Accurate reactions open up the way for more cooperative societies

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We consider a prisoner's dilemma model where the interaction neighborhood is defined by a square lattice. Players are equipped with basic cognitive abilities such as being able to distinguish their partners, remember their actions, and react to their strategy. By means of their short-term memory, they can remember not only the last action of their partner but the way they reacted to it themselves. This additional accuracy in the memory enables the handling of different interaction patterns in a more appropriate way and this results in a cooperative community with a strikingly high cooperation level for any temptation value. However, the more developed cognitive abilities can only be effective if the copying process of the strategies is accurate enough. The excessive extent of faulty decisions can deal a fatal blow to the possibility of stable cooperative relations.

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I. INTRODUCTION

The conundrum of cooperation keeps the scientific community occupied for a long time [1–9]. The way selfish individuals reach the solution to cooperate in order to optimize their own income is a phenomenon widely observable in nature; however, it is rather hard to formulate in a mathematical way. Scientists of many disciplines—mathematics, physics, biology, economy, and, among others, sociology—keep adding contributions to understand this puzzle more thoroughly.

Evolutionary game theory [10–12] offers an excellent playground to test theories and develop models to study the evolution of cooperation. In this framework, it is possible to analyze the behavior of individuals in their natural environment: populations. The harshest dilemma situation to study is offered by the prisoner's dilemma (PD) game [1,3,5]. In this game, both interacting individuals have the option to pay the cost c in order to provide a greater benefit b to the partner. The dilemma lies in the fact that an individual is always better off by not paying the cost, independently of the decision of her partner. If the partner sacrifices the cost, then she would get the whole benefit b, instead of the payoff $b - c$, which she would get for a mutually cooperative choice. If the partner withholds the donation, then she gets nothing, but it is still a better outcome for her than the $-c$ cost for the decision to support a defective partner. However, if both of them follow this rational reasoning, they receive nothing instead of the positive income $b - c$ for mutual cooperation. Thus, the pure mathematical analysis of the PD predicts the reign of defective behavior.

This prediction is quite different from what we observe in nature. Consequently, game theorists improved the model to contain more, realistic elements of situations where cooperation is at stake. The analysis of kin selection [13], reputation effects [14–17], direct and indirect reciprocity [4,18,19], voluntary participation [20,21], and different punishment methods [22–28] could help explain many aspects of a cooperative society. However, our knowledge does not seem to be complete yet. One of the attempts was the introduction of structured populations [29–34]. In these models, individuals cannot interact with any member of the community, only with their acquaintances. As a consequence of this, cooperation could have emerged due to the possibility that cooperative players could form clusters and support each other within the cluster. At the boundaries of the cluster, defective players try to break in but cooperators' higher payoff earned from mutually cooperative individuals can keep the cluster intact. This window of opportunity for cooperation to emerge is, however, a narrow one; it only works for a small set of payoff parameters and even for these values, the level of cooperation is quite low in the whole population. Many different types of population structures such as lattices [29,31], random graphs [35], small world networks [36,37], scale-free networks [38], etc., were studied to describe various connectivity situations and some of them provided additional mechanisms to enhance the level of cooperation due to the specific structure. In the present paper, we use a simpler underlying structure, the square lattice, in order to be able to focus on other features of the model.

In some of our recent papers [39,40], we gave up the traditional game theoretical approach of unconditional strategies and endowed our players with incipient cognitive capabilities. Players were able to distinguish their coplayers, remember their last action and take this last action into account when they decided what move they should choose towards the given coplayer. Even these incipient cognitive abilities were sufficient to tip the scales in favor of cooperation's success. In this paper, we take a further step into this direction by improving the cognitive capabilities of players so that they can take into account not only the opponents' last move but also their own. This seemingly small modification basically doubles the capacity needed to react to different outcomes; the players have to possess a scheme for all four possible outcomes of a one-shot PD interaction. In addition, the memory capacity has to be also improved as now not only the opponent's last move is stored but the player's own decision towards the partner as well. However, this "investment" opens up new possibilities: It enables the fine tuning of decisions, situations can be evaluated more precisely, and the appropriate action can be chosen more accurately. This presents itself in the measure of cooperation: It reaches even higher levels and blooms in any payoff parameter range. The improved efficiency is not without a price, though; it requires accurate data to process. In a noisy environment, where individuals' decisions do not always reflect their intentions, overthinking may be disadvantageous for cooperation.

II. THE MODEL

We consider a spatial PD game where individuals (the players) engage in one-shot games with their neighbors. In a PD game, the two participants have to choose between two options—to cooperate (C) or to defect (D)—and they receive a payoff depending on their simultaneous decisions. A cooperator pays the cost c to grant the partner the greater benefit b. Mutual cooperation (defection) thus yields the reward $R = b - c$ (punishment $P = 0$) to both players, whereas if one player cooperates and the other defects, then the former gets the sucker's payoff $S = -c$ and the latter earns the temptation to defect $T = b$. We have added the original T, R, P, S payoff terminology to make the strategy parameter notation easier to read; moreover, the similarity between the donation game and the PD is more apparent this way. In the donation game scenario, the payoff values automatically satisfy the PD payoff ranking. In order to facilitate the analysis, we fix the reward for mutual cooperation to $b - c = 1$, creating a relation between b and c, thus reducing the number of payoff parameters to one. In this way using only the parameter b, the payoffs are b for a successful defector, 1 for mutual cooperation, 0 for mutual defection, and $1 - b$ for an exploited cooperator.

In the spatial setting, players are located on the nodes of a square lattice. The edges of the lattice define the interaction and imitation neighborhood. Players earn their accumulated payoff from one-shot games with their four neighbors. They are able to distinguish their partners and take different actions towards them. Players have a one-step memory for each of their partners; i.e., they can remember the outcome of their last encounter. The strategy of a player in the spatial position x is thus given by a four-element vector $\mathbf{p}_\mathbf{x} = (p_R, p_S, p_T, p_P)$. Each element of the vector defines the probability of future cooperation with a neighbor from whom Player x earned the payoff R, S, T, or P, respectively, during their last interaction (see Fig. 1 for the explanatory meaning of the parameters). In other words, the four strategy parameters indicate the probability of subsequent cooperation following the four different possible decision pairs (CC, CD, DC, DD) of an interaction. We refer to these strategies as mem-1 strategies [41,42] (indicating their memory capacity) opposing to the *reactive* strategies that can only remember the last decision of their partner [43–45]. This new strategy space is huge; a strategy can be drawn anywhere from the four-dimensional unit cube. For simulation reasons, we discretize the strategy parameter space: Each vector element can take values from 0 to 1 in 0.01 steps; i.e., the strategy vector of a player takes the form $(i \cdot 0.01, j \cdot 0.01, k \cdot 0.01, l \cdot 0.01)$, where $0 \leqslant i,j,k,l \leqslant 100$. This results in more than 10^8 possible strategies in this discretized strategy space. Computer simulations are started from an initial state where players are assigned random strategy parameter values. Given the lack of past encounters at the start of the simulation, individuals cooperate with probability $(p_R + p_S + p_T + p_P)/4$ in the first step or defect with probability $1 - (p_R + p_S + p_T + p_P)/4$. In each interaction, players make decisions depending on their strategy parameters, their last action towards the partner, and the latest action of the partner.

In an elementary simulation step, two neighboring individuals (x and y) are picked randomly and their accumulated payoff

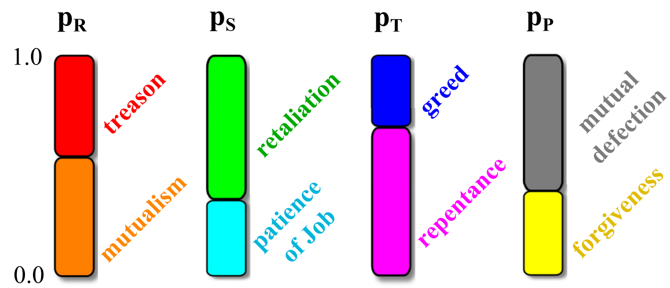

FIG. 1. (Color online) Explanatory meaning of the strategy parameters. The probability of cooperating after a mutually cooperative encounter (p_R) can be described as mutualism, the complementary probability (the probability of defection) $(1 - p_R)$ as treason. Cooperating (p_S) after being a victim of a successful defection is the patience of Job (in line with Christianity's "turn the other cheek"), while paying with defection to the defective partner is qualified as retaliation $(1 - p_S)$. The probability of cooperating after successfully exploiting the partner is the propensity for repentance (p_T) and the probability of continued defection $(1 - p_T)$ can be described as greed. Finally, acting friendly after a mutually defective round (p_P) is a measure of forgiveness, while for keeping up the defective series of decisions $(1 - p_P)$ the term could be "negative mutualism."

is calculated. Player x adopts the strategy of player y according to the pairwise comparison rule with probability $W(x \leftarrow y) = \frac{1}{1+\exp[(P_x - P_y)/K]}$. P_x and P_y stand for the individual payoff of the players and K is associated with errors in decision making that can be originated from various sources such as emotions, free will, fluctuation in the payoffs, and external effects (noise). As a consequence of K, it is possible to occasionally adopt the strategy of a player who performed worse than the focal individual, although most of the time the more successful players are imitated. Whenever a player adopts the strategy of one of her neighbors, the new strategy parameters are determined by a normal distribution with standard deviation σ centered on the adopted values, i.e., $p'_{R_x} = p_{R_y} + \xi_1(\sigma)$ [with $\xi_1(\sigma)$ being a normally distributed random variable with zero mean and standard deviation σ] and likewise for the other three strategy vector elements. This adoption method models a slight blur in the perception and/or an inaccuracy in the copying or succession process. Moreover, it helps avoid the stochastic extinction of strategies and eliminates the pairwise rule's imperfection that it does not introduce new strategies, as this could cause serious issues in the case of so huge a strategy space. In a full Monte Carlo step (MCS), each individual has the chance to change her strategy once on average. We emphasize that this is not an iterated PD game in the sense that players have the option to revise their strategy after each interaction; they do not play long series of PD games with each other before reconsidering their strategies. On the other hand, players play with the same partners all the time as the interaction network is fixed, so in this sense the interactions are repeated.

We have performed extensive computer simulations on a square lattice of the size $N = 100 \times 100$ with periodic boundary conditions taking into account the four nearest neighbors as interaction partners (von Neumann neighborhood). The chosen system size is sufficient as the blurred strategy adoption method prevents the formation of huge clusters of identical players; thus, the correlation length does not approach the system size. Starting from the random initial condition, the system evolves for a transient period of 20 000 generations; then we record the appropriate quantities (number of cooperative and/or defective decisions, strategy parameter values, etc.) in the population for 100 000 MCS and calculate the respective averages. The system reaches the (quasi-)stationary state quite fast, yet, the long averaging time is needed as the selection pressure is low on some of the strategy components and their values drift slowly. Averaging from different initial conditions is not needed because, due to the defined adoption method, the system evolves to the same state. This statement is true even if the population is started from an absolutely defective state, with all players having the ($p_R = 0, p_S = 0, p_T = 0, p_P = 0$) strategy vector. The dynamics and the data collection for reactive strategies were similar to the mem-1 strategy case. The K parameter associated with errors in decision making was set to 0.4, a value that was advantageous for cooperation in the scenario when only unconditional strategies were allowed [31]. The established final state is qualitatively the same for synchronous and asynchronous updating.

III. RESULTS AND DISCUSSION

We have displayed the level of cooperation in Fig. 2 for different σ and temptation values when the population consists of mem-1 or reactive strategies. We have plotted the results for extremely large σ values ($\sigma = 0.03$ and 0.05) to show how detrimental unfaithful copying can prove for cooperation. This effect is present for both reactive and mem-1 strategy types.

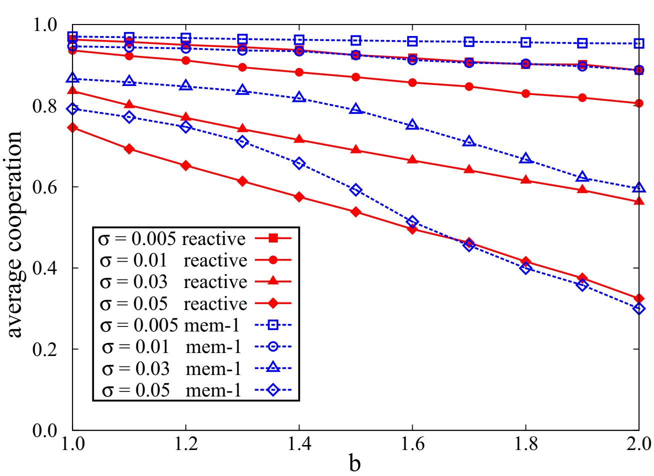

FIG. 2. (Color online) The level of cooperation as a function of the temptation to defect b for different σ values. Dashed blue curves with open symbols show the results for the mem-1 strategies, while solid red curves with solid symbols stand for the less cognitive *reactive* strategies. Blue curves run above red ones (for the same σ parameter): the more accurate handling of specific situations can help create more cooperative societies. However, as the inaccuracy in the copying process grows (increasing σ) the level of cooperation drops for increasing b values.

In comparison to mem-1 strategies, reactive strategies use only two strategy parameters: p, the probability of cooperation after the partner cooperated in the previous round (associated with mutualism), and q, the probability of cooperation after the partner defected in the previous round (associated with forgiveness). As a consequence of this, they cannot assess exactly why a partner made a specific decision towards them. They do not (and cannot) differentiate between situations like rightful retaliation, continuous defection, or treason.

Using these strategy sets, there are no unconditional strategies, and, as such, no "real" cooperators, in the system; therefore, the level of cooperation is defined as the average fraction of cooperative decisions made by the players. It can be seen that mem-1 strategies achieve more cooperative societies for almost any σ and b parameter values; the increased level of incipient cognition enables the more accurate handling of situations and as a consequence it can partly eliminate the decisions originated from faulty perception or "misunderstandings." The measure of cooperation decreases monotonously with the increase of the temptation to defect b. This is a natural effect widely observable in PD models: The support to defective behavior enhances their chances in the evolutionary process. For low σ values, i.e., for faithful copying of more successful strategies, this support has a very small effect on the final outcome of the evolution in the whole b range. The cause for this behavior is that the level of cooperation is very close to 100% in these cases; i.e., individuals hardly ever try to defect. Moreover, individuals use an imitation based strategy adoption rule; consequently, the payoff value for a successful defection (b) does not play a significant role. For higher σ values, however, it begins to act, and cooperation drops significantly with increasing b.

The very high level of cooperation indicates that the interaction pattern largely consists of mutually cooperative relations. In other words, p_R is close to 1 in the whole population. Players with such a strategy parameter decide basically deterministically after a mutually cooperative move with their partner. The effect of increasing σ, an unfaithful strategy adoption, is most prominent in such a case because it can change the deterministic behavior to stochastic. Consequently, occasional defective decisions can break the mutually cooperative chains and move the population from the fully cooperative state. The higher σ is, the higher the probability is for lower p_R parameters to be adopted during a strategy imitation step (cf. the solid red line in Fig. 4) and, as a consequence, more and more "unintentional" defection can occur in the population. With defection present, the evolutionary advantage originating from a higher b can be expressed in the average cooperation level as well; thus, in this case it decreases as b gets larger. For extremely large σ values, this advantage can become so impressive that even full defectors—(0,0,0,0) players—can appear in the system. However, due to the players' capabilities to react to different neighbors differently, they can identify the defectors and can protect the cooperative cluster from them. As a result, a dynamically changing cluster structure can be observed in the population that yields the displayed cooperation levels.

This analysis only considers the final state of the population. What happens during the evolutionary process? In the beginning, in the random initial state, small—more or less—

cooperative groups are formed and with the help of mutually cooperative links from each other, they can achieve a slight payoff advantage and spread their strategy; the population starts to homogenize. When the whole society consists of the same (or a very similar) strategy—this is the case when the simulation is started with exclusively defective players as well—a mutationlike process starts to act: Individuals try to imitate each other and during this process, due to the inherent inaccuracy in perception and/or copying, new breeds emerge. The more successful ones can spread and thus, step by step, the society eventually reaches the final state where the emerging new strategies only represent fluctuations around the optimal strategy. For this reason, the population evolves into the same final state independently of the initial strategy distribution.

Analyzing the stationary strategy parameter values in Fig. 3, it becomes clear why mem-1 strategies can achieve a higher level of cooperation and, as a consequence, higher average payoff on the population level. The main difference between

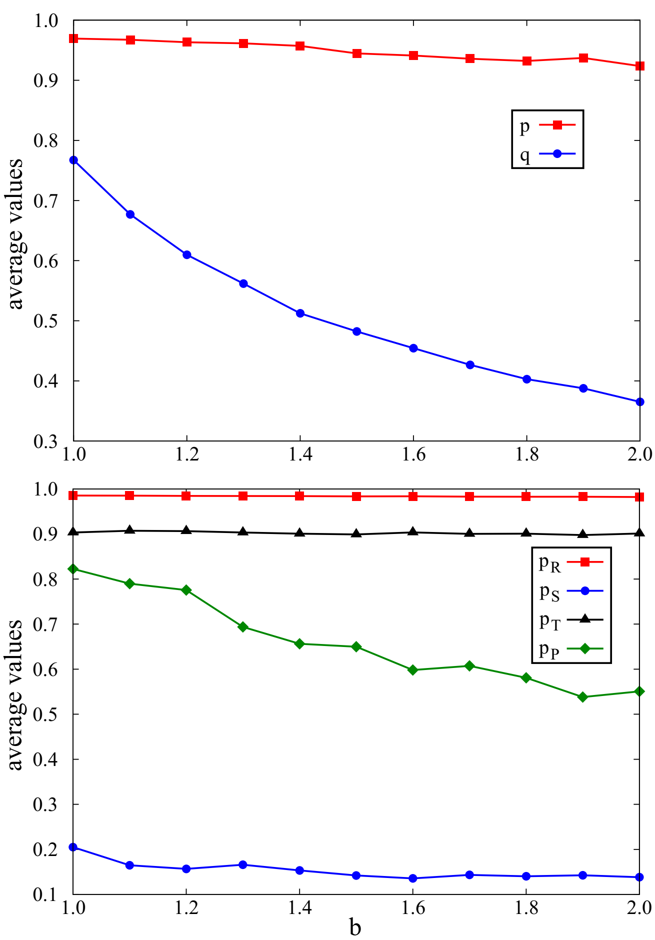

FIG. 3. (Color online) Average strategy parameter values for the reactive (top panel) and mem-1 (bottom panel) strategies as a function of the temptation b for $\sigma = 0.005$. It can be seen that having access to a more cognitive strategy arsenal enables the more accurate handling of specific situations. The average final strategies have similar characteristics in the two cases but the reactive strategies can only give "averaged reactions" to situations where mem-1 strategies fine tune their answers. Note the different scales on the y axis for the two panels.

reactive and mem-1 players is that the latter can differentiate between cases when their opponents defected against them. Exploitation can be met with retaliation (very low p_S value) but mutual defection chains can be interrupted by a milder, forgiving reaction: The average value of p_P is rather high. If the partner does not also switch to cooperation, then in the subsequent round, retaliation can ensue again. Reactive players do not have this option. They have only one parameter (q) handling these cases; thus they either decide to adopt a forgiving behavior to avoid costly defective chains (thereby being vulnerable to ruthless defectors) or they retaliate against most defective moves and risk the possibility of getting into long mutually punishing quarrels with (mostly) cooperative partners who accidentally defected once. As the Figure shows, reactive players try to balance between these two extremes and adopt an average behavior: Parameter q runs between the p_P and the p_S curves. The achieved level of cooperation proves that this behavior can be perfected using more developed behavioral parameters.

Concerning the other two strategy parameters, p_R and p_T, it can be seen that they are both steadily very close to the maximal value 1. We have already examined the very important role of p_R in keeping up the mutual cooperative chains and its vulnerability to σ. The behavior of the strategy parameter p_T—responsible for handling successful exploitations—is slightly more complicated to explain. One could imagine that after a successful defective move it pays off to keep defecting as this is the highest individual payoff that can be attained. However, this payoff cannot be kept up on the population level; thus, the dominant strategy aims to get hold of the second highest payoff. In order to achieve this, it tries to get back to mutual cooperation as soon as possible. The lower values of the other two strategy parameters do not contradict this endeavor as they basically deal with "defensive" tasks: They prevent the exploitation of the strategy.

To sum up, we can state that, in fact, only p_P is affected by the increase of b. This does not happen because of the increase of the temptation to defect ($T = b$) but through the jointly influenced sucker's payoff ($S = 1 - b$). This means that in the decrease of p_P the governing force is the fear from being exploited: Forgiving carries the risk of exploitation if the partner keeps defecting and the cost of being the victim of an exploitation increases with b.

For higher b and σ values the other average strategy parameters start to decline as well; however, it is not the result of the increase in b but the above mentioned appearance of (0,0,0,0) players that pulls down the averages.

It is worth noting that although having similar characteristics, the winning mem-1 strategies do not belong to the family of the recently discovered generous zero-determinant strategies [42,46]; such strategies should satisfy the $p_P + p_R = p_T + p_S$ relation.

Figure 4 illustrates how the strategy parameters are typically distributed in the stationary state. It can be seen that the distribution of p_R is sharply peaked while the other parameters have a flattened distribution. As we already mentioned, p_R is the most used strategy parameter in the stationary state; players mostly cooperate, forming mutually cooperative chains with each other. Thus, a small deviation from the optimal value in this parameter can have serious consequences; an occasional,

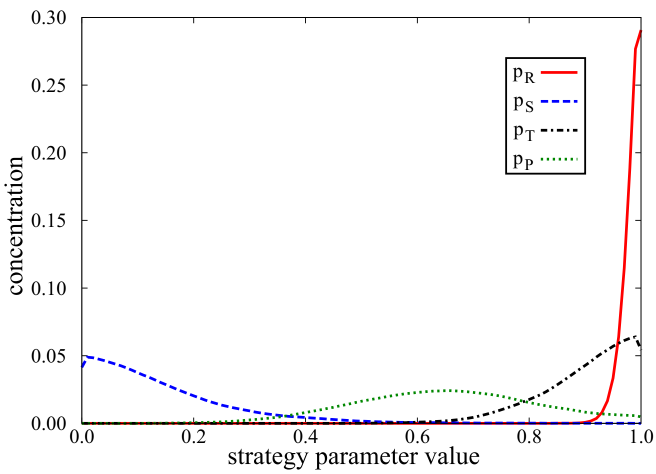

FIG. 4. (Color online) Distribution of the four strategy parameters in the stationary state for $b = 1.5$ and $\sigma = 0.005$. The most frequently used strategy parameter, p_R is exposed to the highest selection pressure. As players mostly mutually cooperate, a small deviation from the optimal value ($p_R = 1$) can have a high impact on their later interaction history. The other strategy parameters are only needed when the chain of mutual cooperation breaks in some form and, as such, the selection pressure is much lower on them; their distribution is flattened. The small cutoffs at the edges are artifacts of the strategy imitation method and do not influence the qualitative behavior of the model.

accidental defective move can cause serious loss for both players. These deviations can come from the mutationlike events of the strategy adoption process. As this kind of decision situation occurs often, players with suboptimal parameters are eliminated promptly. In other words, the selection pressure is high on p_R. The distribution is just as much diffused as the Gaussian strategy adoption (with half-width of σ) forces it to be. On the contrary, mutationlike events in less frequently used strategy parameters can go mostly unnoticed. For example, in a cooperative society, a small change in the p_P parameter (responsible for the handling of mutually defective cases) will not have a big impact; its exact value does not influence the accumulated payoff of the player so significantly. As a consequence, these distributions are much more flattened and broad.

IV. SUMMARY

We have studied a spatial evolutionary PD game with the so called mem-1 players who could recognize their partners; remember the outcome of their last interactions and act taking these information into account. Evolution selects a generous tit-for-tat-like strategy from the mem-1 strategy set that originates its success from the fact that it can give personalized reactions to its partners' decisions and inspire them to cooperate on the long run. We have compared the performance of these strategies to the efficiency of *reactive* strategies where players could only remember the last action of their partners, not the exact outcome of the encounter. We have found that the selected mem-1 strategies can outshine the already great success of reactive strategies and can establish even more cooperative societies. The price for the greater performance comes in the form of larger memory capacity and a more complex strategy description. This increased amount of cognition works great when the information available to the participating individuals is accurate. However, these strategies can prove very vulnerable to errors originated from various sources. Errors in perception or in the strategy adoption process can result in faulty decisions that can cause large turmoil in the interaction pattern and ruin the finely tuned behavioral answers. We have shown that the most important strategy parameter is the one responsible for handling the mutually cooperative scenarios; selection pressure is much lower on the other parameters.

The improvement in the cooperation level is the consequence of a small change in the memory usage of individuals: going from remembering the decision of the partners in the previous round to remembering all decisions in that round. With strategies that can remember the outcomes of more past rounds, this result can probably be enhanced; however, such strategies could be even more susceptible to copying errors, faulty decisions, or external noise due to the increased number of strategy parameters.

ACKNOWLEDGMENTS

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