Chaos
An Interdisciplinary Journal of Nonlinear Science

Where are the coexisting parallel climates? Large ensemble climate projections from the point of view of chaos theory

Cite as: Chaos **33**, 031104 (2023); https://doi.org/10.1063/5.0136719
Submitted: 28 November 2022 • Accepted: 06 February 2023 • Published Online: 13 March 2023

M. Herein, T. Tél and T. Haszpra

Chaos ARTICLE scitation.org/journal/cha

Where are the coexisting parallel climates? Large ensemble climate projections from the point of view of chaos theory

Cite as: Chaos 33, 031104 (2023); doi: 10.1063/5.0136719
Submitted: 28 November 2022 · Accepted: 6 February 2023 ·
Published Online: 13 March 2023

View Online Export Citation CrossMark

M. Herein,[1,2,a)] T. Tél,[1,2] and T. Haszpra[1,2]

AFFILIATIONS

[1]ELKH-ELTE Theoretical Physics Research Group, Pázmány P. stny. 1/A, H-1117 Budapest, Hungary
[2]Department of Theoretical Physics, Eötvös University, Pázmány P. stny. 1/A, H-1117 Budapest, Hungary

[a)]**Author to whom correspondence should be addressed:** matyas.herein@ttk.elte.hu

ABSTRACT

We review the recent results of large ensemble climate projections considering them to be the simulations of chaotic systems. The quick spread of an initially localized ensemble in the first weeks after initialization is an appearance of the butterfly effect, illustrating the unpredictability of the dynamics. We show that the growth rate of uncertainty (an analog of the Lyapunov exponent) can be determined right after initialization. The next phase corresponds to a convergence of the no longer localized ensemble to the time-dependent climate attractor and requires a much longer time. After convergence takes place, the ensemble faithfully represents the climate dynamics. Concerning a credible simulation, the observed signal should then wander within the spread of the converged ensemble all the time, i.e., to behave just as any of the ensemble members. As a manifestation of the chaotic-like climate dynamics, one can imagine that beyond the single, observed time-dependent climate, a plethora of parallel climate realizations exists. Converged climate ensembles also define the probability distribution by which the physical quantities of the different climate realizations occur. Large ensemble simulations were shown earlier to be credible in the sense formulated. Here, in addition, an extended credibility condition is given, which requires the ensemble to be a converged ensemble, valid also for low-dimensional models. Interestingly, to the best of our knowledge, no low-order physical or engineering systems subjected to time-dependent forcings are known for which a comparison between simulation and experiment would be available. As illustrative examples, the CESM1-LE climate model and a chaotic pendulum are taken.

We demonstrate that a plethora of parallel climates can be thought to coexist with our observed reality. We also demonstrate that a decadal characteristic time should be passed after the initialization of a simulation before credible projections can be made. The known chaotic nature of climate dynamics is illustrated by pointing out similarities between a complex state-of-the-art climate model and a simple chaotic pendulum. A more detailed analysis leads, however, to the conclusion that climate modeling can be considered to be moved further ahead than current dynamical system research.

I. INTRODUCTION

In terms of physics, the dynamics of weather and of the fluid components of the climate system are turbulent (Vallis, 2017). They can be considered to be chaotic-like in the sense that they are unpredictable and that they possess an extended chaotic attractor. The dynamics of the entire climate can, therefore, be called chaotic-like, too. The widely used term in climate science "internal variability" (see, e.g., Deser *et al.*, 2012; Collins *et al.*, 2013; Deser, 2020; Ghil and Lucarini, 2020; Bódai *et al.*, 2021; and Lee and Bódai, 2021) might be considered an analog of the existence of a chaotic attractor representing a plethora of permitted states. This observation might have played a role in the appearance and currently increasing role of climate ensemble simulations (Collins, 2007; Deser *et al.*, 2020; Bach *et al.*, 2021; and Maher *et al.*, 2021). Notable examples are single-model initial-condition ensembles, SMILEs (Kay *et al.*, 2015; Kirchmeier-Young *et al.*, 2017; Maher *et al.*, 2019; Swart *et al.*, 2019; Danabasoglu *et al.*, 2020; and Rodgers *et al.*, 2021), and multi-model ensembles (Meehl *et al.*, 2009;

Taylor *et al.*, 2012; Eyring *et al.*, 2016; and Lehner *et al.*, 2020). In our argumentation, we shall rely on SMILEs.

Our aim here is to consider such climate ensembles as special instances of chaos in a huge dimensional phase space. An unusual feature from the point of view of traditional chaos theory is that the system is subjected to parameter drift, as important forcings, like, e.g., the greenhouse gas concentrations, are strongly time-dependent. The emerging literature on chaos in such systems clearly shows that on long term, only an ensemble-based statistical description is meaningful [as suggested already in Romeiras *et al.* (1990)]. We also address the question if a condition for the consistency of a large-scale climate model with the measured reality can be given. Within the realm of single-trajectory simulations, the question is ill-defined and no reliable answer exists. In the class of SMILEs, however, a necessary condition can be and was given, for global quantities at least (Deser *et al.*, 2020; Maher *et al.*, 2021; and Suarez-Gutierrez *et al.*, 2021).

A qualitative picture provided by the "theory of parallel climate realizations" constitutes a proper background regarding the comparison of simulations and reality. This theory (Herein *et al.*, 2017; and Tél *et al.*, 2020) states that it is worth imagining many replicas of the Earth System that evolve in parallel, but differently, although they all are subjected to the same physical laws and to the same time-dependent set of forcings and boundary conditions (e.g., in terms of irradiance and greenhouse gas concentration). This view in itself is a reformulation of the chaotic-like behavior of the dynamics and is also a concept that helps make the term "internal variability" more plausible. We note that the idea of parallel climates appeared in nearly the same formulation as above already in 1978 in a paper by Leith (1978), based on an analogy with classical statistical mechanics.

More generally, the view that instead of a single history, all the possible parallel dynamical evolutions should be considered is well spread in other disciplines, such as nonequilibrium phenomena (Presse *et al.*, 2013) and evolutionary biology (Gould, 1989), and has very recently appeared in chaos theory too (Jánosi *et al.*, 2021).

II. EMERGENCE OF THE PARALLEL REALIZATIONS

We have access to the downloadable meteorological data at NCAR Climate Data Gateway (https://www.earthsystemgrid.org/) produced for the years 1920–2021, where all the 40 members generated by the Community Earth System Model Large Ensemble Project (CESM1-LE) are freely available. CESM1-LE follows a mostly observational based forcing protocol, including greenhouse gas forcing (Kay *et al.*, 2015), as detailed in Sec. S1 of the supplementary material. There is, thus, an opportunity for us to process and visualize these data to check potential analogies with chaos research. As an illustrative variable for this study, we have chosen to investigate the global mean surface temperature.

First, we consider the very beginning of the simulation (the first 30 days) to gain a picture about the reliability of individual trajectory simulations. The initial perturbation in the global surface temperature field (denoted here by TS) between neighboring ensemble members is rather small: on the order of 10^{-14} K, as stated in Kay *et al.* (2015). [We note that this value is three orders of magnitudes smaller than the temperature by means of which the absolute zero K can be approached in experiments (Deppner *et al.*, 2021).] As Fig. 1(a) illustrates, despite these invisible initial differences in TS, the band of all simulations broadens.

This "plume diagram" shows that up to about the 15th day, all curves run practically together and the spread remains below about 0.2 K (after which, a strong broadening occurs). The prediction of the time evolution in the given model and with the chosen initial conditions can, thus, be considered reliable in this period, in the sense that the tiny initial differences do not lead yet to strong deviations. For any longer period, the time evolution of the system can be considered *unpredictable* from the point of view of individual trajectories. The initial spread of the ensemble members, i.e., the unpredictability of the dynamics (Lorenz, 1963; and Ott, 1993), is the origin of internal variability, a feature characterizing the attractor not yet approached here (see Sec. III). This observation illustrates the term "sensitivity to initial conditions," or the "butterfly effect," in the language of the popular culture (see, e.g., Gleick, 1987).

From the point of view of climate projections, the moral of Fig. 1(a) is that none of the 40 members is better than any other. This questions the relevance of any statement concerning the future of the climate if this statement is based on a single realization of any model.

Based on the plume diagram, an important characteristic number of unpredictability can be estimated. A small initial uncertainty Δr_0 is expected to increase exponentially in time and become

$$\Delta r = e^{\lambda t} \cdot \Delta r_0 \tag{1}$$

after time t. Exponent λ can be called the *growth rate of uncertainty*. It measures the rate of separation of initially nearby trajectories and is similar to the Lyapunov exponent, but the latter is defined only on an attractor (Ott, 1993) or on a chaotic saddle (Lai and Tél, 2011). This is not yet the case here since time is too short to judge even if transient chaos has reached a chaotic set. This λ is, thus, at most an initial finite-time Lyapunov exponent.

Prescribing a threshold uncertainty $\Delta r_{\rm th}$ up to which the ensemble is considered to remain together, i.e., up to which the dynamics is predictable, defines the *prediction time* t_p. Note that this term (Ott, 1993; and Tél and Gruiz, 2006) does not imply that anything is correctly predicted for $t < t_{\rm p}$. From (1), we get

$$t_p = \frac{1}{\lambda} ln \frac{\Delta r_{th}}{\Delta r_0}. \tag{2}$$

The choice of the threshold value and the prediction time is somewhat subjective. It can be based on the appearance of a visible but yet small deviation of the ensemble. It is in this sense that based on Fig. 1(a), we choose $\Delta TS_{\rm th} = 0.2$ K, belonging to $t_p = 15$ days, from which with $\Delta TS_0 = 10^{-14}$ K, we get $\lambda \approx 2$ 1/day for the estimated growth rate of uncertainty. This means that the e-folding time, within which the differences between ensemble members grow e times greater, is $1/\lambda \approx 1/2$ day.

Before proceeding, it might be illuminating to consider a typical chaotic low-dimensional model, the pendulum with a periodically moving suspension point, also described in Sec. S1 of the supplementary material. Figure 1(b) exhibits a plot analogous to that of panel 1(a): 40 trajectories of angular velocity ω with a slight difference in their initial value, of an arbitrarily chosen magnitude of 10^{-1} 1/s between neighboring members, are numerically followed in time.

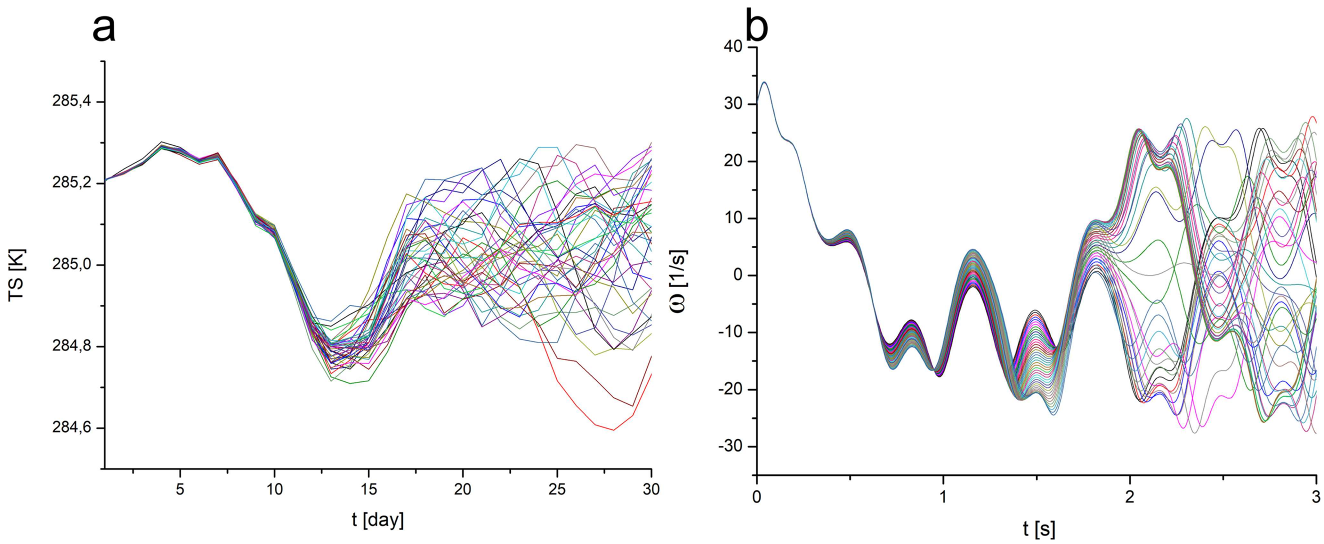

FIG. 1. The spreading of parallel simulations after initialization (a) for the daily global mean surface temperature in climate model CESM1-LE (initial uncertainty between members: $\Delta TS = 10^{-14}$ K) and (b) in the angular velocity of a chaotic pendulum (initial uncertainty: $\Delta\omega = 10^{-1}$ 1/s). The strong sensitivity to initial conditions is evident in both cases. We note that a similar figure to Fig. 1(a) is available at the official portal of NCAR: https://www.cesm.ucar.edu/community-projects/lens/known-issues.

(Since the climate data are dimensional, we decided to simulate the dimensional version of the pendulum equations.) During the first second, or so, the trajectories all remain together, but after this time, a considerable deviation occurs, and all the graphs become distinct. Based on the visual impression provided by Fig. 1(b), the prediction time of the pendulum can be chosen to be about 1.2 s. A threshold uncertainty, $\Delta\omega_{th} = 6\,\mathrm{s}^{-1}$, belongs to this instant. From (2), we find the estimate $\lambda \approx 3.4\,\mathrm{s}^{-1}$ for the growth rate of uncertainty of the pendulum simulation, with an e-folding time of $1/\lambda \approx 0.3$ s.

This argumentation implies that *individual trajectory simulations are unreliable in both cases*: if ensemble data are available, for $t > t_p$, individual trajectories provide a much more restricted characterization than the full ensemble.

It should be emphasized that the physics is drastically different: the climate model has millions of degrees of freedom, while the pendulum is a two degree of freedom problem.

In more qualitative terms, the driven pendulum exhibits irregularity only in time, while spatiotemporal patterns are characteristic features of climate, in harmony with its turbulent character (Janosi and Vattay, 1992; and Vallis, 2017). Qualitatively speaking, the climate system is as complicated as the union of millions of coupled unpredictable dissipative pendulums and also encodes spatial patterns and irregularities. There is, thus, a huge difference between a single driven pendulum and the full climate system; however, they share a particular property of their dynamics, *unpredictability*, which manifests itself in the sudden spreading of the plume diagrams. This illustrates that not only low-dimensional chaos is sensitive to initial conditions, but also much more complicated cases such as turbulence or spatio-temporal systems, which cannot be called chaotic in the traditional sense (see, e.g., Ott, 1993). This is why we are using the term chaotic-like in relation to climate dynamics.

A faithful comparison of the prediction times can only be obtained if these times are given in relation to a characteristic time of the systems. The result will, of course, depend on the forcing period. From the many relevant climatic time scales, a natural choice in the context of centennial climate projections can be 1 year, the period of the annual cycle, while the suspension point of the pendulum example is moving with a period of 0.66 s. Instead of comparing the prediction times (15 days ≈ 1/24 year and 1.2 s) and the periods, a more physical comparison is obtained based on the growth rate of uncertainties since these are expected to be independent of the initial and threshold values. The reciprocal of λ compared with the forcing period leads to a ratio of 1/2 day/1 year = 1/730 ≈ 0.0014 for the climate and 0.3 s/0.66 s = 0.45 for the pendulum. In this comparison, climate turns out to be a factor of 300 less predictable than the pendulum.

III. SPREAD GENERATED BY PARALLEL REALIZATIONS

Turning back to the climate and the different realizations of Fig. 1(a), they obviously can inherit, in the full period investigated, properties of the initial states, e.g., the value of the average global temperature on the first day. It is obvious that one can see parallel realizations from the very beginning of the simulation. These, however, cannot be identified as parallel *climate* realizations since climate is an objective entity, independent of how the simulation is initiated. What we see in Fig. 1(a) is, thus, similar to ensemble weather forecasts, although in terms of a global quantity.

The climate system is dissipative and is, thus, expected to possess an attractor. This is an object that all trajectories converge to. Due to the dissipative nature of the dynamics, initial conditions become only *forgotten* after a time t_c needed to reach this chaotic-like climate attractor as discussed in the literature [see, e.g.,

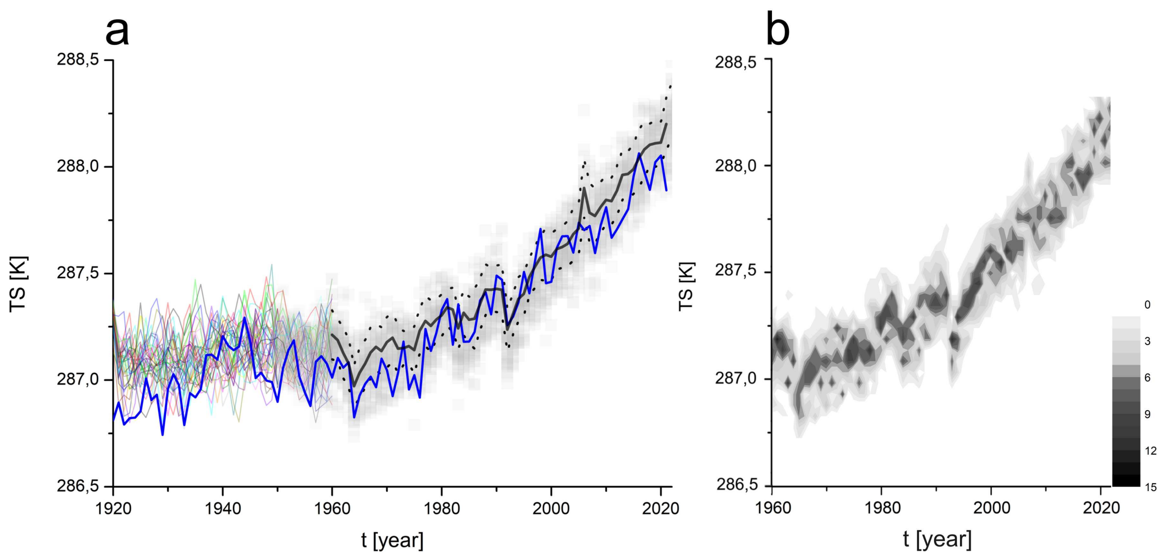

FIG. 2. (a) The annual mean global surface temperature TS of all the 40 members of CESM1-LE. In the period 1920–1960, the individual simulations are colored, while from 1960 on, when the ensemble is converged and simulations represent parallel climate realizations, a single uniform gray shading represents the spread up to 2021. The instantaneous ensemble average is plotted as a black curve, while the range set by the standard deviation of the ensemble corresponds to the distance between the dotted lines. The Met Office Hadley Centre's observation dataset HadCRUT5 is superimposed in blue in the full time window. (b) We also generated the probability distribution, shown here, for the TS values provided by the converged ensemble of CESM1-LE. For more details see Sec. S3 of the supplementary material.

Branstator and Teng, 2010; Herein *et al.*, 2016; Drótos *et al.*, 2017; Haszpra *et al.*, 2020; and Drótos and Bódai, 2022 for a review]. Simulations do indicate that only one attractor exists. Here, we add that the convergence time t_c is by definition different from the prediction time t_p discussed above. Both are, of course, model-dependent. After time t_c, the ensemble traces out the attractor that changes with time, representing climate change. In the literature, special terms are introduced for such time-dependent attracting objects: snapshot (Romeiras *et al.*, 1990) or pullback (Ghil *et al.*, 2008) attractors. We shall call an ensemble *converged*, if it has already approached the attractor, i.e., if it has forgotten the initial conditions. We show that a method easily applicable in practice is that one initializes a second ensemble earlier than the ensemble in question (or the second one with considerable different data, initiated at the same time) and looks for the time instant after which the two ensembles provide practically the same statistics. This time instant is t_c for the ensemble in question. Since we have no access to the runs of CESM1-LE with different initial conditions, this algorithm and further features of the convergence are studied in a different model, in Planet Simulator (Fraedrich *et al.*, 2005), in Secs. S1 and S2 of the supplementary material. In the particular case of CESM1-LE, the convergence time was estimated to be $t_c = 40$ years in Bódai *et al.* (2020).

Before proceeding, we mention that the convergence time to the attractor of the pendulum is about $t_c = 10$ s: the width of the band of trajectories becomes convincingly constant by this time, as visible in Fig. 3 later. In terms of the forcing period of 0.66 s and 1 year, the convergence time of the pendulum and the climate proves to be on the order of 15 and 40 times this period, respectively.

In Fig. 2(a), we plot the TS data of CESM1-LE in this spirit. In the first 40 years, colored curves are used [these are the continuations of those in Fig. 1(a)], although on an annual basis. To express that the regime of reliable climate data (where the ensemble is converged, in other words, the regime where the simulation can be believed to represent parallel climate realizations) starts at about 1960 and after this time, a uniform shading is used. In order to indicate that convergence takes place gradually, we also mark an overlap period of about a decade, where individual realizations are yet visible, but a gray shading is also applied. Gray is not only a new coloring but also an expression that none of the colored curves is better than the other. It is only the ensemble of parallel climate realizations what bears a meaning. The TS axis is divided into small bins of an approximate size of 0.05 K. If a realization happens to yield a value in such a bin, a small rectangle of this height and of a width corresponding to approximately one year is shaded gray, irrespective of the number of realizations falling into this bin. The centers of rectangles are associated with the beginning of each year. In a given year after 1960, the spread of the gray region is the range of the TS values provided by the ensemble of parallel climate realizations. The visualization indicates a trend of a monotonous increase, a climate change, after about 1970.

The ensemble mean is plotted as a black line. Since this is a climate characteristic, we plot it only in the regime of parallel climate realizations. It corresponds to the typical TS value within the ensemble.

In principle, the size of the gray region can depend on time. It is worth emphasizing that here, this width remains practically constant all the time, approximately 0.75 K. Similarly, the distance between the dotted lines, i.e., the twice of the standard deviation value, is also practically the same, approximately 0.3 K. These values characterize the amount of internal variability or the size of the climate attractor (in variable TS). Additional figures illustrating the behavior of the converged ensemble, including the time-dependent distribution of the parallel climates [Fig. 2(b)], are given in Sec. S3 of the supplementary material.

As a consequence of the existence of a multitude of permitted states in chaos, the size of chaotic attractors is nonzero in any variable. Therefore, we can say that the width of the gray region (or that in between the dotted lines), *cannot* be reduced to zero by increasing the size of the ensemble (see Sec. S2 of the supplementary material). This width is an internal property of the dynamics and, for large enough ensembles, does not depend on how many members represent the ensemble. A study by Milinski *et al.* (2020) investigated how large a climate ensemble needs to be in order for a proper characterization of internal variability. They found that a few dozen members are sufficient in the case of global quantities at least, which is also valid for CESM1-LE. The same conclusion is reached in Pierini (2020) in an ensemble description of a low-dimensional ocean model.

IV. COMPARISON WITH DATA, CONSISTENCY WITH REALITY

It is essential to judge models' performance via comparison with actual, observed data. We shall call a model, along with the forcings used, a *credible model* if the ensemble results for a given variable are compatible with the single observed dataset of the same quantity for a long period of time. For the use of a similar terminology, see Deser (2020). As a sufficient condition for credibility, we take global variables. These are the analogues of the few variables present in low-dimensional chaotic systems. An extension for regional behaviors can and should be, of course, a step for the future (see, e.g., Oldenborgh *et al.*, 2013; Deser, 2020; and McKinnon and Deser, 2021).

For judging the credibility of the CESM1-LE model with global variable TS investigated up to now, we use a widely accepted historical dataset, HadCRUT5, for the global annual mean surface temperature. The observed data are also plotted in Fig. 2(a) as a blue continuous line, superimposed on the CESM1-LE realizations. [We note that a similar figure (Fig. 1) can be found in Deser *et al.* (2020b), where another temperature record (Berkeley Earth) and CESM1-LE are compared, not showing, however, the importance of convergence and the instantaneous ensemble-based standard deviation.] It is satisfying to see that in the range where CESM1-LE is converged and can be considered to represent parallel climate realizations (from 1960 onward), the blue curve happens to lie entirely inside the gray range spanned by the ensemble.

The condition of credibility is studied in the recent climate literature (Marotzke and Forster, 2015; Suarez-Gutierrez *et al.*, 2017; Tokarska *et al.*, 2020; Deser, 2020; and Suarez-Gutierrez *et al.*, 2021). The authors agree on that a necessary consistency condition for a given climate model to be credible is that the observed, measured time series lies within the range spanned by its large ensemble realizations over all times, so that it reaches the maxima or minima of the model simulations occasionally (Suarez-Gutierrez *et al.*, 2021).

We emphasize that we totally support the idea that a necessary consistency condition is that the observed values should lie within the ensemble in the sense mentioned above. At the same time, we add *the consistency condition holds only for a converged ensemble* since the lack of the convergence may lead to misleading results (see below and Sec. S2 of the supplementary material).

Note that the blue line of measured data differs from the ensemble mean (black). This is natural, as the former results from the observation of our single reality, and a closer investigation reveals that the blue line differs from *any* of the individual realizations. At the same time, we can say that the observed time series should be similar to an individual realization in a credible model, although it is impossible to say what this individual realization is. One can imagine as if the observed time series appeared to correspond to a random walk between shorter pieces of simulated realizations.

These observations offer a possible answer to the question of what and how one can learn about climates potentially coexisting with the single observed one. As we see, these other "climates" are only accessible via credible simulations, but none of the individual realizations should be taken seriously, rather the spread and other statistical measures of the distribution given by the ensemble. At the end of each year, one can learn what TS values and with what probabilities occurred in the parallel realities forming the basis of a probabilistic climate prediction. Figure 2(b) and S4(b) in the supplementary material show the first determination of the time evolution of instantaneous probability distributions for variable TS gained from the CESM1-LE dataset.

Another determination of these parallel realities is offered by the appearance of Observational Large Ensembles (McKinnon *et al.*, 2017; McKinnon and Deser, 2018; and McKinnon and Deser, 2021), using a climate model to define the typical behavior and apply statistical resampling to observed data in order to simulate internal variability.

Let us finally concentrate on the first 20 years of the period considered in Fig. 2. In this range, the measured blue curve lies somewhat off the band traced out by the simulations. This difference might be attributed to the simulations being as yet away from the climate attractor, the ensemble being not yet converged: the numerical ensemble appears to approach the measured TS time series from above to finally cover it.

V. COMPARISON WITH EXPERIMENTS IN LOW-DIMENSIONAL CHAOTIC SYSTEMS

Here, we discuss briefly the degree and type of agreement between measurements and simulations in chaotic systems. In traditional chaos without parameter drift, attractors are generated with a single *long* trajectory (Ott, 1993). This is known then to coincide with the result following from an ensemble (Eckmann and Ruelle, 1985). Observed chaotic signals of simple systems never coincide with their single-trajectory numerical simulations, as a consequence of unpredictability (Baker and Gollub, 2012). This is in analogy with what we see in Fig. 2(a). In traditional chaos, however, one typically checks if the measured time series is consistent with the shape of the numerically generated chaotic attractor. The attractor is, of course, generated after initial transients die out, that is after *convergence* to the attractor took place. Correspondingly, the initial part of the measured signal is discarded. There are a number of cases that demonstrate [cf., e.g., Fig. 6.3 in Baker and Gollub (2012) for a chaotic pendulum, Figs. 9(a) and 9(f) in Madan and Wu (1993) for Chua's circuit, and Fig. 9 in Ottino *et al.* (1995) for an advection problem] that the measured signal, sampled properly, traces out practically the same pattern as the numerically generated one. It is worth knowing, however, that despite efforts to carry out measurements as precisely as possible, the agreement between simulations (which are unable to describe all fine details) and experiments turns out in some other cases to be only quantitative as, e.g., in Yagasaki (1995), Masoller *et al.* (1998), and Jahanshai *et al.* (2021).

The investigation of non-traditional chaos in problems with parameter drift is, unfortunately, not yet in focus, and consequently, experiments are not yet well spread in this context. Most available examples consider the limit of very slow drift (Chancellor *et al.*, 1996; Chatterjee *et al.*, 2002; and Amon and Lefrance, 2004). An analogy with climate change would require that the change is not adiabatically slow. The only example we are aware of with a fast rate is a paper by Vincze *et al.* (2017), which reports a hydrodynamical problem in rotating tank experiments, mimicking climate change, in a scenario of decreasing horizontal temperature contrast. Unfortunately, no simulation was performed. (In a drift-free version of the rotating tank experiment, however, five different advanced simulations were compared with the measurements (Vincze *et al.*, 2015). The deviations among these and from the measured data were found to be not negligible, not even in relatively simple quantities like the speed of the Rossby wave modes.) We emphasize that to the best of our knowledge, no experiments are known to be conducted with low-dimensional drifting problems of non-negligible drift, the credibility check of which would require an ensemble simulation.

In the lack of measurements, we illustrate what the ensemble approach would look like with our pendulum example. Remember, this is a system without any trend. Here, a single long trajectory approach is equivalent with the ensemble approach. In order to remain compatible with Fig. 2, we should, however, follow an ensemble, not a standard tool in traditional chaos. We take here the ensemble of Fig. 1(b) of the pendulum system and check how the ensemble simulation and a measurement would be related. To this end, one of the simulated trajectories is chosen to correspond to a measured time series in the converged state: $t > t_c$. In Fig. 3, it is clearly visible that the spread is constant in the period $t > t_c = 10$ s, illustrating that a convergence to the attractor has occurred. There is no drift in the system; hence, the band is horizontal and not tilted as in Fig. 2. Nevertheless, the selected trajectory, plotted in blue, stretches across the full ensemble band as time goes on.

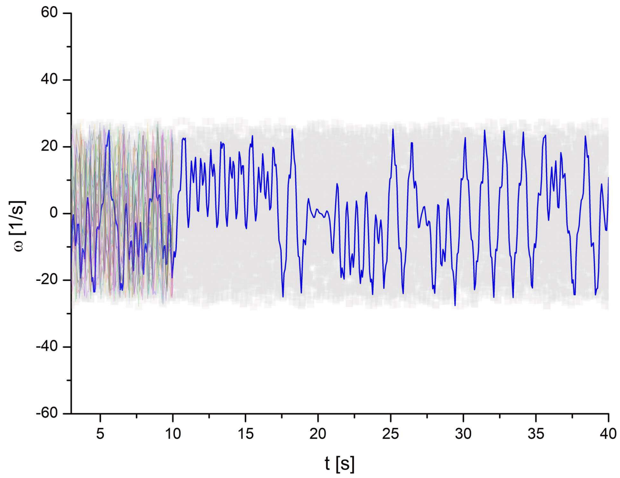

FIG. 3. The spread of all the 40 members of the driven pendulum (gray shading) from 10 to 40 s for the angular velocity [preceded by an overlap period of about ten driving periods, that is, 7 s, where all the individual trajectories are also shown after the end of Fig. 1(b).]. The ω axis is divided into small bins of an approximate size of 0.25 1/s (and 0.07 s for the time axis). In the lack of physical measurement, one ensemble member is considered to represent reality, plotted in blue. Note that the blue curve stretches across the full band similar to Fig. 2.

VI. DISCUSSION

We have investigated initial-condition large ensemble climate projections from a chaos theory point of view. In spite of the huge number of degrees of freedom, surprising similarities are present with low-order chaotic models. We illustrated that none of the individual climate realizations can be considered more reliable than any other; rather, the spread and the probability distribution of a converged ensemble are meaningful only. CESM1-LE is checked to be consistent with the observed global annual mean surface temperature data, but this only holds after a relevant convergence time.

We mention here that a more complete sufficient condition for a climate model being credible (globally at least) requires the agreement between simulated and measured quantities in *all* relevant global quantities (e.g., temperature at any given altitude, mean sea level, ice cover), and the investigated large ensemble must be a converged one again.

In addition we note, since there is a single real climate system only, one cannot argue in favor of using different models for predicting different global quantities, as occurs in certain publications (see, e.g., Suarez-Gutierrez *et al.*, 2021).

As pointed out in Tokarska *et al.* (2020), not all state-of-the-art climate models fulfil the requirement of consistency with reality, even in the single variable TS. Nevertheless, about 10 models have been identified as credible (Papalexiou *et al.*, 2020), including CESM1-LE (Suarez-Gutierrez *et al.*, 2021).

While the evaluation of multi-model averages was widespread over all existing, and therefore not necessarily credible, models earlier (see, e.g., Stocker *et al.*, 2013), the use of a multi-model ensemble restricted to credible models, as suggested in Suarez-Gutierrez *et al.* (2021), can be considered meaningful. Averaging over averages of individual credible ensembles might shed light on uncertainties arising from individually chosen parametrizations in the different models.

Concerning nonlinear systems subjected to a parameter drift of non-negligible rate, a credibility condition has also been formulated here: the measured signal should stretch across the band of a converged ensemble simulation. The credibility of a probabilistic prediction in such a model would imply the appearance of a figure similar to Fig. 2 here, in the case of a monotonic parameter drift. The problem of the large number of global quantities is, however, not present in low-dimensional models with, of course, a low number of variables (in our pendulum, φ and ω).

As a summary, we can conclude that ensemble simulations both in climate dynamics and in low-dimensional systems have three phases when evolving from a localized initial distribution: a first phase where the sensitivity to initial conditions manifests itself in the form of plume diagrams, an intermediate phase where the ensemble converges further toward the time-dependent attractor, and a last, converged phase where the ensemble has practically reached the attractor. Statistically reliable statements can only be obtained from such converged ensembles.

Finally, we emphasize that the lack of ensemble experiments on low-dimensional systems subjected to parameter drift, together with the existence of a number of climate ensemble simulations proven to be credible with the observed mean surface temperature, can lead to the conclusion: due to the nature of climate research, under the condition that these models turn out to be credible in all relevant global quantities, climate modeling can be considered to be moved further than current dynamical system research. Within the latter, the challenge is to conduct careful experiments in chaotic-like systems subjected to parameter drift, comparing the measured dataset with the band produced by an ensemble simulation, and check their consistency.

To answer the question, "Where are the coexisting parallel climates" in climate research, or, in the language of Jánosi *et al.* (2021), "Where are the coexisting parallel dynamical histories" in nonlinear science, we can say that they are present indeed in any credible ensemble simulation, even if not in an individual sense, rather in the form of the probabilities generated by the dynamics.

SUPPLEMENTARY MATERIAL

See the supplementary material for more details on the models used, the convergence and size dependence of the ensemble results, and probabilistic aspects derived from CESM1-LE realizations.

ACKNOWLEDGMENTS

Special thanks are due to C. Deser for the careful reading of the manuscript and providing useful contextual suggestions. Helpful discussions with G. Drótos, I. M. Jánosi, and M. Vincze are acknowledged. We would like to thank D. Jánosi for carrying out the simulation of the driven pendulum. We also thank the observation of one of the anonymous reviewers, which is reflected in the addition of the last but second paragraph of Sec. VI. This work was supported by the National Research, Development and Innovation Office (NKFIH) under Grant Nos. FK135115 (M.H.) and K125171 (T.H., T.T., and M.H.). M.H. was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences. The authors acknowledge the CESM Large Ensemble Community Project and supercomputing resources provided by NSF/CISL/Yellowstone and wish to thank the Climate Data Gateway at NCAR for providing access to the output of the CESM1-LE climate model.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

M. Herein: Conceptualization (equal); Visualization (lead); Writing – original draft (equal); Writing – review & editing (equal). **T. Tél:** Conceptualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **T. Haszpra:** Conceptualization (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The CESM1-LE data can be accessed at http://www.cesm.ucar.edu/projects/community-projects/MMLEA/ (see Deser *et al.*, 2020a). Data that support the findings of this study, including those of pendulum dynamics, are available from the corresponding author upon reasonable request.

REFERENCES

Amon, A. and Lefranc, M., "Topological signature of deterministic chaos in short nonstationary signals from an optical parametric oscillator," Phys. Rev. Lett. **92**, 094101 (2004).

Bach, E., Mote, S., Krishnamurthy, V., Sharma, A. S., Ghil, M., and Kalnay, E., "Ensemble oscillation correction (EnOC): Leveraging oscillatory modes to improve forecasts of chaotic systems," J. Clim. **34**, 5673–5686 (2021).

Baker, G. L. and Gollub, J. P., *Chaotic Dynamics: An Introduction*, 2nd ed. (Cambridge University Press, Cambridge, 2012).

Bódai, T., Drótos, G., Ha, K.-J., Lee, J.-Y., and Chung, E.-S., "Nonlinear forced change and nonergodicity: The case of ENSO-Indian monsoon and global precipitation teleconnections," Front. Earth Sci. **8**, 599785 (2021).

Bódai, T., Drótos, G., Herein, M., Lunkeit, F., and Lucarini, V., "The forced response of the El Niño–Southern Oscillation–Indian Monsoon Teleconnection in ensembles of earth system models," J. Clim. **33**(6), 2163–2182 (2020).

Bódai, T. and Tél, T., "Annual variability in a conceptual climate model: Snapshot attractors, hysteresis in extreme events, and climate sensitivity," Chaos **22**, 023110 (2012).

Branstator, G. and Teng, H., "Two limits of initial-value decadal predictability in a CGCM," J. Clim. **23**, 6292–6311 (2010).

Chancellor, R. S., Alexander, R. M., and Noah, S. T., "Detecting parameter changes using experimental nonlinear dynamics and chaos," J. Vib. Acoust. **118**, 375–383 (1996).

Chatteerjee, A., Cusumano, J. P., and Chelidze, A., "Optimal tracking of parameter drift in a chaotic system: Experiment and theory," J. Sound Vib. **250**, 877–901 (2002).

Collins, M., "Ensembles and probabilities: A new era in the prediction of climate change," Philos. Trans. R. Soc. A **365**(1857), 1957–1970 (2007).

Collins, M., Knutti, R., Arblaster, J., Dufresne, J.-L., Fichefet, T., Friedlingstein, P., Gao, X. *et al.*, "Long-term climate change: Projections, commitments and irreversibility," in *Climate Change 2013—The Physical Science Basis: Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, edited by T. F. Stocker, D. Qin, G.-K. Plattner, M. M. B. Tignor, S. K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex, and P. M. Midgley (Cambridge University Press, 2013), pp. 1029–1136.

Danabasoglu, G., Lamarque, J.-F., Bacmeister, J., Bailey, D. A., DuVivier, A. K., Edwards, J. *et al.*, "The community earth system model version 2 (CESM2)," J. Adv. Model. Earth Syst. **12**, e2019MS001916 (2020).

Deppner, C. *et al.*, "Collective-mode enhanced matter-wave optics," Phys. Rev. Lett **127**, 100401 (2021).

Deser, C., "Certain uncertainty: The role of internal climate variability in projections of regional climate change and risk management," Earth's Future **8**, e2020EF001854, https://doi.org/10.1029/2020EF001854 (2020).
Deser, C., Lehner, F., Rodgers, K. B. *et al.*, "Insights from earth system model initial-condition large ensembles and future prospects," Nat. Clim. Change **10**, 277–286 (2020).
Deser, C., Phillips, A. S., Bourdette, V. *et al.*, "Uncertainty in climate change projections: The role of internal variability," Clim. Dyn. **38**, 527–546 (2012).
Deser, C., Phillips, A. S., Simpson, I. R., Rosenbloom, N., Coleman, D., Lehner, F., Pendergrass, A. G., DiNezio, P., and Stevenson, S., "Isolating the evolving contributions of anthropogenic aerosols and greenhouse gases: A New CESM1 large ensemble community resource," J. Clim. **33**(18), 7835–7858 (2020b).
Drótos, G. and Bódai, T., "On defining climate by means of an ensemble," Earth Space Sci. Open Arch. **20** (2022).
Drótos, G., Bódai, T., and Tél, T., "Probabilistic concepts in a changing climate: A snapshot attractor picture," J. Clim. **28**, 3275–3288 (2015).
Drótos, G., Bódai, T., and Tél, T., "On the importance of the convergence to climate attractors," Eur. Phys. J. Spec. Top. **226**, 2031–2038 (2017).
Eckmann, J. P. and Ruelle, D., "Ergodic theory of chaos and strange attractors," Rev. Mod. Phys. **57**, 617 (1985).
Eyring, V., Bony, S., Meehl, G. A., Senior, C. A., Stevens, B., Stouffer, R. J., and Taylor, K. E., "Overview of the Coupled Model Intercomparison Project Phase 6 (CMIP6) experimental design and organization," Geosci. Model Dev. **9**, 1937–1958 (2016).
Fraedrich, K., Jansen, H., Kirk, E., Luksch, U., and Lunkeit, F., "The planet simulator: Towards a user friendly model," Meteorol. Z. **14**(3), 299–304 (2005).
Ghil, M., Chekroun, M. D., and Simonnet, E., "Climate dynamics and fluid mechanics: Natural variability and related uncertainties," Phys. D **237**, 2111–2126 (2008).
Ghil, M. and Lucarini, V., "The physics of climate variability and climate change," Rev. Mod. Phys. **92**, 035002 (2020).
Gleick, J., *Chaos: Making a New Science* (Viking Press, New York, 1987).
Gould, S. J., *Wonderful Life: The Burgess Shale and the Nature of History* (W. W. Norton & Co, New York, 1989).
Haszpra, T., Herein, M., and Bódai, T., "Investigating ENSO and its teleconnections under climate change in an ensemble view—A new perspective," Earth Syst. Dyn. **11**, 267–280 (2020).
Herein, M., Drótos, G., Haszpra, T., Márfy, J., and Tél, T., "The theory of parallel climate realizations as a new framework for teleconnection analysis," Sci. Rep. **7**, 44529 (2017).
Herein, M., Márfy, J., Drótos, G., and Tél, T., "Probabilistic concepts in intermediate-complexity climate models: A snapshot attractor picture," J. Clim. **29**, 259–272 (2016).
Hurrell, J. W., Holland, M. M., Gent, P. R., Ghan, S., Kay, J. E., Kushner, P. J., Lamarque, J. F., Large, W. G., Lawrence, D., Lindsay, K., Lipscomb, W. H., Long, M. C., Mahowald, N., Marsh, D. R., Neale, R. B., Rasch, P., Vavrus, S., Vertenstein, M., Bader, D., Collins, W. D., Hack, J. J., Kiehl, J., and Marshall, S., "The community earth system model: A framework for collaborative research," Bull. Am. Meteorol. Soc. **94**(9), 1339–1360 (2013).
Jahanshai, H. *et al.*, "Simulation and experimental validation of a non-equilibrium chaotic system," Chaos, Solitons Fractals **143**, 110539 (2021).
Jánosi, D., Károlyi, G., and Tél, T., "Climate change in mechanical systems: The snapshot view of parallel dynamical evolutions," Nonlinear Dyn. **106**, 2781–2805 (2021).
Janosi, I. M. and Vattay, G., "Soft turbulent state of the atmospheric boundary layer," Phys. Rev. A **46**, 6386 (1992).
Kay, J. E. *et al.*, "The community earth system model (CESM) large ensemble project: A community resource for studying climate change in the presence of internal climate variability," Bull. Am. Meteorol. Soc. **96**, 1333–1349 (2015).
Kilic, C., Lunkeit, F., Raible, C. C., and Stocker, T. F., "Stable equatorial ice belts at high obliquity in a coupled atmosphere-ocean model," Astrophys. J. **864**(2), 106 (2018).
Kirchmeier-Young, M. C., Zwiers, F. W., and Gillett, N. P., "Attribution of extreme events in Arctic sea ice extent," J. Clim. **30**(2), 553–571 (2017).
Lai, Y.-C. and Tél, T., *Transient Chaos* (Springer, New York, 2011).
Lamarque, J.-F. *et al.*, "Historical (1850–2000) gridded anthropogenic and biomass burning emissions of reactive gases and aerosols: Methodology and application," Atmos. Chem. Phys. **10**, 7017–7039 (2010).
Lee, J.-Y. and Bódai, T., "Future changes of the ENSO–Indian summer monsoon teleconnection," in *Indian Summer Monsoon Variability*, edited by J. Chowdary, A. Parekh, and C. Gnanaseelan (Elsevier, 2021), Chap. 20, pp. 393–412, ISBN 9780128224021.
Lehner, F., Deser, C., Maher, N., Marotzke, J., Fischer, E., Brunner, L. *et al.*, "Partitioning climate projection uncertainty with multiple large ensembles and CMIP5/6," Earth Syst. Dyn. **11**, 491–508 (2020), Special Issue on Large Ensembles.
Leith, C. E., "Predictability of climate," Nature **276**, 352–355 (1978).
Lorenz, E. N., "Deterministic non-periodic flow," J. Atmos. Sci. **20**, 130–141 (1963).
Madan, R. N. and Wu, C. W., "Introduction to experimental chaos using Chua's circuit," in *Chua's Circuit: A Paradigm for Chaos*, edited by R. N. Madan (World Scientific, Singapore, 1993).
Maher, N. *et al.*, "The Max Planck institute grand ensemble-enabling the exploration of climate system variability," J. Adv. Model. Earth Syst. **11**, 2050–2069 (2019).
Maher, N., Milinski, S., and Ludwig, R., "Large ensemble climate model simulations: Introduction, overview, and future prospects for utilising multiple types of large ensemble," Earth Syst. Dyn. **12**, 401–418 (2021).
Marotzke, J. and Forster, P. M., "Forcing, feedback and internal variability in global temperature trends," Nature **517**, 565 (2015).
Masoller, C., Figliola, A., Giudici, M., Tredicce, J. R., and Abraham, N. B., "Wavelet analysis of low frequency fluctuations of a semiconductor laser," Opt. Commun. **157**, 115–120 (1998).
McKinnon, K. A. and Deser, C., "Internal variability and regional climate trends in an observational large ensemble," J. Clim. **31**, 6783–6802 (2018).
McKinnon, K. A. and Deser, C., "The inherent uncertainty of precipitation variability, trends and extremes due to internal variability, with implications for Eastern U.S.: Water resource," J. Clim. **34**, 9605–9622 (2021).
McKinnon, K. A., Poppick, A., Dunn-Sigouin, E., and Deser, C., "An 'observational large ensemble' to compare observed and modeled temperature trend uncertainties due to internal variability," J. Clim. **30**, 7585–7598 (2017).
Meehl, G. A. *et al.*, "Decadal prediction," Bull. Am. Meteorol. Soc. **90**, 1467–1486 (2009).
Meinshausen, M. *et al.*, "The RCP greenhouse gas concentrations and their extensions from 1765 to 2300," Clim. Change **109**, 213–241 (2011).
Milinski, S., Maher, N., and Olonscheck, D., "How large does a large ensemble need to be?," Earth Syst. Dyn. **11**(4), 885–901 (2020).
Morice, C. P., Kennedy, J. J., Rayner, N. A., Winn, J. P., Hogan, E., Killick, R. E. *et al.*, "An updated assessment of near-surface temperature change from 1850: The HadCRUT5 data set," J. Geophys. Res., Atmos. **126**, e2019JD032361, https://doi.org/10.1029/2019JD032361 (2021).
Oldenburgh, G. J. v., Doblas Reyes, F. J., Drijfhout, S. S., and Hawkins, E., "Reliability of regional climate model trends," Environ. Res. Lett. **8**, 014055 (2013).
Ott, E., *Chaos in Dynamical Systems* (Cambridge University Press, Cambridge, 1993).
Ottino, J. M., Metcalfe, G., Jana, S. C. *et al.*, "Experimental studies of chaotic mixing," in *Proceedings of the 2nd Experimental Chaos Conference*, edited by W. Ditto (World Scientific, Singapore, 1995).
Papalexiou, S. M., Rajulapati, C. R., Clark, M. P., and Lehner, F., "Robustness of CMIP6 historical global mean temperature simulations: Trends, long-term persistence, autocorrelation, and distributional shape," Earth's Future **8**, e2020EF001667, https://doi.org/10.1029/2020EF001667 (2020).
Pierini, S., "Statistical significance of small ensembles of simulations and detection of the internal climate variability: An excitable ocean system case study," J. Stat. Phys. **179**, 1475 (2020).

Presse, S., Ghosh, K., Lee, J., and Dill, K. A., "Principles of maximum entropy and maximum caliber in statistical physics," Rev. Mod. Phys. **85**, 1115–1141 (2013).
Rodgers, K. B., Lee, S.-S., Rosenbloom, N., Timmermann, A., Danabasoglu, G., Deser, C., Edwards, J., Kim, J.-E., Simpson, I. R., Stein, K., Stuecker, M. F., Yamaguchi, R., Bódai, T., Chung, E.-S., Huang, L., Kim, W. M., Lamarque, J.-F., Lombardozzi, D. L., Wieder, W. R., and Yeager, S. G., "Ubiquity of human-induced changes in climate variability," Earth Syst. Dyn. **12**, 1393–1411 (2021).
Romeiras, F. J., Grebogi, C., and Ott, E., "Multifractal properties of snapshot attractors of random maps," Phys. Rev. A **41**, 784 (1990).
Schulzweida, U., *CDO User Guide (Version 2.0.0)* (Zenodo, 2021).
Stocker, T., Qin, D., Plattner, G. K., Tignor, M., Allen, S., Boschung, J., Nauels, A., Xia, Y., Bex, V., and Midgley, P., *IPCC, Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change* (Cambridge University Press, Cambridge, 2013).
Suárez-Gutiérrez, L., Li, C., Thorne, P. W., and Marotzke, J., "Internal variability in simulated and observed tropical tropospheric temperature trends," Geophys. Res. Lett. **44**, 5709–5719, https://doi.org/10.1002/2017GL073798 (2017).
Suárez-Gutiérrez, L., Milinski, S., and Maher, N., "Exploiting large ensembles for a better yet simpler climate model evaluation," Clim. Dyn. **57**, 2557–2580 (2021).
Swart, N. C., Cole, J. N. S., Kharin, V. V., Lazare, M., Scinocca, J. F., Gillett, N. P., Anstey, J., Arora, V., Christian, J. R., Hanna, S., Jiao, Y., Lee, W. G., Majaess, F., Saenko, O. A., Seiler, C., Seinen, C., Shao, A., Sigmond, M., Solheim, L., von Salzen, K., Yang, D., and Winter, B., "The Canadian Earth System Model version 5 (CanESM5.0.3)," Geosci. Model Dev. **12**, 4823–4873 (2019).
Taylor, K. E., Stouffer, R. J., and Meehl, G. A., "An overview of CMIP5 and the experiment design," Bull. Am. Meteorol. Soc. **93**, 485–498 (2012).
Tél, T., Bódai, T., Drótos, G., Haszpra, T., Herein, M., Kaszás, B., and Vincze, M., "The theory of parallel climate realizations," J. Stat. Phys. **179**(5–6), 1496–1530 (2020).
Tél, T. and Gruiz, M., *Chaotic Dynamics* (Cambridge University Press, Cambridge, 2006).
Tokarska, K. B., Stolpe, M. B., Sippel, S., Fischer, E. M., Smith, C. J., Lehner, F., and Knutti, R., "Past warming trend constrains future warming in CMIP6 models," Sci. Adv. **6**(12), eaaz9549 (2020).
Vallis, G. K., *Atmospheric and Oceanic Fluid Dynamics* (Cambridge University Press, Cambridge, 2017).
van Vuuren, D. P. *et al.*, "The representative concentration pathways: An overview," Clim. Change **109**, 5–31 (2011).
Vincze, M., Dan Borcia, I., and Harlander, U., "Temperature fluctuations in a changing climate: An ensemble-based experimental approach," Sci. Rep. **7**(9), 254 (2017).
Vincze, M. *et al.*, "Benchmarking in a rotating annulus: A comparative experimental and numerical study of baroclinic wave dynamics," Meteorol. Z. **23**, 611 (2015).
Yagasaki, K., "Bifurcation and chaos in a quasi-periodically forced beam: Theory, simulation and experiment," J. Sound Vib. **183**, 1–31 (1995).