Three-flavor chiral effective model with four baryonic multiplets within the mirror assignment

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Abstract. We study three-flavor octet baryons by using the so-called extended Linear Sigma Model (eLSM). Within a quark-diquark picture, the requirement of a mirror assignment naturally leads to the consideration of four spin-$\frac{1}{2}$ baryon multiplets. A reduction of the Lagrangian to the two-flavor case leaves four doublets of nucleonic states which mix to form the experimentally observed states $N(939)$, $N(1440)$, $N(1535)$ and $N(1650)$. We determine the parameters of the nucleonic part of the Lagrangian from a fit to masses and decay properties of the aforementioned states. By tracing their masses when chiral symmetry is restored, we conclude that the pairs $N(939)$, $N(1535)$ and $N(1440)$, $N(1650)$ form chiral partners.

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1. Introduction

In these proceedings, which are based on the results of Ref. [1], we focus our attention on baryons consisting of the light quarks u, d, and s and with quantum numbers $J^P = \frac{1}{2}^+$ and $J^P = \frac{1}{2}^-$. First and foremost, this includes the nucleon $N(939)$ and the resonances $N(1440)$, $N(1535)$, $N(1650)$, but also Λ, Σ, and Ξ resonances [2].

The fundamental force of nature describing baryons (and hadrons in general) is Quantum Chromodynamics (QCD), whose Lagrangian is given in terms of quarks and gluons by (e.g. Ref. [3]):

$$\mathcal{L}_{\rm QCD} = \bar{q}\big(i\gamma_\mu D^\mu - m\big)q - \frac{1}{2}\,\mathrm{Tr}(\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}) \ , \tag{1}$$

where $D^\mu = \partial_\mu + ig\mathcal{A}_\mu$ is the covariant derivative and g is the coupling "constant" parametrizing the interaction of the quark fields q with a gluon field $\mathcal{A}_\mu$. The latter is associated with the $SU(3)_c$ gauge field $\mathcal{A}^\mu = A^\mu_a T^a$ for $a = 1, 2, \ldots, 8$], T^a being the $SU(3)$ generators. The Yang-Mills field-strength tensor is $\mathcal{G}_{\mu\nu} = i[D_\mu, D_\nu]/g$. Definitely, the QCD Lagrangian is elegant, compact, and contains only few parameters (the bare quark masses and the coupling g). However, a more detailed study within the framework of renormalization shows that the coupling "constant" of strong interaction depends on momentum scale and becomes arbitrary large in the low-energy regime. This fact forbids the usual approach of perturbation theory and causes QCD to be not analytically solvable. On top of that, confinement implies that only "white", i.e. color singlet states (the hadrons) are the asymptotic states of the theory, and only these can be measured in detectors.

In order to describe hadrons, theoretical physicists had to look for alternatives. One such possibility is given by effective approaches to QCD, such as chiral perturbation theory (e.g. Refs. [4]) or linear Sigma Models (see below). In this process one uses Lagrangians which contain hadrons as degrees of freedom, instead of quarks and gluons as in QCD. To get the basic idea of an effective model, we make a short digression by asking under which conditions a glass of water freezes. The answer is easy: it depends on the temperature of water. But why is this answer so simple? Actually, the exact description of the system requires to solve coupled differential equations for each molecule. This is indeed very complicated, actually impossible also for supercomputers. However, by using the effective description provided by thermodynamics, one can solve (some) problems in a much simpler way. In doing so, we lost the information about the motion of each molecule, but we retained the information that is important for describing the whole system.

Let us then turn back to hadrons with half-integer spin: baryons. A baryon is a very complicated system of quarks and gluons arising from Eq. (1). For a fast moving baryon, as for instance a proton, one may use (generalized) parton distribution functions to analyze its substructure in terms of quarks and gluons, see e.g. Ref. [5]. However, for a baryon in its rest frame, this is a very hard task, possibly even harder than following each molecule of water mentioned above. The use of effective descriptions highly simplify the task. Typically, one uses the concept of 'constituent quarks': an almost massless bare quark is dressed by clouds of gluons and quarks and becomes a quasiparticle with an effective mass of about 300 MeV [3]. A baryon is then described as a bound state of three constituent quarks. Within this framework, also the concept of diquark as the strong correlation of two quarks is important, because often baryons are also regarded as quark-diquark objects.

Certain types of effective models, such as linear Sigma Models, are based on Lagrangians which contain from the very beginning only hadrons (mesons and baryons). Quark and gluon fields do not appear. Yet, these Lagrangians are constructed in such a way that (some of the) symmetries of QCD of Eq. (1) are taken into account at he composite level. In particular, chiral symmetry [3] and its spontaneous and explicit breaking are at the basis of such approaches. Recently, the so-called extended Linear Sigma Model (eLSM) has been developed for both mesons and baryons. It contains scalar, pseudoscalar, vector, and axial-vector mesons. Moreover, it shows chiral symmetry and its breaking but also dilatation symmetry and its anomalous breaking (this is the non-perturbative origin of an energy scale of QCD, the renowned $\Lambda_{QCD} \simeq 200$ MeV).

In the meson sector, the three-flavour case $N_f = 3$ has been investigated in detail in Ref. [6, 7]. Yet, until recently, the baryonic sector was only studied for $N_f = 2$ in Ref. [8]. Here, following Ref. [1], we enlarge (Sec. 2) the eLSM to $N_f = 3$ in the baryonic sector. This is a non-trivial step which naturally leads to the consideration of four baryonic octets. Next, (Sec. 3) we consider the limiting case in which only baryons with quarks u and d are considered and discuss the mixing patterns and the identification of the chiral partner of the nucleon. Finally, in Sec. 4 we present our conclusions and outlooks.

2. The eLSM and its implications

The mesonic part of the eLSM Lagrangian containing (pseudo)scalar and (axial-)vector mesons is given in Refs. [6, 7]. The inclusion of baryons was performed for $N_f = 2$ in Refs. [8] and investigated at finite density in Ref. [9]. Recently, the development of the baryonic eLSM to $N_f = 3$ was undertaken in Ref. [1]. The basic idea is to construct baryonic fields in a chiral quark-diquark picture. For $N_f = 3$ diquarks transform as antiquarks, thus one may construct baryons in a similar way as mesons. This assumption results in matrices with elements of different flavor content, which are related to the octet-baryonic fields:

$$\underbrace{\begin{pmatrix} [d,s] \\ -[u,s] \\ [u,d] \end{pmatrix}}_{\text{diquark}} \underbrace{(u,d,s)}_{\text{quark}} \hat{=} \begin{pmatrix} uds & uus & uud \\ dds & uds & udd \\ dss & uss & uds \end{pmatrix} \sim \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix} . \qquad (2)$$

When enlarging the present discussion by taking into account the chirality of quarks and diquarks (see Ref. [10]) and requiring chiral invariant mass terms, we naturally obtain four baryonic multiplets, two of which transform in a standard way under chiral transformations and two in a so-called "mirror" way. These four multiplets are represented by four matrices analogous to Eq. (2). Two of these matrices labeled N_1 and N_2 behave under chiral transformations as

$$\begin{aligned} N_{1R} &\to U_R N_{1R} U_R^\dagger , \qquad & N_{1L} &\to U_L N_{1L} U_R^\dagger, \\ N_{2R} &\to U_R N_{2R} U_L^\dagger , \qquad & N_{2L} &\to U_L N_{2L} U_L^\dagger , \end{aligned} \qquad (3)$$

where U_L and U_R are 3×3 representation matrices of $U(3)_L$ and $U(3)_R$. The remaining two matrices M_1 and M_2 show a chiral transformation from the left that is 'mirror-like' compared to the aforementioned:

$$\begin{aligned} M_{1R} &\to U_L M_{1R} U_R^\dagger , \qquad & M_{1L} &\to U_R M_{1L} U_R^\dagger \\ M_{2R} &\to U_L M_{2R} U_L^\dagger , \qquad & M_{2L} &\to U_R M_{2L} U_L^\dagger . \end{aligned} \qquad (4)$$

These transformations comply a so-called "mirror assignment" [11] which allows one to introduce chirally invariant baryon mass terms in the Lagrangian. The Lagrangian describing these baryonic fields and their interactions with mesonic degrees of freedom is invariant under chiral symmetry $U(3)_R \times U(3)_L$ as well as parity and charge-conjugation transformations [1]. It reads

$$\begin{aligned} \mathcal{L}_{N_f=3,bar} = \;& \mathrm{Tr}\left\{\bar{N}_{1L} i\gamma_\mu D^\mu_{2L} N_{1L} + \bar{N}_{1R} i\gamma_\mu D^\mu_{1R} N_{1R} + \bar{N}_{2L} i\gamma_\mu D^\mu_{1L} N_{2L} + \bar{N}_{2R} i\gamma_\mu D^\mu_{2R} N_{2R}\right\} \\ & + \mathrm{Tr}\left\{\bar{M}_{1L} i\gamma_\mu D^\mu_{4R} M_{1L} + \bar{M}_{1R} i\gamma_\mu D^\mu_{3L} M_{1R} + \bar{M}_{2L} i\gamma_\mu D^\mu_{3R} M_{2L} + \bar{M}_{2R} i\gamma_\mu D^\mu_{4L} M_{2R}\right\} \\ & - g_N \,\mathrm{Tr}\left\{\bar{N}_{1L} \Phi N_{1R} + \bar{N}_{1R} \Phi^\dagger N_{1L} + \bar{N}_{2L} \Phi N_{2R} + \bar{N}_{2R} \Phi^\dagger N_{2L}\right\} \\ & - g_M \,\mathrm{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{1R} + \bar{M}_{1R} \Phi M_{1L} + \bar{M}_{2L} \Phi^\dagger M_{2R} + \bar{M}_{2R} \Phi M_{2L}\right\} \\ & - m_{0,1} \,\mathrm{Tr}\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\} \\ & - m_{0,2} \,\mathrm{Tr}\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\} \\ & - \kappa_1 \,\mathrm{Tr}\left\{\bar{N}_{1R} \Phi^\dagger N_{2L} \Phi + \bar{N}_{2L} \Phi N_{1R} \Phi^\dagger\right\} - \kappa_1' \,\mathrm{Tr}\left\{\bar{N}_{1L} \Phi N_{2R} \Phi + \bar{N}_{2R} \Phi^\dagger N_{1L} \Phi^\dagger\right\} \\ & - \kappa_2 \,\mathrm{Tr}\left\{\bar{M}_{1R} \Phi M_{2L} \Phi + \bar{M}_{2L} \Phi^\dagger M_{1R} \Phi^\dagger\right\} - \kappa_2' \,\mathrm{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{2R} \Phi + \bar{M}_{2R} \Phi M_{1L} \Phi^\dagger\right\} . \end{aligned} \qquad (5)$$

The traces are invariant under cyclic permutation, which ensures their symmetry under the chiral transformations (3) and (4). The covariant derivatives are given by $D^\mu_{kR} = \partial^\mu - ic_k R^\mu$ and $D^\mu_{kL} = \partial^\mu - ic_k L^\mu$ for $k = 1, \ldots, 4$, where the left- and right-handed matrices L^μ and R^μ represent (axial-)vector mesonic degrees of freedom. (Pseudo)scalar mesonic fields are incorporated via the Φ matrix. The mesonic matrices transform under chiral transformations as $R^\mu \to U_R R^\mu U_R^\dagger$, $L^\mu \to U_L L^\mu U_L^\dagger$, and $\Phi \to U_L \Phi U_R^\dagger$. The mass parameters $m_{0,1}$ and $m_{0,2}$ are particularly important, since they allow to shed light on the origin of the nucleonic masses. They emerge from (dilatation-invariant) interactions upon the condensation of glueball and/or a four-quark states, see e.g. Ref. [9].

The baryonic fields in Eq. (5) are not parity eigenstates, therefore we construct the fields of definite parity,

$$B_N = \frac{N_1 - N_2}{\sqrt{2}}, \qquad B_{N*} = \frac{N_1 + N_2}{\sqrt{2}}, \qquad B_M = \frac{M_1 - M_2}{\sqrt{2}}, \qquad B_{M*} = \frac{M_1 + M_2}{\sqrt{2}}. \tag{6}$$

where now B_N and B_M have positive parity and B_{N*} and B_{M*} have negative parity. In the limit of zero mixing, B_N describes the ground-state baryonic fields of Eq. (2), i.e., $\{N(939)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1338)\}$, B_M the positive-parity fields $\{N(1440), \Lambda(1600), \Sigma(1660), \Xi(1690)\}$, B_{N*} can be assigned to the negative-parity fields $\{N(1535)$, $\Lambda(1670), \Sigma(1620), \Xi(?)\}$ and, finally, B_{M*} to $\{N(1650), \Lambda(1800), \Sigma(1750), \Xi(?)\}$. In general the fields describing physical particles emerge as a mixture of B_N, B_{N*}, B_M, and B_{M*}. The detailed study of this mixing will be performed below for the two-flavor case.

3. Results

In order to determine the twelve parameters of the model, we reduce Eq. (5) to $N_f = 2$. This leaves us with four isodoublets (instead of the baryonic 3×3 matrices), which mix to produce the experimentally observed nucleon $N(939)$, $N(1440)$, $N(1535)$, and $N(1650)$. For the fit, we use thirteen quantities: masses and decay widths of the resonances, the axial coupling constant of the nucleon from [2], as well as the lattice results for the remaining axial coupling constants [12]. Using a standard χ^2-square procedure we found that three acceptable and almost equally deep minima exist, see Ref. [1]. The first two minima lead to small absolute values of $m_{0,1}$ and $m_{0,2}$, while the third one features absolute values of these constants comparable with the nucleon's mass (in agreement with the recent study of Ref. [13]).

Quite remarkably, for all three minima the decay $N(1535) \to N\eta$ cannot be described (it is a factor 10 too small [1]). Thus, further studies are needed to understand $N(1535)$. It may contain a sizable admixture of $s\bar{s}$, see Ref. [14], or the problem might be connected to the role of chiral anomaly in the baryonic sector [15]. The assignment of chiral partners can also be investigated by computing the masses as a function of the chiral condensate φ_N, because masses of chiral partners become degenerate for $\varphi_N \to 0$. For all minima, the result shows that the masses of the $N(939)$ and $N(1535)$ as well as $N(1440)$ and $N(1650)$ merge as $\varphi_N \to 0$, therefore these form two pairs of chiral partners.

Finally, as an illustration, we present here one of the resulting mixing matrices (Minimum 1 in [1]):

$$\begin{pmatrix} N(939) \\ \gamma^5 N(1535) \\ N(1440) \\ \gamma^5 N(1650) \end{pmatrix} = \begin{pmatrix} -\mathbf{0.996} & -0.025 & -0.046 & -0.074 \\ 0.075 & -\mathbf{0.492} & 0.039 & -\mathbf{0.867} \\ -0.050 & -0.057 & \mathbf{0.995} & 0.073 \\ 0.010 & \mathbf{0.869} & 0.086 & -\mathbf{0.488} \end{pmatrix} \begin{pmatrix} B_N \\ \gamma^5 B_{N*} \\ B_M \\ \gamma^5 B_{M*} \end{pmatrix}. \tag{7}$$

In fact, although the mixing has been determined for $N_f = 2$ only, our calculations show that it is a good first approximation for all the members of the octet.

4. Summary and outlook

The eLSM is an effective model of QCD whose building blocks are hadrons. Starting from its Lagrangian, one can perform calculations that are not possible within QCD. In this work we have generalized the eLSM to the three-flavor case. Requiring chirally invariant mass terms one is lead to use the so-called "mirror assignment", and naturally obtains four baryonic multiplets. In order to determine the parameters, we have performed a reduction to the $N_f = 2$ case and a fit to experimentally known quantities. Three minima produce results that are in good agreement with experiment (expect for the decay width $N(1535) \to N\eta$). Furthermore, we concluded that the pairs $N(939)$, $N(1535)$ and $N(1440)$, $N(1650)$ form chiral partners. The most important open problem is to decide which of the minima is preferable. To this end, we will investigate the complete $N_f = 3$ case by performing an overall fit to measure physical quantities. As a consequence, interesting information for both vacuum physics, such as scattering processes involving strange hadrons [16] and at nonzero density, such as the role hyperon in compact stars [17], will be obtained.

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