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AN OPTIMIZED MODEL FOR NEUTROSOPHIC MULTI-CHOICE GOAL PROGRAMMING

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Abstract. Goal Programming (GP), as a generalization of linear programming, is a proper technique for handling Multi-Objective Decision-Making (MODM) problems. Using this technique, we can determine multiple aspiration levels for each objective. The Neutrosophic Set (NS) theory is a powerful tool to deal with indeterminacy and inconsistent data. This paper provides a new idea to formulate Neutrosophic Multi-Choice Goal Programming (NMCGP) problems to simulate real-life situations more realistically. Also, to show the low computational complexity of the proposed model, the Space and Time complexity criteria are used in the Python language. Eventually, a practical mathematical example is solved to present the effectiveness of the proposed model.

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1. INTRODUCTION

Goal Programming (GP) is a helpful technique to obtain the optimal solution to Multi-Objective Decision-Making (MODM) problems with an aspiration level for each target. GP could also be considered an extension of Linear Programming (LP) to process multiple, usually conflicting, objective problems. It enables us to set our aspiration levels for each goal.

The GP problems can be incorporated into two different groups: (1) GP models for crisp Decision-Making (DM) problems (classical goal programming) and (2) GP models for decision-making problems under uncertainty. Most of the existing problems in GP are proposed based on classical goal programming. These kinds of problems were firstly offered by Charnce and Cooper [5] and then developed by Lee [11], Ignazio [7], Tamiz et al.[29], Romero [25], and others [3, 12, 20, 22–24]. The main purpose of GP is to minimize the deviations between the achievement of goals and their aspiration levels. It can be represented as follows:

$$\begin{aligned} &Min \;\; \textstyle\sum_{j=1}^{n} |f_i(X) - g_i|, \\ &s.t. \;\;\; AX = b, \;\; X \geq 0, \end{aligned} \tag{1.1}$$

where g_i is the aspiration level of the ith goal. Due to imprecision in determining aspiration levels of real-world problems, decision-makers are unable to consider a crisp amount of aspiration level for goal programming problems. In this situation, we require the approaches that belong to the second group of GP problems. This group includes the developed approach for goal programming problems under uncertainty. To overcome the current uncertainty, several methods based on fuzzy sets and their extensions have been proposed. Bankian et al. [1], Hocine et al. [6], Kouaissah and Hocine [10], Kamal et al. [8], Maity et al. [13], Keshteli and Nasseri [9], and many others proposed different techniques to deal with fuzzy goal programming problems. However, the fuzzy sets are still unable to handle indeterminate and inconsistent information that commonly exists in natural systems. From this point of view, Smarandache in [28] suggested a concept of Neutrosophic Set (NS) theory. As another deployment of FS theory, NS theory is a powerful tool to handle uneven, indistinctive, and defective data. The degree of indeterminacy in neutrosophic theory is considered as an independent factor between zero and one, which has a significant contribution in decision making. A NS N can be characterized by three membership functions: truth, indeterminacy, and falsity membership functions that are entirely independent of each other. Wang et al. [30] proposed the Single-Valued Neutrosophic Sets (SVNSs) as an instance of the NSs to handle practical problems from a scientific and engineering perspective. Recently, SVNS has become an important research topic and attracted much attention in decision-making problems. By applying a bidirectional measure on decision-making problems, Ye [32] proposed a new approach to handle MAGDM problems under a neutrosophic environment. Zhang et al. [33] presented two aggregation operators based on neutrosophic values, such as neutrosophic weighted arithmetic operator and neutrosophic weighted geometric operator, and applied them to multi-criteria decision-making problems. Moreover, Ye in [31] proposed a correlation coefficient of SVNSs and the cross-entropy measure of SVNSs and used them for decision-making under the Neutrosophic environment. By introducing the cosine measures for evaluating linguistic neutrosophic numbers, Shi and Ye [27] proposed a novel strategy to solve neutrosophic DM problems. Ranking of the NS is an essential part of the MAGDM process. By defining a score function for ranking the SVNNs, Sahin [26] proposed a strategy for Multi-Attribute Group Decision-Making (MAGDM) under the neutrosophic environment. By highlighting the shortcomings of the existing score functions, Nancy and Garg [19] introduced a modified score function for ranking the SVNNs and then applied it to neutrosophic decision-making problems. Using a bidirectional projection measure, Pramanik et al. [21] proposed a novel neutrosophic approach to handle teacher selection problems. Nafei et al. [17] proposed a novel approach for decision-making with group recommendations based on NSs. Meanwhile, Nafei et al. [14], by developing a new Hamming distance between SVNNs, proposed an extension of the TOPSIS method for MAGDM based on SVNSs where the information about attributes are expressed by decision-makers based on neutrosophic numbers. This research presents a new method for solving Neutrosophic Multi-Choice Goal Programming (NMCGP) problems. The advantage of using NMCGP is its ability to simulate all aspects of real-life situations in our models. The remainder of this research is marshaled as follows. Section 2 presents some practical definitions of Neutrosophic Sets and other essential concepts. In Section 3, the Neutrosophic Multi-Choice Goal Programming formulation is described. In Section 4, we first explain our method using a numerical example and then present the calculation processing of the model. The conclusions are discussed in Section 5.

2. PRELIMINARIES

This section briefly reviews some necessary backgrounds and preliminaries of neutrosophic sets, single-valued neutrosophic sets, and other essential details.

Definition 1 ([18])**.** A neutrosophic set U in domain X $(x \in X)$ is characterized by truth $T_U(x) : X \to]0^-, 1^+[$, indeterminacy $I_U(x) : X \to]0^-, 1^+[$ and falsity $F_U(x) : X \to]0^-, 1^+[$ membership functions. There is no restriction on the sum of $T_U(x), I_U(x)$, and $F_U(x)$. Therefore, $\bar{0} \leq T_N(x) + I_N(x) + F_N(x) \leq 3^+$.

Definition 2 ([2, 15])**.** Assume that X is a domain. A Single-Valued Neutrosophic Set (SVNS) U through X taking the form $U = \{x, T_U(x), I_U(x), F_U(x); x \in X\}$, where $T_U(x) : X \to [0,1], I_U(x) : X \to [0,1]$ and $F_U(x) : X \to [0,1]$ with $0 \leq T_U(x) + I_U(x) + F_U(x) \leq 3$ for all $x \in X$. Also, $T_U(x), I_U(x)$ and $F_U(x)$ represent the truth membership degree, the indeterminacy membership degree and the falsity membership degree, respectively, of x to U.

Definition 3 ([16])**.** Assume that U and V are two NSs. For all $x \in X$, U is included in V, if and only if:

$$\begin{aligned} &Inf\ T_U(x) \leq Inf\ T_V(x), \\ &Sup\ T_U(x) \leq Sup\ T_V(x), \\ &Inf\ I_U(x) \geq Inf\ I_V(x), \\ &Sup\ I_U(x) \geq Sup\ I_V(x), \\ &Inf\ F_U(x) \geq Inf\ F_V(x), \\ &Sup\ F_U(x) \geq Sup\ F_V(x). \end{aligned} \tag{2.1}$$

Definition 4 ([15])**.** Let U be a neutrosophic set. The complement of U is interpreted by U^c and it could be defined as

$$T_U^c(x) = \{1^+\} - T_U(x), I_U^c(x) = \{1^+\} - I_U(x), F_U^c(x) = \{1^+\} - F_U(x),\ \forall x \in X.$$

Therefore, $U^c = \{x, F_U(x), 1 - I_U(x), T_U(x); x \in X\}$.

3. NEUTROSOPHIC MULTI-CHOICE GOAL PROGRAMMING

Decision-Making is a substantial and essential part of daily life that can be applied in various areas, such as society, economics, management, military, and engineering technology. In most cases, it is intricate for decision-makers to accurately reveal a preference when solving Multi Attribute Decision Making (MADM) problems with imprecise, vague, and incomplete information.

Multi-Objective Decision-Making as a subset of decision science is an essential part of daily life that can be applied to various problems. The Goal Programming approach is a powerful technique to address these kinds of problems. Sometimes, decision-makers prefer to have more than one aspiration level for each goal because of some specific situations. To handle this problem, Chang in [4] proposed a Multi-Choice Goal Programming model that is characterized based on crisp decision-making problems. In some conditions, the objectives can have an ambiguous level. In this case, we present the Neutrosophic Multi-Choice Goal Grogramming (NMCGP) model approach that is a powerful tool to solve this kind of unclear difficulties. A NMCGP problem could be formulated as follows:

$$\begin{aligned} &Min\ \ \textstyle\sum_{i=1}^{n} \left| f_i(X) - g_{i1} \text{ or } g_{i2} \ \text{ or } ...g_{im} \right|, \\ &s.t. \quad AX = b,\ X \geq 0, \end{aligned} \tag{3.1}$$

where $f_i(X)$ is a linear function of x_i $(1 \leq i \leq n)$ for the ith goal and g_{ij} $(1 \leq i \leq n$ and $1 \leq j \leq m)$ is the jth aspiration level of the ith goal.

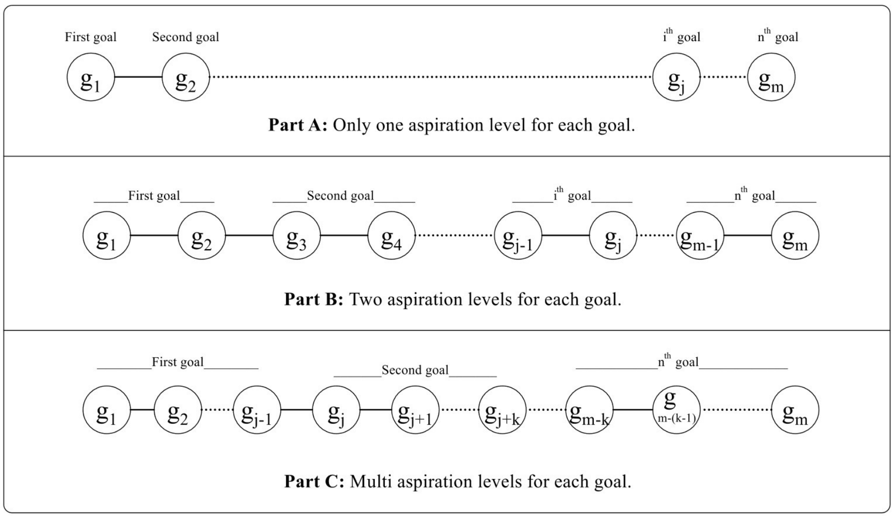

FIGURE 1. Neutrosophic aspirational levels for Multi-Objective Goal Programming problems

Three different classes for NMCGP problems can be considered as shown in (Fig. 1):

(I) Problems with one neutrosophic aspiration level for each goal (Part A),
(II) Problems with two neutrosophic aspiration levels for each goal (Part B),
(III) Problems with more than two (multiple) neutrosophic aspiration levels for each goal (Part C).

In order to model these problems, we present the truth, the indeterminacy, and the falsity membership functions for the ith neutrosophic goal as follows (Fig. 2):

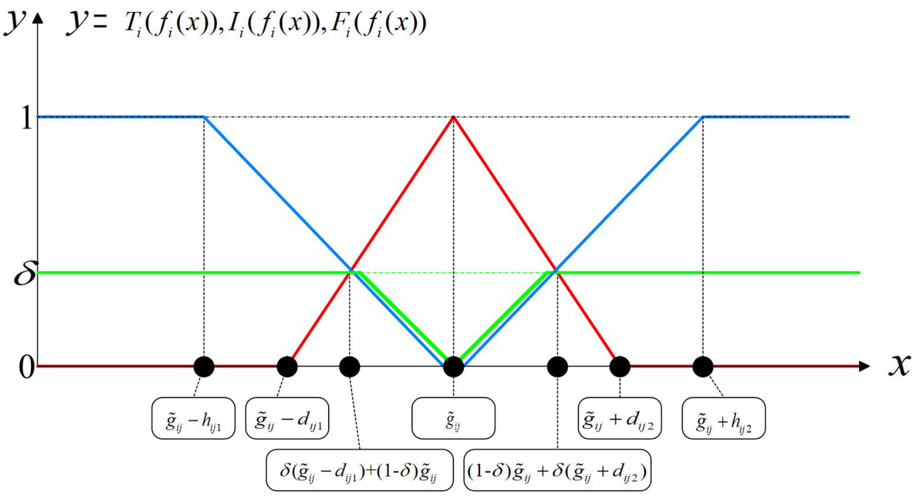

FIGURE 2. Truth, indeterminacy, and falsity membership functions for NMCGP

$$T_i(f_i(x)) = \begin{cases} \sum_{j=1}^{m} \frac{f_i(x)-\tilde{g}_{ij}}{d_{ij1}} S_{ij}(B), & \tilde{g}_{ij} - d_{ij1} \le f_i(x) \le \tilde{g}_{ij}, \\ 1, & f_i(x) = \tilde{g}_{ij}, \\ \sum_{j=1}^{m} \frac{\tilde{g}_{ij}-f_i(x)}{d_{ij2}} S_{ij}(B), & \tilde{g}_{ij} \le f_i(x) \le \tilde{g}_{ij} + d_{ij2}, \\ 0, & O.W. \end{cases} \tag{3.2}$$

$$I_i(f_i(x)) = \begin{cases} \sum_{j=1}^{m} \frac{\tilde{g}_{ij}-f_i(x)}{d_{ij2}} S_{ij}(B), & \delta(\tilde{g}_{ij} - d_{ij1}) + (1-\delta)\tilde{g}_{ij} \le f_i(x) \le \tilde{g}_{ij}, \\ \delta, & f_i(x) = \tilde{g}_{ij}, \\ \sum_{j=1}^{m} \frac{f_i(x)-\tilde{g}_{ij}}{d_{ij2}} S_{ij}(B), & \tilde{g}_{ij} \le f_i(x) \le (1-\delta)\tilde{g}_{ij} + \delta(\tilde{g}_{ij} + d_{ij2}), \\ \delta, & O.W. \end{cases} \tag{3.3}$$

$$F_i(f_i(x)) = \begin{cases} \sum_{j=1}^{m} \frac{\tilde{g}_{ij}-f_i(x)}{h_{ij1}} S_{ij}(B), & \tilde{g}_{ij} - h_{ij1} \le f_i(x) \le \tilde{g}_{ij}, \\ 0, & f_i(x) = \tilde{g}_{ij}, \\ \sum_{j=1}^{m} \frac{f_i(x)-\tilde{g}_{ij}}{h_{ij2}} S_{ij}(B) & \tilde{g}_{ij} \le f_i(x) \le \tilde{g}_{ij} + h_{ij2}, \\ 0, & O.W. \end{cases} \tag{3.4}$$

where d_{ij1} and d_{ij2} are the negative and positive tolerances for the truth membership function in the jth aspiration level of the ith goal, respectively. While h_{ij1} and h_{ij2} are the negative and positive tolerances for the falsity membership function in the jth aspiration level of the ith goal, respectively. $S_{ij}(B)$ is a function of binary serial numbers (i.e., a concept of height-balanced binary tree) attached to different aspiration levels. As shown in Fig. 3, a binary tree with n binary variables can represent 2^n aspiration levels.

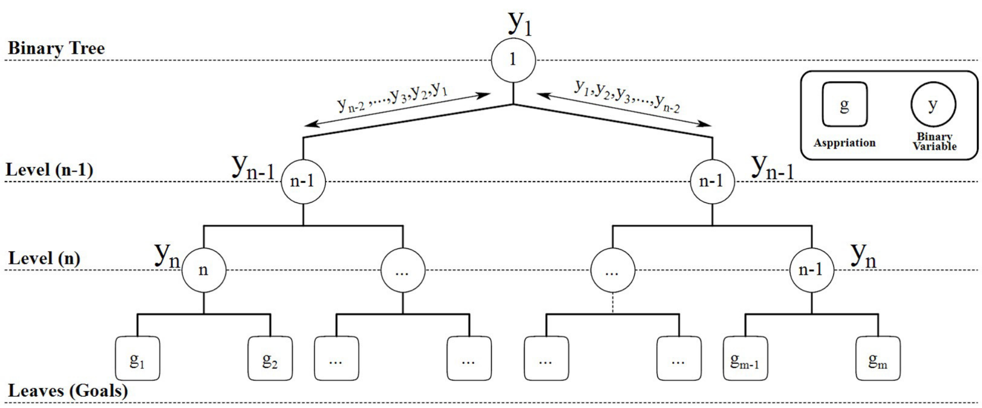

FIGURE 3. The concept of height-balanced binary tree

Nevertheless, the Neutrosophic Multi-Choice Poal Programming problem can be modeled as follows:

$$\begin{aligned} & Max\, T_i(f_i(x)),\ Min\, I_i(f_i(x)),\ Min\, F_i(f_i(x)), \\ & s.t. \\ & \quad T_i(f_i(x)) \ge F_i(f_i(x)), \\ & \quad T_i(f_i(x)) \ge I_i(f_i(x)), \\ & \quad 0 \le T_i(f_i(x)) + F_i(f_i(x)) + I_i(f_i(x)) \le 3, \\ & \quad T_i(f_i(x)), F_i(f_i(x)), I_i(f_i(x)) \ge 0, \\ & \quad AX \ge b, \\ & \quad X \ge 0, x \in X, X \in F,\ \ i = 1, \ldots, n. \end{aligned}$$

The previous NMCGP model is reformulated as follows:

$$\begin{aligned}
&Max\, T_i(f_i(x)),\ Min\, I_i(f_i(x)),\ Min\, F_i(f_i(x)),\\
&\ s.t.\\
&\qquad T_i(f_i(x)) \geq F_i(f_i(x)),\\
&\qquad T_i(f_i(x)) \geq I_i(f_i(x)),\\
&\qquad 0 \leq T_i(f_i(x)) + F_i(f_i(x)) + I_i(f_i(x)) \leq 3,\\
&\qquad T_i(f_i(x)), F_i(f_i(x)), I_i(f_i(x)) \geq 0,\\
&\qquad AX \geq b,\\
&\qquad X \geq 0, x \in X, X \in F,\ \ i = 1, \ldots, n.
\end{aligned}$$

Because $Max\ \alpha_i$ is equivalent to $Min\ (1-\alpha_i)$ where $0 \leq \alpha_i \leq 1$, the above model can be described as below:

$$\begin{aligned}
&Max\, T_i(f_i(x)),\ Min\, I_i(f_i(x)),\ Min\, F_i(f_i(x)),\\
&\ s.t.\\
&\qquad T_i(f_i(x)) \geq F_i(f_i(x)),\\
&\qquad T_i(f_i(x)) \geq I_i(f_i(x)),\\
&\qquad 0 \leq T_i(f_i(x)) + F_i(f_i(x)) + I_i(f_i(x)) \leq 3,\\
&\qquad T_i(f_i(x)), F_i(f_i(x)), I_i(f_i(x)) \geq 0,\\
&\qquad AX \geq b,\\
&\qquad X \geq 0, x \in X, X \in F,\ \ i = 1, \ldots, n,
\end{aligned}$$

where $\theta_i = 1-\alpha_i$. The summary of the proposed model is shown in Fig. 4. To present a clear description of the proposed approach for solving the NMCGP problems, assume that we have three different goals with.

1. Only one neutrosophic aspiration level for each goal. This case can be modeled as:

$$\begin{aligned}
&Min\, Z = \sum_{i=1}^{3} (\theta_i + \beta_i + \gamma_i),\\
&s.t.\\
&\qquad \theta_i \geq 1 - \frac{f_1(x) - \tilde{g}_1}{d_1^-},\ \theta_i \geq 1 - \frac{\tilde{g}_1 - f_1(x)}{d_1^+},\ \beta_1 \geq \frac{\tilde{g}_1 - f_1(x)}{d_1^-},\\
&\qquad \beta_1 \geq \frac{f_1(x) - \tilde{g}_1}{d_1^+},\ \gamma_1 \geq \frac{\tilde{g}_1 - f_1(x)}{h_1^-},\ \gamma_1 \geq \frac{f_1(x) - \tilde{g}_1}{h_1^+},\\
&\qquad \alpha_2 \leq \frac{f_2(x) - \tilde{g}_2}{d_2^-},\ \alpha_2 \leq \frac{\tilde{g}_2 - f_2(x)}{d_2^+},\ \beta_2 \geq \frac{\tilde{g}_2 - f_2(x)}{d_2^-},
\end{aligned}$$

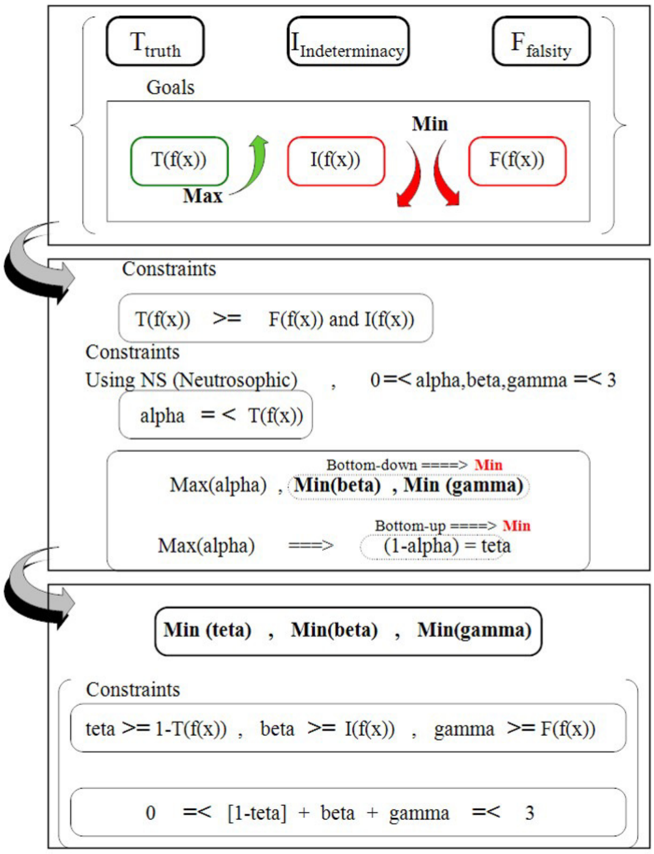

FIGURE 4. The framework of the proposed model

$$
\begin{aligned}
&\beta_2 \geq \frac{f_2(x)-\tilde{g}_2}{d_2^+},\ \gamma_2 \geq \frac{\tilde{g}_2-f_2(x)}{h_2^-},\ \gamma_2 \geq \frac{f_2(x)-\tilde{g}_2}{h_2^+},\\
&\alpha_3 \leq \frac{f_3(x)-\tilde{g}_3}{d_3^-},\ \alpha_3 \leq \frac{\tilde{g}_3-f_3(x)}{d_3^+},\ \beta_3 \geq \frac{\tilde{g}_3-f_3(x)}{d_3^-},\\
&\beta_3 \geq \frac{f_3(x)-\tilde{g}_3}{d_3^+},\ \gamma_3 \geq \frac{\tilde{g}_3-f_3(x)}{h_3^-},\ \gamma_3 \geq \frac{f_3(x)-\tilde{g}_3}{h_3^+},\\
&AX \geq b,\ 0 \leq (1-\theta_1)+\beta_1+\gamma_1 \leq 3,\ 0 \leq (1-\theta_2)+\beta_2+\gamma_2 \leq 3,\\
&0 \leq (1-\theta_3)+\beta_3+\gamma_3 \leq 3,\ 1-\beta_1 \geq \theta_1,\ 1-\gamma_1 \geq \theta_1,\\
&1-\beta_2 \geq \theta_2,\ 1-\gamma_2 \geq \theta_2,\ 1-\beta_3 \geq \theta_3,\\
&1-\gamma_3 \geq \theta_3,\ 0 \leq \theta_1,\beta_1,\gamma_1 \leq 1,\ 0 \leq \theta_2,\beta_2,\gamma_2 \leq 1,\ 0 \leq \theta_3,\beta_3,\gamma_3 \leq 1,\\
&X \geq 0,\ x \in X,\ X \in F\ (F \text{ is a feasible set}).
\end{aligned}
$$

2. Two neutrosophic aspiration levels for each goal. It belongs to a case of neutrosophic goal programming problems with an either-or selection. Assume that the target in the first goal is to select an appropriate Neutrosophic aspiration level from either $\tilde{g}_1$ or $\tilde{g}_5$. Also, the target in the second goal is to select an appropriate neutrosophic aspiration level from either $\tilde{g}_2$ or $\tilde{g}_4$. Finally, the third goal's target is to select an appropriate neutrosophic aspiration level from either $\tilde{g}_3$ or $\tilde{g}_6$. This case can be modeled as follows:

$$
\begin{aligned}
& Min\, Z = (\theta_1+\beta_1+\gamma_1)+(\theta_2+\beta_2+\gamma_2)+(\theta_3+\beta_3+\gamma_3),\\
& s.t.\\
&\qquad \theta_1 \geq 1-\left(\frac{f_1(x)-\tilde{g}_1}{d_{11}^-}y_1+\frac{f_1(x)-\tilde{g}_5}{d_{15}^-}(1-y_1)\right),\\
&\qquad \theta_1 \geq 1-\left(\frac{\tilde{g}_1-f_1(x)}{d_{11}^+}y_1+\frac{\tilde{g}_5-f_1(x)}{d_{15}^+}(1-y_1)\right),\\
&\qquad \beta_1 \geq \frac{\tilde{g}_1-f_1(x)}{d_{11}^-}y_1+\frac{\tilde{g}_5-f_1(x)}{d_{15}^-}(1-y_1),\\
&\qquad \beta_1 \geq \frac{f_1(x)-\tilde{g}_1}{d_{11}^+}y_1+\frac{f_1(x)-\tilde{g}_5}{d_{15}^+}(1-y_1),\\
&\qquad \gamma_1 \geq \frac{\tilde{g}_1-f_1(x)}{h_{11}^-}y_1+\frac{\tilde{g}_5-f_1(x)}{h_{15}^-}(1-y_1),\\
&\qquad \gamma_1 \geq \frac{f_1(x)-\tilde{g}_1}{h_{11}^+}y_1+\frac{f_1(x)-\tilde{g}_5}{h_{15}^+}(1-y_1),\\
&\qquad \theta_2 \geq 1-\left(\frac{f_2(x)-\tilde{g}_2}{d_{22}^-}y_2+\frac{f_2(x)-\tilde{g}_4}{d_{24}^-}(1-y_2)\right),\\
&\qquad \theta_2 \geq 1-\left(\frac{\tilde{g}_2-f_2(x)}{d_{22}^+}y_2+\frac{\tilde{g}_4-f_2(x)}{d_{24}^+}(1-y_2)\right),\\
&\qquad \beta_2 \geq \frac{\tilde{g}_2-f_2(x)}{d_{22}^-}y_2+\frac{\tilde{g}_4-f_2(x)}{d_{24}^-}(1-y_2),\\
&\qquad \beta_2 \geq \frac{f_2(x)-\tilde{g}_2}{d_{22}^+}y_2+\frac{f_2(x)-\tilde{g}_4}{d_{24}^+}(1-y_2),\\
&\qquad \gamma_2 \geq \frac{\tilde{g}_2-f_2(x)}{h_{22}^-}y_2+\frac{\tilde{g}_4-f_2(x)}{h_{24}^-}(1-y_2),\\
&\qquad \gamma_2 \geq \frac{f_2(x)-\tilde{g}_2}{h_{22}^+}y_2+\frac{f_2(x)-\tilde{g}_4}{h_{24}^+}(1-y_2),\\
&\qquad \theta_3 \geq 1-\left(\frac{f_3(x)-\tilde{g}_3}{d_{33}^-}y_3+\frac{f_3(x)-\tilde{g}_6}{d_{36}^-}(1-y_3)\right),
\end{aligned}
$$

$$
\begin{aligned}
&\theta_3 \geq 1-\left(\frac{\tilde{g}_3 - f_3(x)}{d_{33}^{+}}y_3 + \frac{\tilde{g}_6 - f_3(x)}{d_{36}^{+}}(1-y_3)\right),\\
&\beta_3 \geq \frac{\tilde{g}_3 - f_3(x)}{d_{33}^{-}}y_3 + \frac{\tilde{g}_6 - f_3(x)}{d_{36}^{-}}(1-y_3),\\
&\beta_3 \geq \frac{f_3(x) - \tilde{g}_3}{d_{33}^{+}}y_3 + \frac{f_3(x) - \tilde{g}_6}{d_{36}^{+}}(1-y_3),\\
&\gamma_3 \geq \frac{\tilde{g}_3 - f_3(x)}{h_{33}^{-}}y_3 + \frac{\tilde{g}_6 - f_3(x)}{h_{36}^{-}}(1-y_3),\\
&\gamma_3 \geq \frac{f_3(x) - \tilde{g}_3}{h_{33}^{+}}y_3 + \frac{f_3(x) - \tilde{g}_6}{h_{36}^{+}}(1-y_3),\\
&AX \geq b,\\
&0 \leq (1-\theta_1)+\beta_1+\gamma_1 \leq 3,\\
&0 \leq (1-\theta_2)+\beta_2+\gamma_2 \leq 3,\\
&0 \leq (1-\theta_3)+\beta_3+\gamma_3 \leq 3,\\
&1-\beta_1 \geq \theta_1,\ 1-\gamma_1 \geq \theta_1,\ 1-\beta_2 \geq \theta_2,\\
&1-\gamma_2 \geq \theta_2,\ 1-\beta_3 \geq \theta_3,\ 1-\gamma_3 \geq \theta_3,\\
&0 \leq \theta_1,\beta_1,\gamma_1 \leq 1,\ 0 \leq \theta_2,\beta_2,\gamma_2 \leq 1,\\
&0 \leq \theta_3,\beta_3,\gamma_3 \leq 1,\ X \geq 0,\ x \in X,\\
&X \in F\ (F \text{ is a feasible set}),
\end{aligned}
$$

where y_1, y_2 and y_3 represent the binary variables.

3. Multi neutrosophic aspiration levels for each goal. It belongs to the case of neutrosophic goal programming problems with multi-choice selection. Here we arbitrarily considered three neutrosophic aspiration levels for each goal. Assume that the target in the first goal is to select an appropriate neutrosophic aspiration level between $\tilde{g}_1$, $\tilde{g}_5$ and $\tilde{g}_8$. Also, the target in the second goal is to select an appropriate neutrosophic aspiration level between $\tilde{g}_2$, $\tilde{g}_4$ and $\tilde{g}_7$. Finally, the target in the third goal is to select an appropriate neutrosophic aspiration level between $\tilde{g}_3$, $\tilde{g}_6$ and $\tilde{g}_9$. This case can be modeled as follows:

$$
\begin{aligned}
&Min\ Z = (\theta_1+\beta_1+\gamma_1)+(\theta_2+\beta_2+\gamma_2)+(\theta_3+\beta_3+\gamma_3),\\
&s.t.\\
&\quad \theta_1 \geq 1-\left(\frac{f_1(x)-\tilde{g}_1}{d_{11}^{-}}y_1y_2 + \frac{f_1(x)-\tilde{g}_5}{d_{15}^{-}}y_1(1-y_2) + \frac{f_1(x)-\tilde{g}_8}{d_{18}^{-}}y_2(1-y_1)\right),\\
&\quad \theta_1 \geq 1-\left(\frac{\tilde{g}_1-f_1(x)}{d_{11}^{+}}y_1y_2 + \frac{\tilde{g}_5-f_1(x)}{d_{15}^{+}}y_1(1-y_2) + \frac{\tilde{g}_8-f_1(x)}{d_{18}^{+}}y_2(1-y_1)\right),
\end{aligned}
$$

$$\beta_1 \geq \frac{\tilde{g}_1 - f_1(x)}{d_{11}^-} y_1 y_2 + \frac{\tilde{g}_5 - f_1(x)}{d_{15}^-} y_1(1-y_2) + \frac{\tilde{g}_8 - f_1(x)}{d_{18}^-} y_2(1-y_1),$$

$$\beta_1 \geq \frac{f_1(x) - \tilde{g}_1}{d_{11}^+} y_1 y_2 + \frac{f_1(x) - \tilde{g}_5}{d_{15}^+} y_1(1-y_2) + \frac{f_1(x) - \tilde{g}_8}{d_{18}^+} y_2(1-y_1),$$

$$\gamma_1 \geq \frac{\tilde{g}_1 - f_1(x)}{h_{11}^-} y_1 y_2 + \frac{\tilde{g}_5 - f_1(x)}{h_{15}^-} y_1(1-y_2) + \frac{\tilde{g}_8 - f_1(x)}{h_{18}^-} y_2(1-y_1),$$

$$\gamma_1 \geq \frac{f_1(x) - \tilde{g}_1}{h_{11}^+} y_1 y_2 + \frac{f_1(x) - \tilde{g}_5}{h_{15}^+} y_1(1-y_2) + \frac{f_1(x) - \tilde{g}_8}{h_{18}^+} y_2(1-y_1),$$

$$\theta_2 \geq 1 - \left(\frac{f_2(x) - \tilde{g}_2}{d_{22}^-} y_3 y_4 + \frac{f_2(x) - \tilde{g}_4}{d_{24}^-} y_3(1-y_4) + \frac{f_2(x) - \tilde{g}_7}{d_{27}^-} y_4(1-y_3) \right),$$

$$\theta_2 \geq 1 - \left(\frac{\tilde{g}_2 - f_2(x)}{d_{22}^+} y_3 y_4 + \frac{\tilde{g}_4 - f_2(x)}{d_{24}^+} y_3(1-y_4) + \frac{\tilde{g}_7 - f_2(x)}{d_{27}^+} y_4(1-y_3) \right),$$

$$\beta_2 \geq \frac{\tilde{g}_2 - f_2(x)}{d_{22}^-} y_3 y_4 + \frac{\tilde{g}_4 - f_2(x)}{d_{24}^-} y_3(1-y_4) + \frac{\tilde{g}_7 - f_2(x)}{d_{27}^-} y_4(1-y_3),$$

$$\beta_2 \geq \frac{f_2(x) - \tilde{g}_2}{d_{22}^+} y_3 y_4 + \frac{f_2(x) - \tilde{g}_4}{d_{24}^+} y_3(1-y_4) + \frac{f_2(x) - \tilde{g}_7}{d_{27}^+} y_4(1-y_3),$$

$$\gamma_2 \geq \frac{\tilde{g}_2 - f_2(x)}{h_{22}^-} y_3 y_4 + \frac{\tilde{g}_4 - f_2(x)}{h_{24}^-} y_3(1-y_4) + \frac{\tilde{g}_7 - f_2(x)}{h_{27}^-} y_4(1-y_3),$$

$$\gamma_2 \geq \frac{f_2(x) - \tilde{g}_2}{h_{22}^+} y_3 y_4 + \frac{f_2(x) - \tilde{g}_4}{h_{24}^+} y_3(1-y_4) + \frac{f_2(x) - \tilde{g}_7}{h_{27}^+} y_4(1-y_3),$$

$$\theta_3 \geq 1 - \left(\frac{f_3(x) - \tilde{g}_3}{d_{33}^-} y_5 y_6 + \frac{f_3(x) - \tilde{g}_6}{d_{36}^-} y_5(1-y_6) + \frac{f_3(x) - \tilde{g}_9}{d_{39}^-} y_6(1-y_5) \right),$$

$$\theta_3 \geq 1 - \left(\frac{\tilde{g}_3 - f_3(x)}{d_{33}^+} y_5 y_6 + \frac{\tilde{g}_6 - f_3(x)}{d_{36}^+} y_5(1-y_6) + \frac{\tilde{g}_9 - f_3(x)}{d_{39}^+} y_6(1-y_5) \right),$$

$$\beta_3 \geq \frac{\tilde{g}_3 - f_3(x)}{d_{33}^-} y_5 y_6 + \frac{\tilde{g}_6 - f_3(x)}{d_{36}^-} y_5(1-y_6) + \frac{\tilde{g}_9 - f_3(x)}{d_{39}^-} y_6(1-y_5),$$

$$\beta_3 \geq \frac{f_3(x) - \tilde{g}_3}{d_{33}^+} y_5 y_6 + \frac{f_3(x) - \tilde{g}_6}{d_{36}^+} y_5(1-y_6) + \frac{f_3(x) - \tilde{g}_9}{d_{39}^+} y_6(1-y_5),$$

$$\gamma_3 \geq \frac{\tilde{g}_3 - f_3(x)}{h_{33}^-} y_5 y_6 + \frac{\tilde{g}_6 - f_3(x)}{h_{36}^-} y_5(1-y_6) + \frac{\tilde{g}_9 - f_3(x)}{h_{39}^-} y_6(1-y_5),$$

$$\gamma_3 \geq \frac{f_3(x) - \tilde{g}_3}{h_{33}^+} y_5 y_6 + \frac{f_3(x) - \tilde{g}_6}{h_{36}^+} y_5(1-y_6) + \frac{f_3(x) - \tilde{g}_9}{h_{39}^+} y_6(1-y_5),$$

$$AX \geq b,$$

$$y_1 + y_2 \geq 1,\ y_3 + y_4 \geq 1,\ y_5 + y_6 \geq 1,$$

$$0 \le (1-\theta_1)+\beta_1+\gamma_1 \le 3,\ 0 \le (1-\theta_2)+\beta_2+\gamma_2 \le 3,$$
$$0 \le (1-\theta_3)+\beta_3+\gamma_3 \le 3,$$
$$1-\beta_1 \ge \theta_1, 1-\gamma_1 \ge \theta_1,\ 1-\beta_2 \ge \theta_2,$$
$$1-\gamma_2 \ge \theta_2,\ 1-\beta_3 \ge \theta_3,\ 1-\gamma_3 \ge \theta_3,$$
$$0 \le \theta_1,\beta_1,\gamma_1 \le 1,\ 0 \le \theta_2,\beta_2,\gamma_2 \le 1,\ 0 \le \theta_3,\beta_3,\gamma_3 \le 1,$$
$$X \ge 0,\ x \in X,\ X \in F\ (F \text{ is a feasible set}).$$

4. Numerical illustration

A factory produces three different products such as P_1, P_2 and P_3. To produce each product we need three resources as Z_1, Z_2, and Z_3. The value of these resources is shown in Table 1. Assume that there exist six different markets willing to buy some products. The information about products and demand for them is presented in Table 2. We expect that the profit earned by selling the products should be at least 2000 dollars. The factory's policy is to select only one applicant to sell each product.

Remark 1. In this example, the negative and positive tolerances for acceptance and rejection of each goal are considered equal values.

TABLE 1. The values of required resources to produce a product

Products	**Markets**	**Demands**	***d***	***h***	**Profit**
P1	Market 1	80	7	5	20
	Market 2	90	5	2	
	Market 3	50	4	1	
P2	Market 4	28	4	9	70
	Market 5	26	6	7	
P3	Market 6	10	5	4	85

TABLE 2. Information about products

Resources	**Products**		
	P1	**P2**	**P3**
Z1	4	5	9
Z2	7	6	2
Z3	3	8	7
Capacity	500	600	400

This NMCGP problem can be formulated as follows:

$$Min\ Z = (\theta_1+\beta_1+\gamma_1)+(\theta_2+\beta_2+\gamma_2)+(\theta_3+\beta_3+\gamma_3)$$

$s.t.$

$$
\begin{aligned}
&\theta_1 \geq 1-\left(\frac{P_1-80}{7}y_1y_2+\frac{P_1-90}{5}y_1(1-y_2)+\frac{P_1-50}{4}y_2(1-y_1)\right),\\
&\theta_1 \geq 1-\left(\frac{80-P_1}{7}y_1y_2+\frac{90-P_1}{5}y_1(1-y_2)+\frac{50-P_1}{4}y_2(1-y_1)\right),\\
&\beta_1 \geq \frac{80-P_1}{7}y_1y_2+\frac{90-P_1}{5}y_1(1-y_2)+\frac{50-P_1}{4}y_2(1-y_1),\\
&\beta_1 \geq \frac{P_1-80}{7}y_1y_2+\frac{P_1-90}{5}y_1(1-y_2)+\frac{P_1-50}{4}y_2(1-y_1),\\
&\gamma_1 \geq \frac{80-P_1}{5}y_1y_2+\frac{90-P_1}{2}y_1(1-y_2)+\frac{50-P_1}{1}y_2(1-y_1),\\
&\gamma_1 \geq \frac{P_1-80}{5}y_1y_2+\frac{P_1-90}{2}y_1(1-y_2)+\frac{P_1-50}{1}y_2(1-y_1),\\
&\theta_2 \geq 1-\left(\frac{P_2-28}{4}y_3+\frac{P_2-26}{6}(1-y_3)\right),\\
&\theta_2 \geq 1-\left(\frac{28-P_2}{4}y_3+\frac{26-P_2}{6}(1-y_3)\right),\\
&\beta_2 \geq \frac{28-P_2}{4}y_3+\frac{26-P_2}{6}(1-y_3),\\
&\beta_2 \geq \frac{P_2-28}{4}y_3+\frac{P_2-26}{6}(1-y_3),\\
&\gamma_2 \geq \frac{28-P_2}{9}y_3+\frac{26-P_2}{7}(1-y_3),\\
&\gamma_2 \geq \frac{P_2-28}{9}y_3+\frac{P_2-26}{7}(1-y_3),\\
&\theta_3 \geq 1-\left(\frac{P_3-10}{5}\right),\ \theta_3 \geq 1-\left(\frac{10-P_3}{5}\right),\ \beta_3 \geq \frac{10-P_3}{5},\\
&\beta_3 \geq \frac{P_3-10}{5},\ \gamma_3 \geq \frac{10-P_3}{4},\ \gamma_3 \geq \frac{P_3-10}{4},\\
&20P_1+70P_2+85P_3 \geq 2000,\\
&x_{11}+x_{12}+x_{13} \leq 500,\ x_{21}+x_{22}+x_{23} \leq 600,\ x_{31}+x_{32}+x_{33} \leq 400,\\
&P_1 \leq \frac{x_{11}}{4}, P_1 \leq \frac{x_{12}}{7}, P_1 \leq \frac{x_{13}}{3},\ P_2 \leq \frac{x_{21}}{5}, P_2 \leq \frac{x_{22}}{6}, P_2 \leq \frac{x_{23}}{8},\\
&P_3 \leq \frac{x_{31}}{9}, P_3 \leq \frac{x_{32}}{2}, P_3 \leq \frac{x_{33}}{7},\\
&y_1+y_2 \geq 1,\ 0 \leq (1-\theta_1)+\beta_1+\gamma_1 \leq 3,\\
&0 \leq (1-\theta_2)+\beta_2+\gamma_2 \leq 3,\ 0 \leq (1-\theta_3)+\beta_3+\gamma_3 \leq 3,\\
&1-\beta_1 \geq \theta_1,\ 1-\gamma_1 \geq \theta_1,\ 1-\beta_2 \geq \theta_2,
\end{aligned}
$$

$$1-\gamma_2 \geq \theta_2,\ 1-\beta_3 \geq \theta_3,\ 1-\gamma_3 \geq \theta_3,$$
$$0 \leq \theta_1, \beta_1, \gamma_1 \leq 1,\ 0 \leq \theta_2, \beta_2, \gamma_2 \leq 1,\ 0 \leq \theta_3, \beta_3, \gamma_3 \leq 1,$$
$$X \geq 0,\ x \in X.$$

The optimal solutions of the given problem are $P_1 = 0.20, P_2 = 5.13, P_3 = 22.22$, $y_1 = 2.87, y_2 = 1.23$, and $y_3 = 0.36$. In this regard, the obtained degrees for truth, indeterminacy, and falsity of first, second, and third neutrosophic goals are respectively obtained as $(\theta_1, \beta_1, \gamma_1) = (0.7, 0.4, 0.5), (\theta_2, \beta_2, \gamma_2) = (0.3, 0.5, 0.5)$, $(\theta_3, \beta_3, \gamma_3) = (0.1, 0.8, 0.8)$. Subsequently, $Z = 4.6$.

4.1. *Calculation Complexity Analysis*

The Space Complexity of our proposed model indicates how much memory space is required. This is intended to solve computational problems by considering input functions and typescripts and the amount of space that needs to be run is in Gigabytes. It is usually used for complex problems with many goals.

The proposed method is implemented on MathPy 2.7.3 on Python 3.03, and Ubuntu 20.0 OS on a virtual machine in VMbox (VM). The simulation machine has 16 G RAM, Intel i4 processor running at 3.2 GHz. All tests are based on three blocks with a block size of the 128 MB distributed file system. To investigate the Space Complexity (SC) of the proposed method, we have used Calculation Processing (CP) in each part of the method. To run the simulation. In this case, the evaluation based on SC to organize different products has been provided. We have considered different cases for P_1, P_2, P_3 and their combinations to evaluate the SC based on memory usage. The meaning of simulation steps is finishing each part of CP. As shown in Part A, Fig. 5, the details of memory usage are illustrated. The SCP (Space of CP) for various P_i $(1 \leq i \leq 3)$ is 3 GHz memory and computational execution time is 12 minutes. In this case, we considered 10 steps of calculations for simulation. With increasing the steps of simulation from 10 to 20, we can observe the increases in SCP and time complexity (Part B, Fig. 5). Subsequently, by increasing the steps of simulation from 20 to 25steps, we can observe the increases in SC to 10 G and increases in TC to 32 min (Part C, Fig. 5). In summary, this computational process shows that the proposed method can be used on ordinary machines by finding many targets and indicates a low computational Memory and CPU space.

5. Conclusion

This paper proposed a new model and a new method for solving Neutrosophic multi-choice goal programming problems to increase accuracy and reduce doubts and mistakes in the decision-making process. NMCGP, as a generalization of classical MCGP and Fuzzy MCGP, is a helpful technique to mark indeterminacy as an independent component in the Decision-Making process. Using NMCGP, we can consider all aspects of real-world problems in determining our goals' aspiration levels.

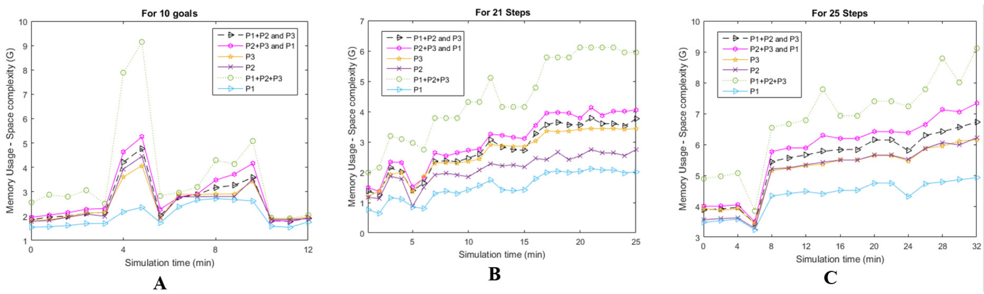

FIGURE 5. Different steps of calculations for simulation

The proposed approach can help us solve problems in all fields, including transportation, management, production, marketing, etc. In this respect, a practical example is presented to illustrate the applicability of the proposed method. The computational process shows that the proposed model can be used on ordinary machines by finding many targets and indicates a low computational memory and CPU space.

REFERENCES

[1] B. Bankian-Tabrizi, K. Shahanaghi, and M. S. Jabalameli, "Fuzzy multi-choice goal programming," *Applied Mathematical Modelling*, vol. 36, no. 4, pp. 1415–1420, 2012, doi: 10.1016/j.apm.2011.08.040.

[2] S. Broumi, A. Dey, M. Talea, A. Bakali, F. Smarandache, D. Nagarajan, M. Lathamaheswari, and R. Kumar, "Shortest path problem using Bellman algorithm under neutrosophic environment," *Complex & Intelligent Systems*, vol. 5, no. 4, pp. 409–416, 2019, doi: 10.1007/s40747-019-0101-8.

[3] C.-T. Chang, "On the mixed binary goal programming problems," *Applied Mathematics and Computation*, vol. 159, no. 3, pp. 759–768, 2004, doi: 10.1016/j.amc.2003.11.001.

[4] C.-T. Chang, "Multi-choice goal programming," *Omega*, vol. 35, no. 4, pp. 389–396, 2007, doi: doi.org/10.1016/j.omega.2005.07.009.

[5] A. Charmes and W. Cooper, *Management models and industrial applications of linear programming. Ed.* John Wiley and Sons Inc. New York, 1961.

[6] A. Hocine, Z.-Y. Zhuang, N. Kouaissah, and D.-C. Li, "Weighted-additive fuzzy multi-choice goal programming (WA-FMCGP) for supporting renewable energy site selection decisions," *European Journal of Operational Research*, vol. 285, no. 2, pp. 642–654, 2020, doi: 10.1016/j.ejor.2020.02.009.

[7] J. P. Ignizio, *Introduction to linear goal programming*. Sage Beverly Hills, CA, 1985.

[8] M. Kamal, S. Gupta, P. Chatterjee, D. Pamucar, and Z. Stevic, "Bi-level multi-objective production planning problem with multi-choice parameters: a fuzzy goal programming algorithm," *Algorithms*, vol. 12, no. 7, p. 143, 2019, doi: 10.3390/a12070143.

[9] G. R. Keshteli and S. H. Nasseri, "A multi-parametric approach to solve flexible fuzzy multi-choice goal programming," *Punjab University Journal of Mathematics*, vol. 51, no. 12, 2020.

[10] N. Kouaissah and A. Hocine, "Optimizing sustainable and renewable energy portfolios using a fuzzy interval goal programming approach," *Computers & Industrial Engineering*, vol. 144, p. 106448, 2020, doi: 10.1016/j.cie.2020.106448.

[11] S. M. Lee, *Goal programming for decision analysis*. Auerbach Publishers Philadelphia, 1972.

[12] H.-L. Li, "An efficient method for solving linear goal programming problems," *Journal of Optimization Theory and Applications*, vol. 90, no. 2, pp. 465–469, 1996, doi: 10.1007/BF02190009.

[13] G. Maity and S. K. Roy, "Solving fuzzy transportation problem using multi-choice goal programming," *Discrete Mathematics, Algorithms and Applications*, vol. 9, no. 06, p. 1750076, 2017, doi: 10.1142/S1793830917500768.

[14] A. Nafei, Y. Gu, and W. Yuan, "An extension of the TOPSIS for multi-attribute group decision making under neutrosophic environment," *Miskolc Mathematical Notes*, vol. 22, no. 1, pp. 393–405, 2022, doi: 10.18514/MMN.2021.3332.

[15] A. Nafei, C.-Y. Huang, S. P. Azizi, and S.-C. Chen, "An optimized method for solving membership-based neutrosophic linear programming problems," *Studies in Informatics and Control*, vol. 31, no. 4, pp. 45–52, 2022, doi: 10.24846/v31i4y202205.

[16] A. Nafei, A. Javadpour, H. Nasseri, and W. Yuan, "Optimized score function and its application in group multiattribute decision making based on fuzzy neutrosophic sets," *International Journal of Intelligent Systems*, vol. 36, no. 12, pp. 7522–7543, 2021, doi: 10.1002/int.22597.

[17] A. Nafei, W. Yuan, and H. Nasseri, "Group multi-attribute decision making based on interval neutrosophic sets," *Studies in Informatics and Control*, vol. 28, no. 3, pp. 309–316, 2019, doi: 10.24846/v28i3y201907.

[18] A. Nafei, W. Yuan, and H. Nasseri, "A new method for solving interval neutrosophic linear programming problems," *Gazi University Journal of Science*, vol. 33, no. 4, pp. 796–808, 2020, doi: 10.35378/gujs.689125.

[19] Nancy and H. Garg, "An improved score function for ranking neutrosophic sets and its application to decision-making process," *International Journal for Uncertainty Quantification*, vol. 6, no. 5, 2016, doi: 10.1615/Int.J.UncertaintyQuantification.2016018441.

[20] B. B. Pal, B. N. Moitra, and U. Maulik, "A goal programming procedure for fuzzy multiobjective linear fractional programming problem," *Fuzzy Sets and Systems*, vol. 139, no. 2, pp. 395–405, 2003, doi: 10.1016/S0165-0114(02)00374-3.

[21] S. Pramanik, R. Roy, and T. K. Roy, "Teacher selection strategy based on bidirectional projection measure in neutrosophic number environment," *Peer Reviewers*, p. 29, 2017.

[22] M. A. Ragusa and F. Wu, "Global regularity and stability of solutions to the 3D double-diffusive convection system with Navier boundary conditions," *Advances in Differential Equations*, vol. 26, no. 7/8, pp. 281–304, 2021, doi: ade/ade026-0708-281.

[23] M. A. Ragusa and F. Wu, "Regularity criteria for the 3D magneto-hydrodynamics equations in anisotropic Lorentz spaces," *Symmetry*, vol. 13, no. 4, p. 625, 2021, doi: 10.3390/sym13040625.

[24] M. A. Ragusa and F. Wu, "Regularity criteria via one directional derivative of the velocity in anisotropic Lebesgue spaces to the 3D Navier-Stokes equations," *Journal of Mathematical Analysis and Applications*, vol. 502, no. 2, pp. 125–286, 2021, doi: 10.1016/j.jmaa.2021.125286.

[25] C. Romero, "Extended lexicographic goal programming: a unifying approach," *Omega*, vol. 29, no. 1, pp. 63–71, 2001.

[26] R. Şahin, "Multi-criteria neutrosophic decision making method based on score and accuracy functions under neutrosophic environment," *arXiv preprint arXiv:1412.5202*, 2014.

[27] L. Shi and J. Ye, "Cosine measures of linguistic neutrosophic numbers and their application in multiple attribute group decision-making," *Information*, vol. 8, no. 4, p. 117, 2017, doi: 10.3390/info8040117.

[28] F. Smarandache, *A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic*. American Research Press, Rehoboth, 1999.

[29] M. Tamiz, D. Jones, and C. Romero, "Goal programming for decision making: An overview of the current state-of-the-art," *European Journal of operational research*, vol. 111, no. 3, pp. 569–581, 1998.

[30] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Technical Sciences and Applied Mathematics*, vol. 10, pp. 10–14, 2012.

[31] J. Ye, "Single valued neutrosophic cross-entropy for multicriteria decision making problems," *Applied Mathematical Modelling*, vol. 38, no. 3, pp. 1170–1175, 2014, doi: 10.1016/j.apm.2013.07.020.

[32] J. Ye, "Bidirectional projection method for multiple attribute group decision making with neutrosophic numbers," *Neural Computing and Applications*, vol. 28, no. 5, pp. 1021–1029, 2017, doi: 10.1007/s00521-015-2123-5.

[33] H.-y. Zhang, J.-q. Wang, and X.-h. Chen, "Interval neutrosophic sets and their application in multicriteria decision making problems," *The Scientific World Journal*, vol. 2014, 2014, doi: 10.1155/2014/645953.

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