

The unique dynamical system underlying RR Lyrae pulsations

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Abstract. Hydrodynamic models of RR Lyrae pulsation display a very rich behaviour. Contrary to earlier expectations, high order resonances play a crucial role in the nonlinear dynamics representing the interacting modes. Chaotic attractors can be found at different time scales: both in the pulsation itself and in the amplitude equations shaping the possible modulation of the oscillations. Although there is no one-to-one connection between the nonlinear features found in the numerical models and the observed behaviour, the richness of the found phenomena suggests that the interaction of modes should be taken seriously in the study of the still unsolved puzzle of Blazhko effect. One of the main lessons of this complex system is that we should rethink the simple interpretation of the observed effect of resonances.

1. Introduction

Recent space- and ground-based observations of RR Lyrae stars have resulted in a golden age of stellar pulsation studies. A new level of complexity is uncovered; new pieces of the great puzzle of RR Lyrae variability have been found: e.g. unexpected pulsation modes and resonances. One of the most interesting features among the new findings is the period doubling visible in most of the Blazhko modulated stars (Szabó et al. 2010). Furthermore this effect is clearly the consequence of a high order (9:2) resonance of the fundamental mode with a strange 9th overtone (Kolláth et al. 2011).

In addition, observations and hydrodynamic models suggest that a third mode may play an important role in the dynamics. Considering that the interaction of two modes leads to a rare and unique dynamical system, the interaction of the base system with additional modes may produce even more interesting behaviour.

The final question remains still unsolved: whether the very complex interaction of modes can solve the mystery of the Blazhko effect. Anyway, the dynamics of RR Lyrae pulsations provide useful lessons in general in relation to nonlinear processes in nature. In this paper we concentrate only one aspect of the complex dynamical system underlying RR Lyrae pulsation: how the resonance appears in modulated solutions. This question is investigated both for the amplitude equations and hydrodynamic calculations.

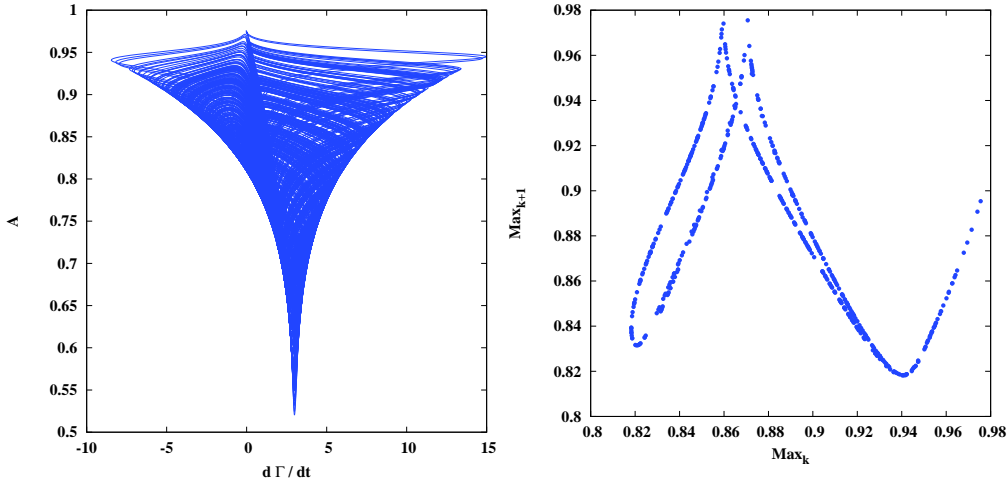


Figure 1. The chaotic attractor of the resonant amplitude equations (left) and corresponding first return map (right).

2. A rare chaotic attractor related to the 9:2 resonance

The period doubling bifurcation found in the hydrodynamic models is a consequence of the resonant interaction of the 9th overtone with the fundamental mode. This effect can be understood by a simplified model of pulsations: if the solution of the amplitude equations with half-integer resonances is a nonzero fixed point (i.e. the amplitudes of both modes are constant) this system provides a natural description of period doubling. Buchler & Kolláth (2001) demonstrated, that the same amplitude equations have solutions that may explain the modulation of RR Lyrae stars, furthermore this modulation can be chaotic. In this paper we use a simplified version of the resonant amplitude equations which is capable to present the major properties of the system:

$$\begin{aligned}
 \frac{dA_0}{dt} &= (1 - A_0^2 - rA_9^2)A_0 + cA_0^8A_9^2 \cos(\Gamma) \\
 \frac{dA_9}{dt} &= (\kappa - A_9^2 - sA_0^2)A_9 + bA_0^9A_9 \cos(\Gamma) \\
 \frac{d\Gamma}{dt} &= 2\Delta - 9cA_0^7A_9^2 \sin(\Gamma) - 2bA_0^9 \sin(\Gamma),
 \end{aligned}
 \tag{1}$$

where A_0 , A_9 are the amplitudes of the fundamental mode and the 9th overtone; c , b , r , s and κ are constant parameters. Γ is the phase differences of the modes, its derivative gives the instantaneous frequency difference from the resonance: $d\Gamma/dt = 2\omega_0 - 9\omega_9$. The constant term which forces the system to deviate from the resonance is given by Δ . Please note that we reduced the number of coefficients by setting the time scale to the growth rate of the fundamental mode and by normalizing the amplitudes.

The strange attractor of the system and the first return map calculated from the maxima of the fundamental mode amplitude variation are displayed in

Fig. 1. The parameters used for the figure are the following: $c = 1.5$, $b = 10$, $r = s = 1.0$, $\kappa = -0.1$ and $\Delta = 1.5$. This chaotic attractor represents a very rare type of dynamics: a double cusp map, where the two leaves of the attractor do not superpose exactly. The only known system of this type is the one proposed by Rössler & Ortoleva (1978). It is remarkable that the interaction of two modes results in such a rich dynamical phenomena.

We have to note that modulated solutions of the amplitude equations exist only for non-zero values of the offset parameter Δ . When Δ is small, the resonance locks and a clean period doubled state is obtained. One of the most important pieces of information one can learn from Fig. 1 is that $d\Gamma/dt$ has a non zero mean and a wide distribution. It clearly demonstrates that in the Fourier spectra of such systems, broader and offset structures are expected at the half-integer frequencies instead of single peaks.

3. Modulation and resonances in hydrodynamic models

There exist period doubled solutions in RR Lyrae hydrodynamic models, but no model has been found with Blazhko-like modulation. However, Smolec & Moskalik (2014) found both phenomena in BL Herculis models, indicating that such behaviour is expected in hydrodynamic calculations. Moreover, it has been already demonstrated by Plachy et al. (2013) that period doubling coexists with chaotic and multimode pulsations in RR Lyrae models.

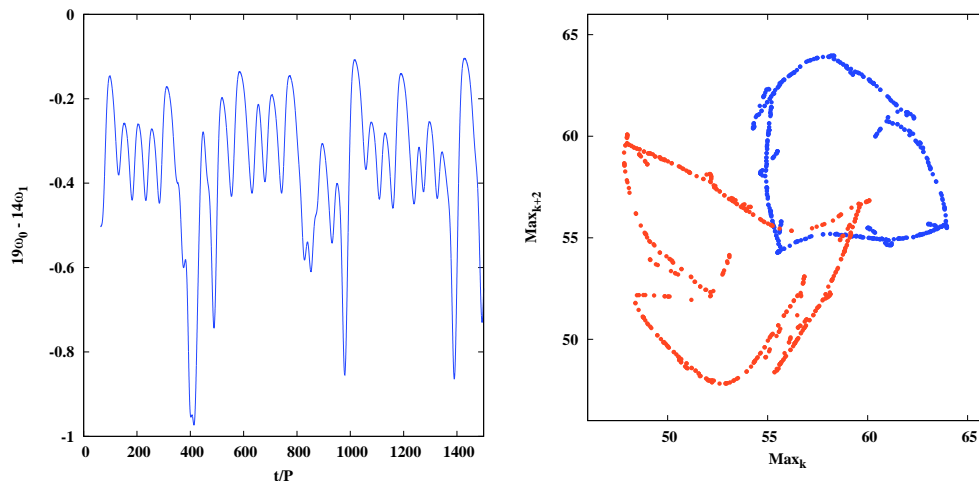


Figure 2. The variation of the relative phase in a hydrodynamic model close to a 14:19 resonance (left) and the corresponding first return map of the radius maxima (right).

When a third mode appears in the interplay of oscillations, the number of possible solutions is increased. For example the interaction of the fundamental mode with the strange mode alters the stability properties of the fundamental mode and makes it more sensitive to other pulsation modes, e.g. our model calculations demonstrated that the period doubled version of the same oscillation now may lose its stability with respect to the first overtone. While in normal double mode scenario of the fundamental mode and the first overtone there is

only one possible solution where both modes coexist, in the presence of the 9th overtone there is a possibility for additional solutions with a double mode character. Figure 2 in Plachy et al. (2013) displays the first return maps calculated from the radius variation of models representing this kind of behaviour.

The right panel in Figure 2 in this paper displays a similar model. By plotting the maxima against the second previous maximum values, the period-doubling structure can be easily recognized (the two branches representing period doubling are plotted with different colours). The substructure of the individual branches are related to the effect of the first overtone which is in a close 14:19 resonance with the fundamental mode. The agglomerations (7 in both branches) in the otherwise smooth return maps are the signs of the still active resonant interaction. In exact resonance the diagram reduces to 14 points in the plot. The instantaneous frequencies are calculated from the radius variation, and the offset from the resonance centre is plotted in the left panel of the figure. Similarly to the 9:2 resonant amplitude equations, an even higher order resonance can play a role in shaping the light variation. And more importantly, an offset from the resonance center results in a slight modulation of the fundamental mode amplitude and a nonzero mean in the $19\omega_0 - 14\omega_1$ off-resonance frequency difference.

4. Conclusion

It is demonstrated that high order resonances play an important role in the dynamics of RR Lyrae model pulsations. However, the exact, phase locked resonance is just one of the possible solutions. In general, when modulation emerges in a resonant system, the behaviour of the dynamics is significantly changed. The main characteristics of these systems are:

- If the frequency offset from the resonance reaches a specific level, the phases are not locked to each other. Even then, fingerprints of the resonance are still observable in the resulting variations.
- The peaks in the Fourier transform of modulated resonant systems do not coincide exactly with the resonant frequencies (e.g. the half integer ones), but there is an offset compared to the resonance centre.
- Broad structures dominate the spectrum instead of single resonant peaks.

Acknowledgments

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