = 0$ ("outward" rotation of the magnetization) or $\chi = \pi$ ("inward" rotation of the magnetization) depends on the sign of the Dzyaloshinskii constant in model (2), described later, and endows the Néel skyrmions with a magnetic monopole moment on the contrary to Bloch skyrmions with a magnetic toroidal moment²⁴. Moreover, Néel skyrmions are dressed with an electric polarization^{25,26} and induce internal magnetic charges due to the non-zero divergence of the rotating magnetization vector⁴.

In the C_{nv} and D_{2d} symmetry classes investigated here, no Lifshitz invariants along the high-symmetry axis, the z direction, are present³. Therefore, only modulated magnetic structures with wave vectors perpendicular to the z axis are favoured by the DMI. As it will be shown in this work, in magnetic fields tilted with respect to the high-symmetry axis both the Néel skyrmions and antiskyrmions²¹ shift their cores with respect to the inplane component of the field instead of coaligning their axes with the field as it happens for Bloch skyrmions in cubic helimagnets, the so-called locking of the skyrmion axes.

LI (1) define a set of competing modulated phases in the phase diagrams specific to the different crystallographic classes. For the C_{nv} and D_{2d} symmetry classes, usually only spirals, skyrmions (or antiskyrmions), and collinear spin-structures are identified 12,21 . There is no competing conical state for fields applied along the highsymmetry axis. Although a transverse conical state arises for fields spanning large angles with the high-symmetry $axis^{22}$, but this is fundamentally different from the longitudinal conical state in cubic helimagnets, where the **q**-vector co-aligns with the field. The lack of the longitudinal conical state may lead to wide stability regions of skyrmions even in bulk materials with C_{nv} and D_{2d} symmetry classes. In contrast in bulk cubic helimagnets, the longitudinal conical phase is present for arbitrary direction of the magnetic field and restricts the region of skyrmion stability to a small pocket near the ordering temperatures $(A-phase)^{10,11}$.

In the present paper, we investigate the robustness of Néel skyrmions with displaced cores in oblique magnetic fields and the role of the uniaxial anisotropy (UA). Lacunar spinels GaV_4S_8 and $GaV_4Se_8^{12,22,27}$ bear uniaxial anisotropy of easy-axis and easy-plane type, respectively, as they undergo a Jahn-Teller transition from a room-temperature cubic structure into a lowtemperature rhombohedral polar (C_{3v}) structure^{25,26,28}. From the experimental phase diagrams 12,22,23 , showing a temperature dependent ratio of h_{c1}/h_{c2} , where h_{c1} is the field of the first-order phase transition between spirals and SkL and h_{c2} is the field of SkL to ferromagnetic state transition, one can conclude that the magnitude of effective UA varies with temperature (see also Fig. 7). In particular, the magnetic phase diagrams of GaV_4S_8 obtained for different orientations of the magnetic field (see e.g. Fig. 2 in Ref. 12) exhibit tri-critical points and by their topology resemble the theoretical phase diagrams constructed in Refs. 29 and 30. Therefore in the present paper, we construct the phase diagrams (Fig. 6) on the plane spanned by the axial and in-plane components of the magnetic field for both signs and various values of the UA. We show that the easy-axis anisotropy decreases the region of SkL stability and promotes the formation of ISs, whereas the easy-plane anisotropy enhances the SkL stability. We also discuss the internal structure of modulated states and the way how the theoretical phase diagrams could be set up in correspondence with the experimental ones. We also study the deformation of antiskyrmions in oblique magnetic fields, realized in Heusler compounds with tetragonal crystal structure. This compound family provide a perfect platform for the design of magnetic configurations with variable uniaxial anisotropy 21 .

II. THE MODEL

The magnetic energy density of a non-centrosymmetric ferromagnet with C_{nv} and D_{2d} symmetry can be written as the sum of the exchange, the DMI, Zeeman, and the anisotropy energy density contributions, correspondingly:

$$w = \sum_{i,j} (\partial_i m_j)^2 + w_{DMI} - \mathbf{m} \cdot \mathbf{h} - k_u m_z^2.$$
(2)

where spatial coordinates \mathbf{x} are measured in units of the characteristic length of modulated states L = A/D. A > 0 is the exchange stiffness, D is the Dzyaloshinskii constant. The unit vector along the magnetization \mathbf{M} is $\mathbf{m} = \mathbf{M}/|\mathbf{M}|$. $\mathbf{h} = \mathbf{H}/H_0$ is the applied magnetic field, $H_0 = D^2/A|\mathbf{M}|$. The magnetic field has a tilt angle α with the z axis, i.e.

$$h_z = h \cos \alpha, \ h_\perp = h \sin \alpha. \tag{3}$$

The UA $k_u = K_u M^2 A/D^2$. In the case of Néel skyrmions, k_u is the effective uniaxial anisotropy which includes the intrinsic uniaxial anisotropy (K_u) and the anisotropy due to the stray field energy⁴.

The DMI energy density has the following form:

$$C_{nv}: m_x \partial_x m_z - m_z \partial_x m_x + m_y \partial_y m_z - m_z \partial_y m_y, D_{2d}: m_x \partial_y m_z - m_z \partial_y m_x + m_y \partial_x m_z - m_z \partial_x m_y$$
(4)

where $\partial_x = \partial/\partial x$, $\partial_y = \partial/\partial y$. In both cases, only modulated phases with the propagation directions perpendicular to the polar axis of C_{nv} or to the tetragonal axis in magnets with D_{2d} symmetry, i.e. in the *xy*-plane of model (2), are favored.

Functional (2) includes only the main energy contributions essential to stabilize modulated states and an additional UA. As the main goal of the paper is to investigate the role of UA in the stability regions of skyrmions with shifted cores, we do not take into account other anisotropic contributions. In particular, besides the

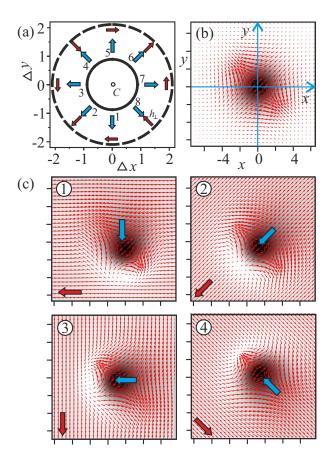


FIG. 3. (Color online) Isolated antiskyrmions in an oblique magnetic field plotted for magnets with D_{2d} symmetry. (a) Displacement of skyrmion centers plotted for $\alpha = 0.7$ (solid line) and $\alpha = \pi/2$ (dashed line). The magnitude of the field is h = 0.6 for both cases. The distance Δ by which the skyrmions shift their cores with respect to the point C is tuned by the value of α . Blue and red arrows in each point of the curve show the directions of the skyrmion shift and the direction of the field, respectively. (b) The spin structure of an isolated antiskyrmion in a field coaligned with its axis, the $\overline{4}$ axis of the host crystal. The color scale indicates m_z component of the magnetization (the color scale is the same as in Fig. 2). In-plane components of the magnetization are shown with thin red arrows in magnetic fields coaligned with its core. The x and y axes are selected in a way that xzand yz planes are the mirror planes of the D_{2d} symmetry. Grating period of x and y axes is 2. The point C in (a) is the center of an antiskyrmion. (c) The spin structure of isolated antiskyrmions with displaced cores. The in-plane projection of the magnetic field h_{\perp} is shown by thick red arrows.

single-ion type anisotropy, considered here, we neglect the difference in exchange couplings for two sets of bonds present in lacunar spinels $\text{GaV}_4\text{Se}_8^{22}$ and $\text{GaV}_4\text{S}_8^{12}$ as well as the exchange anisotropy. Moreover, we do not capture the influence of thermal fluctuations near the ordering temperatures.

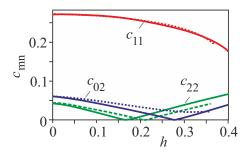


FIG. 4. (Color online) Coefficients c_{11} (red line), c_{02} (blue line), and c_{22} (green line) of the fast Fourier transform (5) for "stright" skyrmions with $\alpha = 0$ (solid lines) and elongated skyrmions with $\alpha = 0.9$ (dotted lines) See text for details. c_{11} is the amplitude of the fundamental harmonics (six-spot pattern characteristic to SkL), while c_{02} and c_{22} are the amplitudes of higher-order harmonics.

III. RESULTS AND DISCUSSION

A. Chiral modulations in C_{nv} magnets

Three magnetic phases obtained as the solutions of the model (2) can be identified as follows:

(i) Spirals. For the case of C_{nv} magnets the spirals represent cycloids with the rotation plane of the magnetization containing the wave vector \mathbf{q} and the high-symmetry (C_n) axis. For D_{2d} crystals, the type of the spiral depends on the in-plane orientation of the \mathbf{q} -vector: it is a helicoid (a Bloch spiral) for $\mathbf{q} \parallel x$ and $\mathbf{q} \parallel y$, while cycloids are formed for diagonal directions as can be discerned in the corresponding structure of the antiskyrmion in Fig. 3. The x and y axes are selected in a way that xz and yz planes are the mirror planes of the D_{2d} symmetry.

(ii) Skyrmions. Skyrmions can exist either in the form of a thermodynamically stable SkL (the corresponding regions are red shaded in the phase diagrams of Figs. 5 and 6) or as metastable IS within the stable homogeneous state (the corresponding regions are blue-shaded in Figs. 5 and 6). Isolated Néel skyrmions in bulk systems with C_{nv} have not been experimentally observed yet, although they were reported in thin films with interface-induced DMI^{8,30}. Isolated antiskyrmions in bulk Heusler alloys were reported in Ref. 21.

When the magnetic field is parallel with the highsymmetry (C_n) axis, i.e. $h_{\perp} = 0$, the period of the cycloid and its anharmonicity is gradually increased with increasing field. When the magnetic field is perpendicular to the high-symmetry axis, i.e. $h_z = 0$, a transverse conical state emerges, with a uniform magnetization developing parallel to the field, besides the magnetization component rotating in the plane perpendicular to the field. The opening angle of the cone is decreased with increasing field. As the field points in a general oblique angle, the modulated phases are skewed. In particular, skewed cycloids with the **q**-vectors along y axis, when h_{\perp} is along the x axis, can be expanded by cycloids with

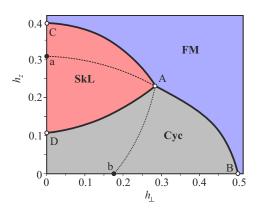


FIG. 5. (Color online) The phase diagram for model (2) plotted for zero UA $k_u = 0$ on the plane (h_{\perp}, h_z) of the field components. Filled areas indicate the regions of global stability for the SkL (red) and spirals (gray). In the saturated state (blue area), skyrmions exist as isolated objects. Thin dashed line a - A designates the phase transition between the metastable spiral state and the polarized ferromagnetic phase. Thin dotted line b - A is the line of the elliptical instability of the metastable SkL with respect to the spiral state (see text for details).

magnetization rotating in the yz plane (period of such a cycloid gradually increases with increasing h_z -component of the field), plus a transverse component oscillating in the xy plane and a uniform component.

(iii) Angular homogeneous phases. The magnetization of these field polarized ferromagnetic phases is tilted away from the z axis in the presence of oblique fields.

B. The internal structure of skewed skyrmions

(i) Skewed Néel skyrmions. The shape of the Néel skyrmions in tilted magnetic fields, as shown in Fig. 3, deforms in the following way: the cores of the skyrmions are shifted from the center of the unit cell of the SkL, but the lattice retains the stability against transformation into spirals. Whether the skyrmion centers shift along or opposite to h_{\perp} depends on the sign of the Dzyaloshinskii constant D in Eq. (2): Néel skyrmions with the "in-ward" sense of the magnetization rotation (Fig. 2 (a)) have a core displacement along h_{\perp} , whereas Néel skyrmions with the "out-ward" magnetization rotation (Fig. 2 (b)) have a core shift opposite to h_{\perp} . The effect is easily understood, as the in-plane component of the magnetic field h_{\perp} increases the area, where the magnetization points along the field. The corresponding shift of skyrmion cores is marked by blue arrows in Fig. 2 (a) and (b).

Figs. 2 (c) and (d) show the displacement of skyrmion centers as a function of the orientation of the in-plane field component for h = 0.2, $\alpha = 0.7$. The values Δx and Δy , the components of the core shift along the two axis, are relatively small as compared with the period of a spiral in zero magnetic field. The displacement of

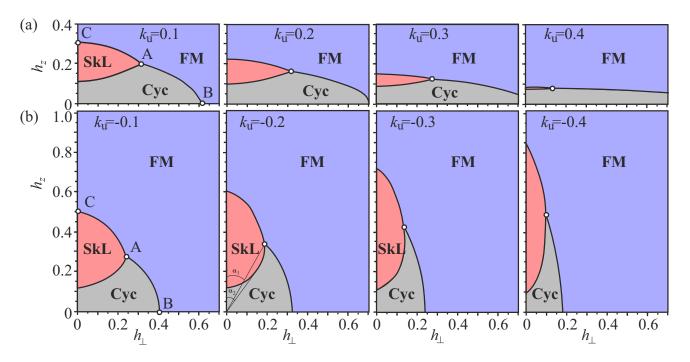


FIG. 6. (Color online) Magnetic phase diagrams as obtained for model (2) on the $h_{\perp} - h_z$ plane of the field components with the variable constant of the uniaxial anisotropy (k_u) for the easy-axis (a) and easy-plane (b) cases. h_{\perp} and h_z stand for the magnetic field components perpendicular and along the high-symmetry (C_n) axis. The second graph in (b) also shows how the critical angles α_1 and α_2 signifying the onset of the reentrant phenomenon are defined. In the interval $\alpha_1 - \alpha_2$, with increasing field the SkL transforms back to a skewed spiral state via the first-order phase transition.

the core is of the same magnitude, irrespective of the orientation of the in-plane component of the field and does not reflect the six-fold symmetry of the SkL unit cell.

In Ref. 31, it was shown that the skewed skyrmions have anisotropic inter-skyrmion interaction, i.e. the interaction energy between two skyrmions depends on their relative orientation and leads to the distorted triangular lattice. Nevertheless, the skyrmion-skyrmion interaction remains repulsive with the strongest repulsion between skyrmions in the direction where they are elongated³¹. This phenomenon singles out the Néel skyrmions from the Bloch skyrmions in B20 compounds: while Boch skyrmions orient their cores along the magnetization of the ferromagnetic host phase, skyrmions in magnets with axially symmetric crystal structure experience an orientational confinement, with their cores nearly fixed to the direction of the high-symmetry axis. Therefore for B20 materials, it is instructive first to make a full analysis of the magnetization processes for homogeneous states³² and to define the global and local minima of the energy functional (2). Then, the stable (metastable) SkL will orient their axes along the corresponding homogeneous states and form multiple skyrmionic domains with nontrivial topological boundaries between them.

In thin films of B20 materials, the reorientation process of the Bloch skyrmions in the tilted fields is a more delicate issue, leading to an undulated shape of the skyrmion cylinders³³. Such a skyrmion deformation must minimize, in particular, contributions from so called surface twists, modifying the structure of skyrmions near the surface and even leading to their thermodynamical stability (see Ref. 34 for details), and from the demagnetization effects. In Ref. 35 the process of skyrmion core rotation in tilted magnetic fields has been observed in thin epitaxial films of MnSi. The increase of the angle α results in the effective increase of the film thickness for skyrmions and, thus, in the appearance of the conical phase which is stable for thicker MnSi-films (in accordance with the theoretical phase diagram constructed in Ref. 34).

(ii) Skewed antiskyrmions. In case of the antiskyrmions the core displacement direction depends on the orientation of the in-plane field component h_{\perp} with respect to the coordinate axes. The modulations propagating parallel to these axes are helical, while the modulations propagating along the ±45 degree lines are cycloidal (Fig. 3 (b)). The core shift is perpendicular to the field if h_{\perp} is coaligned with the coordinate axes, i.e. for $h_x = 0, h_y = h_{\perp}$ (or $h_x = h_{\perp}, h_y = 0$), corresponding to points 1,3,5,7 in Fig. 3 (a) and (c). The core shift occurs along and opposite to the field in points 2, 6 and 4,8 of Fig. 3 (a) and (c), respectively.

The distance Δ , by which the skyrmion centers are displaced from their original position (point *C*) in the oblique magnetic field, depends on the value of the field and the angle α : Fig. 3 (a) shows two curves for $\alpha = 0.7$ (solid line) and $\alpha = \pi/2$ (dashed line), with h = 0.6 in both cases..

C. Fourier components of skewed SkL

The internal structure of the skyrmion lattice can also be characterized by the behavior of the Fourier components in oblique magnetic fields. Fig. 4 shows field dependence of the coefficients c_{11} (red line), c_{02} (blue line), and c_{22} (green line) for "stright" skyrmions with $\alpha = 0$ (solid lines) and elongated skyrmions with $\alpha = 0.9$ (dotted lines). The Fourier components are introduced according to

$$m_z(x,y) = \sum_{mn} c_{mn} e^{i(mx+ny)},$$

$$c_{mn} = \frac{1}{S} \int \int m_z(x,y) e^{i(mx+ny)} dx dy$$
(5)

where S is the surface area of the corresponding unit cell for SkL. The "stright" and elongated skyrmions can be distinguished by the behavior of their c_{02} coefficients: for $\alpha = 0.9$ these coefficients never become zero as is the case for $\alpha = 0$.

The coefficients c_{22} and c_{02} for "stright" skyrmions become zero for some values of the field and are small in the field region around h = 0.2. In Ref. 36 (see Fig. 10³⁶), it was shown that for the case of B20 magnets at the field h = 0.2 the difference between the energy densities of the SkL and the conical phase is minimal. On this ground, it was suggested that the skyrmion lattice is stabilized with respect to the cones exactly around this field value and underlies the phenomenon of A-phase¹⁰. Analysis of the Fourier components (5) may justify the use of the triple-q ansatz in this field interval¹⁰. Near the transition into the ferromagnetic state, however, the values of higher harmonics become significant and the ansatz in the form of the triple-q becomes incorrect.

D. Phase diagrams of states

By comparing the equilibrium energies of spirals, SkL and polarized FM states, we construct the phase diagrams on the planes (h_{\perp}, h_z) of the field components for different values of the uniaxial anisotropy k_u . The areas of magnetic states corresponding to the global minimum of the energy functional are indicated by different colors.

We start the analysis of the phase diagrams with the $k_u = 0$ case. The SkL is thermodynamically stable within a curvilinear triangle A - C - D (red-shaded region) with vertices A(0.29, 0.23), D(0, 0.11), and C(0, 0.4)^{4,29}. The line A - D is the line of the first-order phase transition between spirals and SkL. At the line A - C the SkL transforms into the polarized FM state whereas at the line b - A (thin dotted line) the metastable SkL undergoes an elliptical instability toward the spiral state (an elliptical instability of ISs was considered in Refs. 30 and 37). Therefore, the point A is a cusp point in which different lines characterizing different processes joint. The point A also corresponds to the threshold angle α_{max} of

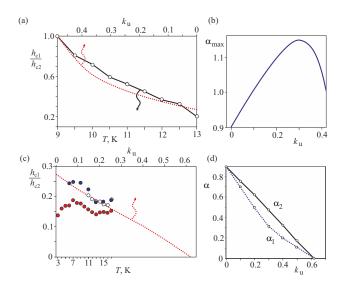


FIG. 7. (Color online) (a) the temperature-dependent (lower scale, black line with circular markers) ratio h_{c1}/h_{c2} as obtained from the experimental phase diagrams in Ref. 12 and plotted together with the UA-dependent ratio obtained theoretically from model (2) (upper scale, red dotted line). (b) the maximal angle α_{max} of the tilted magnetic field corresponding to the point A at the phase diagram of Fig. 6 (a) and plotted in dependence on the constant k_u of UA. For α exceeding this value, no SkL can be realized. (c) the ratio h_{c1}/h_{c2} plotted for the easy-plane UA: open circular markers show the values from Ref. 22, blue and red circular markers - from Ref. 23. (d) k_u -dependence of the angles α_1 and α_2 signifying the onset and ending of the reentrant phenomenon at which the spiral state may appear from the skyrmion lattice at the increasing magnetic field.

an oblique magnetic field, where the thermodynamically stable SkL state still emerges. As a metastable solution, SkL exists even in a perpendicular magnetic field in the range 0 - b of Fig. 5. For $\alpha > \alpha_{max}$ only skewed spirals (gray-shaded region) can be observed culminating in a conical phase at the point B (B(0.5, 0) for $k_u = 0$). The analysis of the type of the phase transition between skewed spirals and FM states as observed along the line a - A - B has been done in Ref. 38.

For the easy-axis uniaxial anisotropy $(k_u > 0$ in Eq. (2), Fig. 6 (a)), the existence range of the skyrmion lattice is the largest for the field co-aligned with the magnetic easy axis (point C in Fig. 6) and decreases with increasing tilting angle α . The angle α_{max} is a function of the uniaxial anisotropy k_u as shown in Fig. 7 (b). Since the skyrmion cores are aligned along the easy axes, therefore, the stronger the anisotropy is, the higher field is needed to shift the skyrmionic cores and the larger α_{max} can be achieved. At the same time, the UA suppresses the modulated phases. Thus, the temperature or UA-dependence of α_{max} shows a maximum at $k_u = 0.3$, which is realized at T=11K in GaV₄S₈.

In Fig. 7 (a) for $\alpha = 0$, we plot the ratio h_{c1}/h_{c2} : a red dotted line has been obtained from the theoretical

phase diagram in Ref. 29 and has been plotted as a function of k_u (corresponding to upper scale), while the solid black line (with circular markers) is the temperature dependence of h_{c1}/h_{c2} as extracted from the experimental data of Refs. 12 and 22 for GaV_4S_8 and GaV_4Se_8 in $H \parallel [111]$ (lower scale). By this, we make a correspondence between the experimental phase diagrams obtained for lacunar spinels GaV_4S_8 and GaV_4Se_8 and the theoretical phase diagrams in Fig. 6. Value $h_{c1}/h_{c2} = 1$ in Fig. 7 (a) corresponds to the three-critical point with $k_u^{cr} = 0.475$ and temperature $T^{cr} = 9$ K for GaV₄S₈¹². For $T < T^{cr}$, skyrmions are suppressed and only cycloids are observed. For T < 5 K ($k_u > 0.617$) only the ferromagnetic state is present in the phase diagram. Thus the increasing UA first suppresses the SkL and eventually the cycloids.

In GaV₄S₈¹² at the structural transition, the lattice is stretched along one of the four < 111 > body diagonals. The four distortion directions correspond to the magnetic easy axes of the four structural domains coexisting below 44 K. The magnetic state in each domain depends on the strength and orientation of the magnetic field with respect to the corresponding easy axis. In Ref. 12 the magnetic field was applied along the following directions in the cubic setting: <100 > ($\alpha = 0.955$), <110 > ($\alpha =$ 0.615, $\alpha = \pi/2$), and <111 > ($\alpha = 0$, $\alpha = 1.23$). The experimentally observed $\alpha_{max} = 1.23$ is slightly larger than the theoretical value in Fig. 7 (b) which could be explained, e.g., by the exchange anisotropy contribution or the slight rotation of Néel skyrmions neglected in our theory.

For the easy-plane anisotropy $(k_u < 0$ in Eq. (2), Fig. 6 (b)), the region of the skyrmion stability is significantly enhanced for $\alpha = 0$. Indeed, it was reported recently in GaV₄Se₈^{22,23} that SkL remains stable down to zero Kelvin. This corroborates with the prediction by Randeria and co-workers as applied for SkL in polar materials with Rashba-type spin-orbit interaction⁴⁰.

The angular stability of SkL, however, impairs, as the α_{max} decreases with k_u as discerned in Fig. 7 (d). This is accompanied by the re-entrance phenomenon in some interval of angles α : two successive first-order phase transitions from the spiral state to SkL and back take place with increasing magnetic field. In Fig. 6 (b) α_1 corresponds to the point A and signifies the onset of the re-entrant spiral state, α_2 coincides with the farthest point of the SkL stability region and signifies the limiting angle of the tilted field after which the re-entrance phenomenon disappears. According to Fig. 7 (d) the largest interval $\Delta \alpha = \alpha_2 - \alpha_1$ is reached for $k_u = 0.3$. No experimental observation of the predicted re-entrance phenomenon has been reported yet.

The values of h_{c1}/h_{c2} extracted from the experimental data on GaV₄Se₈ in Ref. 22 (white circles in Fig. 7 (c)) give the values of k_u in the diapason 0.15–0.25. Only the temperature range 11–17K was taken into account since for T < 11K some additional unknown phases appear which complicates the assignment of h_{c1} and h_{c2} . These

values slightly differ from the values extracted from Ref. 23 (blue and red circles in Fig. 7 (c)). No new phases below T < 11K have been reported in Ref. 23.

The structure of IS surrounded by the canted ferromagnetic phase (blue-shaded regions in Figs. 5 and 6) was recently studied in Ref. 39. Such IS has asymmetric magnetic structure and exhibit anisotropic interskyrmion potential due to the intricate domain-wall region, which connects the core of the IS with the embedding canted ferromagnetic phase. In Ref. 31 the shape of isolated skyrmions has been investigated for the field $h \approx 0.4$ and $k_{\mu} = 0$ in the units of Eq. (2), i.e., at the boundary between the skyrmion lattice and the homogeneous phase. With the increasing angle α , as seen from the phase diagram, one gets onto the region of the stable skewed spiral. Therefore, a clear coexistence phase of the isolated skyrmion and the skewed spiral can be reached. For h > 0.5 even for $\alpha = \pi/2$ ISs exist as localized entities within the homogeneously magnetized in-plane phase as also demonstrated for antiskyrmions in Fig. 3 (a).

The theoretical phase diagrams in Fig. 6 reproduce the main aspects of the experimental phase diagrams in Refs. 12 and 22 for both GaV₄Se₈ and GaV₄S₈ as compared with the isotropic case: i) the ratio of the critical fields required to reach the ferromagnetic state along and perpendicular to the polar c axis (i.e. the ratio h_C/h_B), ii) the extended or reduced angular stability range of SkL for the tilted fields in case of UA of easy-axis or easy-plane type, iii) the suppression or elongation of the stability range of the cycloidal states against fields perpendicular to the polar axis (point B). In particular, the value of the UA $k_u = 0.25$ reproduces well the experimentally constructed phase diagram in Ref. 22 and was chosen to match the ratio h_C/h_B in Fig. 6 (b) and the temperature value in Fig. 7 (c).

We therefore argue that the role played by the UA is different in chiral B20 magnets and in considered materials with C_{nv} and D_{2d} symmetry. In the first case, the UA of the easy-axis type (introduced by the axial deformation of the originally cubic structure) is invoked to suppress the conical phase²⁹. SkL becomes thermodynamically stable in some interval of anisotropy parameters although in the rest of the phase diagram SkL remains a metastable state. For the UA of the easy-plane type, only the conical phase is the global minimum of the functional (2). To stabilize SkL in this case, one should find corresponding directions of an applied magnetic field. In particular, cigar-like skyrmions can be stabilized with their axes lying in the easy plane together with the co-aligned magnetic field³⁶.

In the case of C_{nv} and D_{2d} acentric magnets, the UA modifies correspondingly the regions of the SkL stability promoting either ISs (easy-axis type) or enhancing the stability range of SkL (easy-plane type). The easyplane anisotropy may also lead to a stability of nonaxisymmetric IS with unique properties³⁹ as compared to ordinary axisymmetric skyrmions³⁰.

IV. CONCLUSIONS

In conclusion, we present theoretical studies on the robustness of skyrmions in tilted magnetic fields with special attention on the role of uniaxial magnetic anisotropy, which turns out to be a key factor governing the stability range of the modulated states. Together with earlier complementary studies on the cubic chiral magnets, our results on axially symmetric non-centrosymmetric magnets establish a consistent picture about the role of uniaxial anisotropy in the stability of modulated phases including spiral, skyrmion and antiskyrmion lattices. Our findings can be tested experimentally in non-centrosymmetric magnets lacking the Lifshitz invari-

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ants along z, e.g. with C_{nv} , D_{2d} , and S_4 crystallographic symmetry. We also described the deformations of skyrmions and antiskyrmions, both in lattices and individual ones, when the magnetic field is tilted away from this unique axis, which is the high-symmetry axis in these crystallographic classes.

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