A game theoretic framework for controlling the behavior of a content seeking to be popular on social networking sites

Khadija Touya, Hamid Garmani, Mohamed Baslam, Rachid El Ayachi, and Mostafa Jourhmane

***Abstract*—Over the years, people are becoming more dependent on Online Social Networks, through whom they constitute various sorts of relationships. Furthermore, such areas present spaces of interaction among users; they send more messages and posts showing domains they are interested in to guarantee the level of their popularity. This popularity depends on its own rate, the number of comments the posted topic gets but; also on the cost a user has to pay to accomplish his task on this network. However, the selfish behavior of those subscribers is the root cause of competition over popularity among those users. In this paper, we aim to control the behavior of a social networks users who try their best to increase their popularity in a competitive manner. We formulate this competition as a non-cooperative game. We propose an efficient game theoretical model to solve this competition and find a situation of equilibrium for the said game.**

***Index Terms*—Social network, game theory, popularity, utility function, content, publishing distribution, number of comments, price of anarchy, Nash equilibrium, best response.**

I. Introduction

Generally, social network users who have a specific type of relationship try to maintain these relational links; but they also seek to achieve a particular place within the network they use. Moreover, by posting content that guarantees a good reputation within the social network, each user looks to maximize his profit in terms of popularity so he becomes more popular, then he will attract more users to interact with him and precisely make them react to content he publishes on his own news feed, on the pages and the walls of the groups of his network. As users reveal self-promotions in their target to reach their objectives, many competitions are happening between those users; each one seeks to maximize his profit. One of these competitions is the conflict over popularity. Indeed, to have a good level of popularity, users think about publishing more content to be on the top of a timeline. This selfish behavior causes competition between published content.

Thus, popularity on social networks has become a topic of interest for researchers who want to establish analyses and studies regarding these areas of interaction [1], [2], [3], [4], [5], [6], [7], [8].

The authors are with the TIAD Laboratory, Computer Science Department, Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Beni Mellal, Morocco (e-mail: {ouya.khadija, garmani.hamid}@gmail.com)

DOI: 10.36244/ICJ.2023.2.7

Currently, various works are done in this field and many approaches are proposed to predict, optimize and estimate popularity in online social networks [9], [10], [11]. Thus, in [12], Reiffers-Masson et al. worked on solving the popularity optimization problem. They presented an approach based on flow control. Firstly, the authors developed a mathematical model of popularity, then they proved the equivalence of the popularity maximization problem with a pseudo concave optimization problem, and finally, they provided an algorithm converging to the optimal solution. In [13], authors presented a game-theoretic approach to model the competition for the popularity of contents in social networks; thus, they formulated the interaction between contents in the form of a non-cooperative game where they took into consideration the rate as the main parameter, but the price and the cost of creating the content also influence the utility function. This analysis is based on the work done by Altman in [14], where the author treated a competitive situation about popularity among service providers, he modeled this competition as a non-cooperative game and considered the creation of content by service providers and the use of the acceleration method; this tool ensuring the evolution of the content's popularity in an increasing way.

In this context, the game theory seems to be the most effective tool to solve this kind of competition; it is an approach that offers new perspectives and mechanisms beyond what classical technics could do.

In this work, we focus on solving the popularity competition problem, formulating it as a non-cooperative game where the players are the content published on the social network walls. We model the game, prove the existence and uniqueness of the Nash equilibrium and then propose an efficient algorithm to learn the equilibrium point.

To achieve the goal of our theoretical approach, we organized the rest of this paper as follows: in section II, we propose the model of the non-cooperative game between contents shared by selfish and competitive information providers, and then present an efficient analysis employing technics from algorithmic game theory, in particular, the best response algorithm ensuring fast convergence to the Nash equilibrium. In section III, we present some results illustrating the proposed theoretical approach. Finally, we close our study with a general conclusion in section IV.

II. PROBLEM MODELISATION

In this section, we formulate the interaction between selfish information providers (IPs), sharing content or posting messages, as a non-cooperative game between the contents shared on social network walls.

To set up our model, we consider a game that describes a social network with N contents (players). For each content, there is an appropriate strategy to guarantee the maximization of its popularity. This maximization is achieved taking into consideration strategies of other contents.
Let λ_i be the publishing distribution of content i, γ_i the price it must pay to be published.
And α_i the number of comments it will acquire and β_i the price to pay for getting a comment.

We analyze the set game, starting with the development of the utility function, then proving the existence and the uniqueness of the Nash equilibrium, and ending with the best response algorithm that guarantees convergence to the equilibrium situation, already noticed.

Utility model

We consider that contents shared within a social network are actors who do their best to improve their results in terms of popularity within this structure of interactions.

$G = [\mathscr{N}, \{\Lambda_i, \Theta_i\}, \{U_i\ (.)\}]$ denotes the non-cooperative game of publishing distribution and number of acquired comments, $\mathscr{N}$ is the set of contents, Λ_i, Θ_i is the set of strategies appropriate to the publishing distribution of content i and the comments it has accumulated, $\Lambda_i = [0, \lambda_i^{max}]$, $\Theta_i = [0, \alpha_i^{max}]$ and $U_i(.\ )$ is its utility function which he seeks to maximize by choosing the best strategy.

There is a tie between the publishing distribution, acquiring comments, and the popularity of the specific content published on the walls of social networks. Therefore, the utility function, of each content looking to gain popularity, depends on its revenue both in terms of the publishing distribution and in terms the number of comments it requires within the social network it propagates. But also, on the prices, it has to be distributed and to achieve a comment. Formally, the objective function of content i is defined as follows:

$$U_i(\lambda, \alpha, t) = P_0 + p_{ip}(t) \times \sum_j \lambda_j + p_{ic}(t) \times \sum_j \alpha_j - \gamma_i \lambda_i^2 - \beta_i \alpha_i^2 \tag{1}$$

Where :

The first term P_0 is a positive constant ensuring non-negative popularity. The second term, $p_{ip}(t) \times \sum_j \lambda_j$, is the impact of other contents' publishing distribution on the revenue of the content i in terms of the publishing distribution. The third term, $p_{ic}(t) \times \sum_j \alpha_j$, denotes the impact of the number of comments other agents accumulate on the revenue of the content i concerning the number of acquired comments. The fourth term, $i\lambda_i^2$, is the cost the content i has to be distributed and the last one, $\beta_i \alpha_i^2$, is the cost the same content has to pay to acquire a comment.

Following the work made in [12], arrivals are considered Poisson Point Processes. Therefore, both $p_{ip}\,(t)$ and $p_{ic}\,(t)$ follow an exponential distribution with rates, respectively, λ_i and α_i. Then the two parameters are expressed as follows:

$p_{ip}\,(t) = \int_0^T \lambda_i e^{-\lambda_i t}\ dt$ is the probability for the content i to be distributed.

$p_{ic}\,(t) = \int_0^T \alpha_i e^{-\alpha_i t}\ dt$ is the probability for the content i to acquire a comment.

When T tends to infinity, the probabilities are calculated as shown below:

$$p_{ip}\,(t) = \int_0^\infty \lambda_i e^{-\lambda_i t}\ dt\ = \left[-e^{-\lambda_i t}\right]_0^\infty\ = 1$$

$$p_{ic}\,(t) = \int_0^\infty \alpha_i e^{-\alpha_i t}\ dt\ = \left[-e^{-\alpha_i t}\right]_0^\infty\ = 1$$

Based on these results, we describe the utility function using the following formula:

$$U_i(\lambda, \alpha) = P_0 + \sum_j \lambda_j + \sum_j \alpha_j - \gamma_i \lambda_i^2 - \beta_i \alpha_i^2 \tag{2}$$

III. GAME ANALYSIS

Given that the competition between contents is becoming more popular, the natural solution of this non-cooperative game will be allowed by the Nash equilibrium, which is considered to be a strategic profile such that no content can unilaterally increase its revenue. Using the tools of concave game theory, we prove the existence and the uniqueness of the Nash equilibrium point as presented in [15]. We assume that a non-cooperative game G is concave if the utility functions of all players are strictly concave with respect to their corresponding strategies [15].

According to [15], a Nash equilibrium exists for a concave game if the space of joint strategies is compact and convex, and the utility function that a given player seeks to maximize is concave with respect to his own strategy and continuous at any point in the space of strategies through the understudy system.

Let φ be the weighted sum of utility functions with non-negative weights, it is defined by the following formula:

$$\varphi = \sum_{i=1} x_i U_i \tag{3}$$

To ensure the uniqueness of the Nash equilibrium, φ must be diagonally strictly concave. Where, the concept of strict diagonal concavity means that the control an individual content has over its utility function is greater than the control that others have over it. Thus, the uniqueness of the existing equilibrium is demonstrated using the pseudo-gradient of the weighted sum of utility functions discussed in [15].

A. Publishing distribution game

The game G of publishing distribution is defined for fixed values $\alpha_i \in \Theta_i$ such as: $G(\alpha) = [\mathscr{N}, \{\Lambda_i\}, \{U_i(., \alpha)\}]$

Definition 1: Publishing distributions' vector $\lambda^* = (\lambda_1^*, ..., \lambda_N^*)$ is a Nash equilibrium if for each $i \in \{1, ..., N\}$,

$$U_i\left(\lambda_i^*, \lambda_{-i}^*\right) = \max_{\lambda_i \in \Lambda_i} U_i(\lambda_i, \lambda_{-i}^*) \tag{4}$$

In other words, the definition 1 shows, clearly, that by reaching the equilibrium point, no source could obtain a benefit by changing its strategy unilaterally (individually).

Theorem 1: For the game $G(\alpha)$ which is concave, the Nash equilibrium exists and it is unique.

Proof 1: To prove the existence of the equilibrium point, we mention that the strategy space of each content Λ_i is defined by all frequencies in the closed interval bounded by the minimum and maximum frequencies. Thus, the joint strategy space Λ_i is non-empty, convex, and compact. Moreover, the utility functions are concave with respect to λs as shown by the second derivative test:

$$\frac{\partial^2 U_i(\lambda, \alpha)}{\partial^2 \lambda_i} = -2\gamma_i < 0 \tag{5}$$

Having reached the negativity of the second derivative, we focus on the proof of the uniqueness of the equilibrium situation. Following [15], we define the weighted sum of the users' utility functions as follows:

$$\varphi(\lambda, x) = \sum_{i=1}^{N} x_i U_i(\lambda_i, \lambda_{-i}) \tag{6}$$

The pseudo-gradient of the equation (6) is given by :

$$g(\lambda, x) = [x_1 \nabla U_1(\lambda_1, \lambda_{-1}), \quad \dots \quad , x_N \nabla U_N(\lambda_N, \lambda_{-N})]^T$$

Thus, the Jacobian matrix J of the pseudo-gradient is expressed as follows:

$$J = \begin{pmatrix} x_1 \frac{\partial^2 U_1}{\partial \lambda_1^2} & \cdots & x_1 \frac{\partial^2 U_1}{\partial \lambda_1 \partial \lambda_N} \\ \vdots & \ddots & \vdots \\ x_N \frac{\partial^2 U_N}{\partial \lambda_N \partial \lambda_1} & \cdots & x_N \frac{\partial^2 U_N}{\partial \lambda_N^2} \end{pmatrix}$$

$$J = \begin{pmatrix} -2x_1\gamma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -2x_N\gamma_N \end{pmatrix}$$

The found matrix J is a diagonal matrix with negative diagonal elements, so we can say that J is negatively defined. Similarly, $[J + J^T]$ is negatively defined, and according to [15], the weighted sum of utility functions is diagonally strictly concave. Thus, the game $G(\alpha)$ admits a unique Nash equilibrium.■

B. Number of acquired comments game

The game G of the number of acquired comments is defined for fixed values $\lambda_i \in \Lambda_i$ such as: $G(\lambda) = [\mathscr{N}, \{\Theta_i\}, \{U_i(\lambda, .)\}]$

Definition 2: The number of acquired comments' vector $\alpha^* = (\alpha_1^*, ..., \alpha_N^*)$ is a Nash equilibrium if for each $i \in \{1, ..., N\}$,

$$U_i\left(\alpha_i^*, \alpha_{-i}^*\right) = \max_{\alpha_i \in \Theta_i} U_i(\alpha_i^*, \alpha_{-i}^*) \tag{7}$$

As it is already mentioned (in definition 1), the definition 2 allows to conclude that also no content (player) has the advantage to change its strategy individually, as it is already noted in the last part reserved to the game of publishing distribution of contents.

Theorem 2: For the game $G(\lambda)$ which is concave, Nash equilibrium existes and it is unique.

Proof 2: Calculating the second derivative of the utility function with respect to the number of comments, we find that :

$$\frac{\partial^2 U_i(\lambda, \alpha)}{\partial \alpha_i^2} = -2\beta_i < 0 \tag{8}$$

Ensuring the existence of the Nash equilibrium, we turn to the proof of its uniqueness. Following [15], we define the weighted sum of users' utility functions as follows:

$$\varphi(\alpha, x) = \sum_{i=1}^{N} x_i U_i(\alpha_i, \alpha_{-i}) \tag{9}$$

The pseudo-gradient of the equation (9) is given by :

$$g(\alpha, x) = [x_1 \nabla U_1(\alpha_1, \alpha_{-1}), \quad \dots \quad , x_N \nabla U_N(\alpha_N, \alpha_{-N})]^T$$

Thus, the Jacobian matrix J of the pseudo-gradient is expressed as follows:

$$J = \begin{pmatrix} x_1 \frac{\partial^2 U_1}{\partial \alpha_1^2} & \cdots & x_1 \frac{\partial^2 U_1}{\partial \alpha \partial \alpha_N} \\ \vdots & \ddots & \vdots \\ x_N \frac{\partial^2 U_N}{\partial \alpha_N \partial \alpha_1} & \cdots & x_N \frac{\partial^2 U_N}{\partial \alpha_N^2} \end{pmatrix}$$

$$J = \begin{pmatrix} -2x_1\beta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -2x_N\beta_N \end{pmatrix}$$

The found matrix J is a diagonal matrix with negative diagonal elements, so we allowed to say that J is negatively defined. Similarly, $[J + J^T]$ is negatively defined, and according to [15], the weighted sum of utility functions is diagonally strictly concave. Thus, the game $G(\lambda)$ admits a unique Nash equilibrium. ■

C. Learning the Nash equilibrium

The above sections show that the Nash equilibrium exists and it is unique. Indeed, we will develop a distributed algorithm that converges to the Nash equilibrium of the publishing distribution set and the number of acquired comments. The algorithm 1 details the work done to learn the Nash equilibrium.

Algorithm 1 Best Response Algorithm

1: Initialization of publishing distribution and number of acquired comments vectors λ and α, randomly;

2: For each content $i \in \mathcal{N}$, at the iteration t :

$$a)\quad \lambda_i(t+1) = \underset{\lambda_i \in \Lambda_i]}{\text{argmax}} \left(U_i(\lambda_i, \lambda_{-i})\right).$$

$$b)\quad \alpha_i(t+1) = \underset{\alpha_i \in \Theta_i]}{\text{argmax}} \left(U_i(\alpha_i, \alpha_{-i})\right).$$

3: **IF** $\forall i \in \mathcal{N}$,
$|\lambda_i(t+1) - \lambda_i(t)| < \epsilon$ and $|\alpha_i(t+1) - \alpha_i(t)| < \epsilon$, STOP.

4: **ELSE**, $t \leftarrow t+1$ and go back to step (2)

D. Price of anarchy

The price of Anarchy was introduced by Koutsoupias and Papadimitriou, in their work [16]. Then the book [17] developed central ideas of it, but also multiple kinds of works have been produced around this concept. Furthermore, the work established by Roughgarden and Tardos [18] made this main measure of loss of equilibrias efficiency more popular. In fact, they employed the price of anarchy in atomic and nonatomic congestion games. Moreover, PoA has also appeared for facilitating a location in [17] and for creating a network in [19]. Thereby, the price of anarchy is considered to be a tool for resolving the issue of multiple equilibriums adopting a worst-case approach. This approach defines that loss as the worst-case ratio comparing the global efficiency measure at an outcome, to the optimal value of that efficiency measure. In a noncooperative game, the price of anarchy is defined as the ratio between the worst utility function value of equilibrium and the one of an optimal outcome. According to [20], the inefficiency caused by the players self-promotion is measured as the quotient between the social welfare, that Maille and Tuffin presented as the sum of the utilities of all providers in the system) in [21], obtained at the Nash equilibrium and the maximum value of the social welfare, as shown in (10).

$$\begin{cases} PoA_\lambda = \frac{\min W_{NE}(\lambda)}{\max W(\lambda)} \\ PoA_\alpha = \frac{\min W_{NE}(\alpha)}{\max W(\alpha)} \end{cases} \tag{10}$$

Where:

$$\begin{cases} \max W(\lambda) = \max_\lambda \sum_{i=1}^N U_i(\lambda) \\ \max W(\alpha) = \max_\alpha \sum_{i=1}^N U_i(\alpha) \end{cases}$$

is a system presenting the social welfare function for each parameter,
and

$$\begin{cases} W_{NE}(\lambda) = \sum_{i=1}^N U_i(\lambda^*) \\ W_{NE}(\alpha) = \sum_{i=1}^N U_i(\alpha^*) \end{cases}$$

is a system presenting the sum of utilities of all contents at a frequency Nash equilibrium and a number of acquired comments Nash equilibrium.

IV. NUMERICAL INVESTIGATIONS

We propose to numerically study the interaction game between the contents on the walls and the news feeds of a social network, taking into account the previous expressions of the utilities. To illustrate our work and show how to take advantage of our theoretical analysis, we perform the numerical part considering the best response algorithm and the expression of the utility function of each content.

To do so, we consider a system with two contents; two players seeking to maximize their respective revenues. Each content varies its own decision parameters - publishing distribution and the number of comments it receives - taking into account those of its opponent.

Taking into account the expression of the utility function given by the equation (2), we start with the graphical representation of this function, in the case of the publishing distribution game on the one hand and the number of comments game on the other hand.

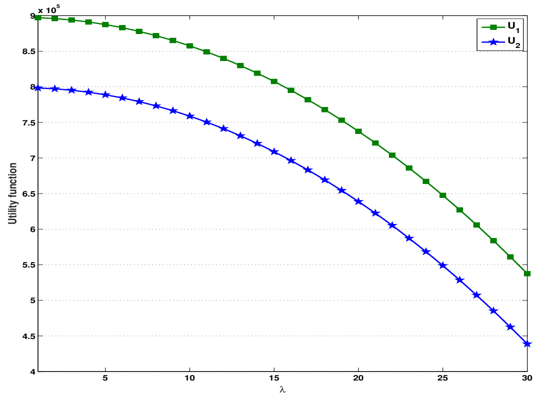

Fig. 1: Publishing distribution game: Utility function with respect to λ.

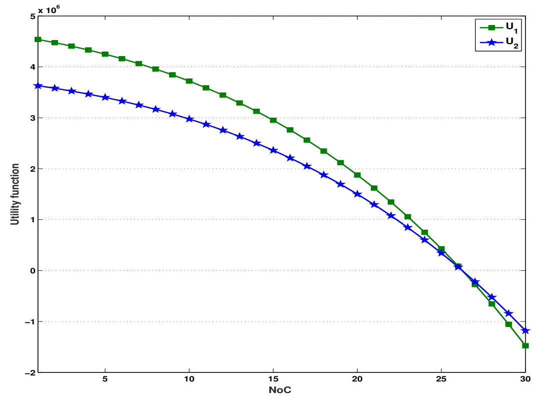

Fig. 2: Number of acquired comments game: Utility function with respect to Noc.

Figures 1 and 2 present the evolution of the utility functions of the contents as a function of the two parameters; the publishing frequency and the number of acquired comments. We notice that the plotted curves reveal the concavity of the function for all the values of the decision parameters already mentioned (λ and NoC). Therefore, the Nash equilibrium for both cases exists and it is also unique.

Concerning the learning of the Nash equilibrium of publishing frequency and also of the number of comments, we use the best response algorithm (algorithm 1) .

Figures 3 and 4 illustrate the convergence to the Nash equilibrium of posting frequency and to the Nash equilibrium of number of comments acquired. We remark that this convergence is ensured after a few rounds (almost 5 iterations are sufficient to reach the Nash equilibrium), which means that the learning speed of the Nash equilibrium is relatively high. These numerically completed investigations reinforce, then, the results that we have already proved theoretically.

We move on to study the impact of the publishing price and the comment acquisition price on the performance of the treated system.

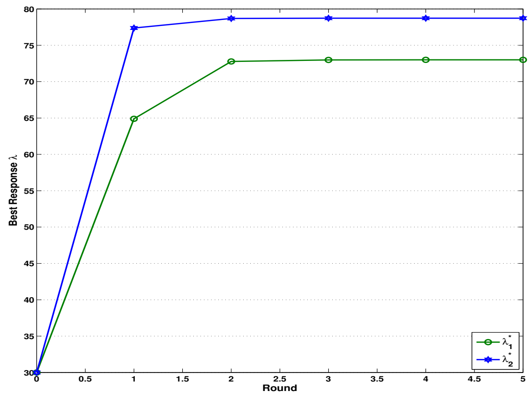

Fig. 3: Publishing frequency game: Convergence to the Nash equilibrium of λ.

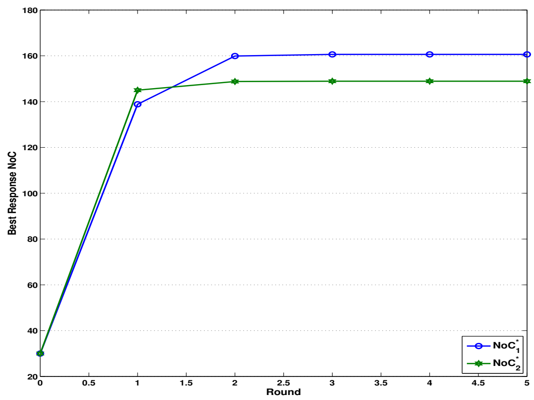

Fig. 4: Number of acquired comments game: Convergence to the Nash equilibrium of NoC.

Figures 5 and 6 describe the impact of the publishing price on the Nash equilibrium utility function and publishing distribution. With increasing values of the price of publishing content on the timelines of a social network γ, the equilibrium utility functions and publishing frequencies decrease. While when the price is low, the publishing frequencies and utility functions are higher. As a result, increasing the publishing price γ leads to the adoption of a content publishing frequency which is lower.

On the other hand, the price of obtaining a comment has a large influence on the values of the utility functions but also on the number of acquired comments (NoC) at the equilibrium.

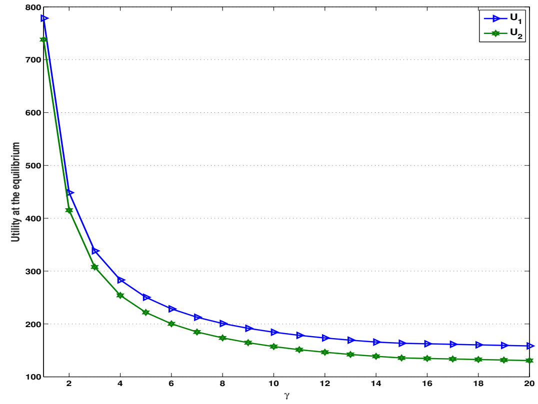

Fig. 5: Publishing distribution game: Impact of the price γ on the utility function at the equilibrium.

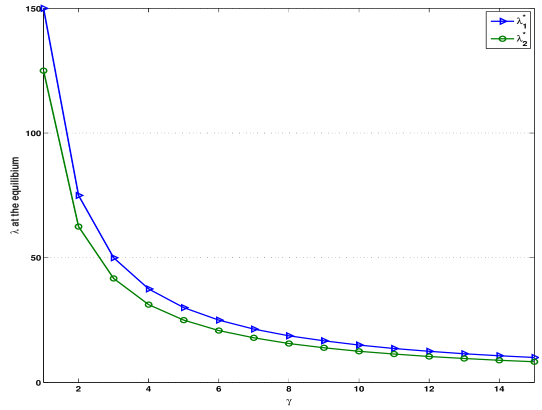

Fig. 6: Publishing distribution game: Impact of the price γ on the publishing distribution at the equilibrium.

The curves, in Figures 7 and 8, illustrate the impact of the price of acquiring a comment, respectively, on the utility function and the number of acquired comments of both of contents (players) at Nash equilibrium. According to the graphs, increasing the price of obtaining a comment β leads to the decrease of the values at Nash equilibrium concerning the utility function and the number of aquired comments. So, with a low price of getting a comment, the number of comments obtained and the utility function at the Nash equilibrium have higher values for both players which encourages the contents to look for acquiring more comments. Whereas, for a very high price, the contents concur to weaken the number of acquired comments.

To measure the efficiency of the Nash equilibrium, in our study, we propose to analyze the evolution of the price of anarchy as a function of the prices γ and β.

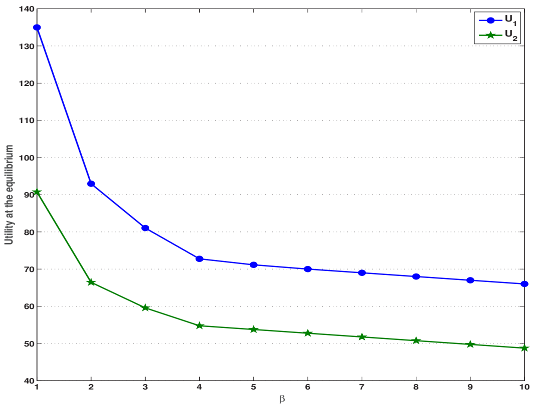

Fig. 7: NoC game : Impact of the price β on the utility function at the equilibrium.

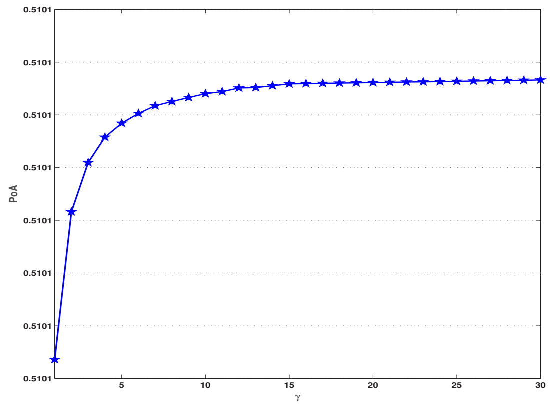

Fig. 9: Publishing game: Price of anarchy with respect to the price γ.

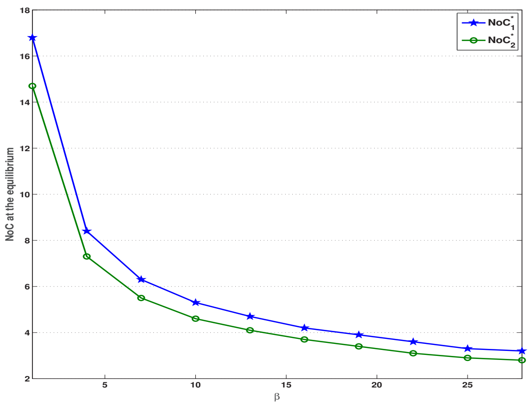

Fig. 8: NoC game : Impact of the price β on the NoC at the equilibrium.

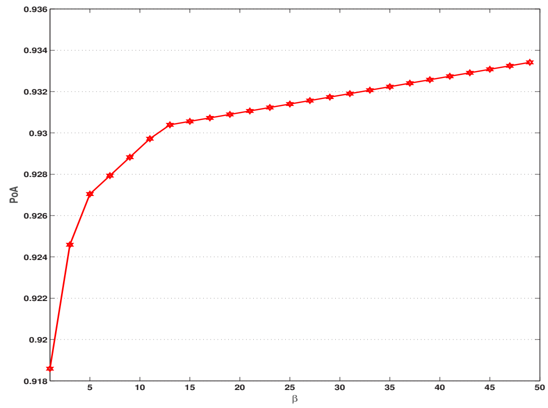

Fig. 10: NoC game: Price of anarchy with respect to the price β.

The impact of prices (publishing price and comment acquisition price) on the efficiency of the studied system is represented in Figures 9 and 10. The two curves reveal that the PoA increases with respect to the publishing price (comment acquisition price). For a publishing price and a comment acquisition price which are low, PoA is low. Subsequently, players behave in a highly selfish and individual manner to maximize their gain in terms of popularity. However, by increasing prices, the PoA increases and thus the players move, in their decision making, towards the Nash equilibrium strategies of publishing frequency and strategies of number of acquired comments. Thus, the Nash equilibrium is fair and socially efficient in both cases.

V. CONCLUSION

In this paper, we proposed a theoretical approach using game theory to model the interactions between contents posted on social network news feeds as players through a class of two-parameter Nash equilibrium models. The model is based on a simple function describing the behavior of the contents. In fact, we took into account the characteristics of the content itself (its publishing frequency and the number of comments it receives), but also those of other contents sharing the same structure. We proved the existence and the uniqueness of the Nash equilibrium point and developed the distributed best response algorithm that allows to learn this equilibrium point in a finite number of iterations. In short, the numerically obtained results validate the work established to study user reactions; they can be extended to general considerations on networks. In this context, the existence and the uniqueness of the Nash equilibrium, in the proposed approach, help us to confirm the stability within the under study social network. Since the progression of popularity is considered as a primordial step that cannot be ignored in order to see one's web visibility explode, information providers employ it to dominate social networks they are subscribed to. Then maximize their profit while communicating through the network.

REFERENCES

[1] B. Ribeiro and C. Faloutsos, "Modeling Website Popularity Competition in the Attention-Activity Marketplace," *arXiv:1403.0600 [physics]*, Mar. 2014, DOI: 10.1145/2684822.2685312.

[2] A. Matakos and A. Gionis, "Tell me something my friends do not know: Diversity maximization in social networks," *arXiv:1811.10354 [cs]*, 2018 DOI: 10.1109/ICDM.2018.00048.

[3] Y. Ma, J. He, and Q. Yu, "Modeling on social popularity and achievement: A case study on table tennis," *Physica A: Statistical Mechanics and its Applications*, vol. 524, pp. 235–245, 2019, DOI: 10.1016/j.physa.2019.04.007.

[4] S. Dhamal, W. Ben-Ameur, T. Chahed, and E. Altman, "A two phase investment game for competitive opinion dynamics in social networks," *Information Processing & Management*, vol. 57, no. 2, p. 102064, 2020, DOI: 10.1016/j.ipm.2019.102064.

[5] K. Touya, M. Baslam, R. El Ayachi, and M. Jourhmane, "A Game theoretic approach for competition over visibility in social networks," *Bulletin of Electrical Engineering and Informatics*, vol. 8, no. 2, pp. 674–682, 2019, DOI: 10.11591/eei.v8i2.728.

[6] X. Li, I. Kawachi, O. M. Buxton, S. Haneuse, and J.-P. Onnela, "Social network analysis of group position, popularity, and sleep behaviors among us adolescents," *Social Science and Medicine*, vol. 232, pp. 417–426, 2019, DOI: 10.1016/j.socscimed.2019.05.026.

[7] Q. Cao, H. Shen, J. Gao, B. Wei, and X. Cheng, "Popularity prediction on social platforms with coupled graph neural networks," in *Proceedings of the 13th international conference on web search and data mining,* 2020, pp. 70–78, DOI: 10.1145/3336191.3371834.

[8] S. C. Montgomery, M. Donnelly, P. Bhatnagar, A. Carlin, F. Kee, and R. F. Hunter, "Peer social network processes and adolescent health behaviors: A systematic review," *Preventive medicine*, vol. 130, p. 105900, 2020, DOI: 10.1016/j.ypmed.2019.105900.

[9] K. Lerman and T. Hogg, "Using a Model of Social Dynamics to Predict Popularity of News," *arXiv:1004.5354 [cs]*, Apr. 2010, DOI: 10.1145/1772690.1772754.

[10] Z. Bao, Y. Liu, Z. Zhang, H. Liu, and J. Cheng, "Predicting popularity via a generative model with adaptive peeking window," *Physica A: Statistical Mechanics and its Applications*, vol. 522, pp. 54–68, May 2019, DOI: 10.1016/j.physa.2019.01.132.

[11] D. Wu, B. Liu, Q. Yang, and R. Wang, "Social-aware cooperative caching mechanism in mobile social networks," *Journal of Network and Computer Applications*, vol. 149, p. 102457, 2020, DOI: 10.1016/j.jnca.2019.102457.

[12] A. R. Masson, Y. Hayel, E. Altman, and G. Martel, "A Generalized Fractional Program for Maximizing Content Popularity in Online Social Networks," in *2018 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining (ASONAM)*. Barcelona: IEEE, Aug. 2018, pp. 869–872, DOI: 10.1109/ASONAM.2018.8508480.

[13] S. Hafidi, M. Baslam, and R. E. Ayachi, "Analysis of Competition Fronting the Popularity of Content in Social Networks," vol. 6, no. 2, pp. 86–94, 2017, DOI: 10.11591/ijict.v6i2.pp86-94, number: 2.

[14] E. Altman, "A semi-dynamic model for competition over popularity and over advertisement space in social networks," in *Proceedings of the 6th International Conference on Performance Evaluation Methodologies and Tools*. Cargse, France: IEEE, 2012, DOI: 10.4108/valuetools.2012.250334.

[15] J. B. Rosen, "Existence and Uniqueness of Equilibrium Points for Concave N-Person Games," *Econometrica*, vol. 33, no. 3, p. 520, 1965, DOI: 10.2307/1911749.

[16] E. Koutsoupias and C. Papadimitriou, "Worst-case equilibria," *Computer science review*, vol. 3, no. 2, pp. 65–69, 2009, DOI: 10.1016/j.cosrev.2009.04.003.

[17] A. Vetta, "Nash equilibria in competitive societies, with applications to facility location, traffic routing and auctions," pp. 416–425, 2002, DOI: 10.1109/sfcs.2002.1181966.

[18] T. Roughgarden and . Tardos, "How bad is selfish routing?" *Journal of the ACM (JACM)*, vol. 49, no. 2, pp. 236–259, 2002, DOI: 10.1145/506147.506153.

[19] E. Anshelevich, A. Dasgupta, E. Tardos, and T. Wexler, "Near-optimal network design with selfish agents," pp. 511–520, 2003, DOI: 10.1145/780542.780617.

[20] L. Guijarro, V. Pla, J. Vidal, and J. Martinez-Bauset, "Analysis of price competition under peering and transit agreements in internet service provision to peer-to-peer users," in *IEEE Consumer Communications and Networking Conference (CCNC2011), Las Vegas, Nevada USA*, 2011, pp. 9–12, DOI: 10.1109/ccnc.2011.5766356.

[21] P. Maill and B. Tuffin, "Analysis of price competition in a slotted resource allocation game," pp. 888–896, 2008, DOI: 10.1109/infocom.2007.141.

Khadija Touya received her Ph.D. degree in 2022 from the Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Morocco. Her research interests include network economics, network security, applications of game theory in social networks, wireless networks, and radio resource management.

Hamid Garmani is a Professor of computer science in the Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Morocco. His research interests include network economics, network security, applications of game theory in wireless networks, and radio resource management.

Mohamed Baslam is a Professor of computer science in the Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Morocco. His current research interests include performance evaluation and optimization of networks based on game-theoretic and queuing models, applications in communication/transportation and social networks, such as wireless flexible networks, bio-inspired and selforganizing networks, and economic models of the Internet and yield management.

Rachid El Ayachi is a Professor of computer science in the Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Morocco. His research interests include image processing, machine learning, natural language processing, Network Engineering Games, social networks and neutrality issues.

Mostafa Jourhmane is a professor of mathematics and computer science in the Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Morocco. His research interests include Numerical analysis, classification, image processing, neural networks, big data, Network Engineering Games, social networks and their control and the analysis through game theoretical models of network and neutrality issues.