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Application of mathematical models to Euro Standard passenger car curves for predicting the future of the market

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Abstract
In Europe, vehicle emissions are regulated by the Euro Standard legislation. Governments of different countries force automotive industries to adapt their vehicles to comply with these standards. The legislation is regularly updated to reduce vehicle emissions and control the issue of gas emissions. This study aims to examine previous regulations and adapt three different mathematical models to their market curves. The information collected proposes three possible predictions for the future Euro Standard. This will aid the anticipation of future regulations and enable adaptation to the demands of the future market of passenger cars.
Keywords
Euro Standard, mathematical model, prediction

1. Introduction

Predicting the future market penetration of passenger cars is crucial for several reasons:

- Business planning: Automakers, suppliers, and related industries must make strategic decisions regarding production, capacity, and investments. Accurate predictions of market penetration help them allocate resources effectively, optimise production levels, and plan for potential shifts in demand.
- Financial performance: Accurate market penetration predictions can impact the financial performance of companies. How well companies anticipate market trends and adapt their strategies can affect stock prices and investor confidence.
- Technology development: The automotive industry is rapidly evolving, with advancements in electric vehicles (EVs), autonomous driving, connectivity, and alternative fuel technologies. Predicting market penetration helps companies align their research and development efforts with future demand for these technologies.
- Policy and regulations: Governments and regulatory bodies use market penetration projections to design policies, incentives, and regulations that encourage the adoption of cleaner and more efficient vehicles. Accurate predictions can help set realistic targets for emissions reduction and sustainable transportation goals.
- Environmental impact: The transportation sector significantly contributes to greenhouse gas emissions and air pollution. Predicting future market penetration of electric and other eco-friendly vehicles helps assess the potential environmental impact and plan for a more sustainable future.
- Infrastructure planning: Adopting new vehicle technologies requires infrastructure development, such as charging stations for EVs or hydrogen refuelling stations for fuel cell vehicles. Predictions help stakeholders plan and invest in the necessary infrastructure to support changing market needs.
- Consumer behaviour: Understanding future market penetration helps automakers anticipate consumer preferences and demands. This insight allows them to tailor their marketing strategies, product offerings, and pricing to meet customer expectations.
- Competitor analysis: Market penetration predictions provide valuable information for competitive analysis. Companies can gauge their market share and understand how their competitors may respond to changing market dynamics.

Predicting the future market penetration of passenger cars is essential for making informed decisions, staying competitive, promoting sustainable growth, and contributing to a more environmentally friendly transportation system.

Currently, the latest standard in force is Euro Standard 6d. However, the study will focus on the Euro 3, Euro 4,Euro 5 and Euro 6 Standards to have a transparent and robust analysis. Their limits are the acceptable amount of exhaust gases to be used as a solid basis for upper estimation. Regulated emissions include carbon monoxide (CO), hydrocarbons (HC), nitrogen oxides (NO_X) and particulate matter (PM). Some of these pollutants contribute to the greenhouse effect, while others harm human health by causing respiratory and other problems (*Bereczky and Török, 2011*). The limits are measured as an emitted gram of pollutant per kilometre driven (*Tánczos and Török, 2008*). There is no fixed period, but from time to time, the regulations are revised, and the limits are tightened according to scientific evidence and environmental goals *(Figure 1)*. The regulations are adapted to the type of vehicle, distinguishing between petrol and diesel-powered vehicles:

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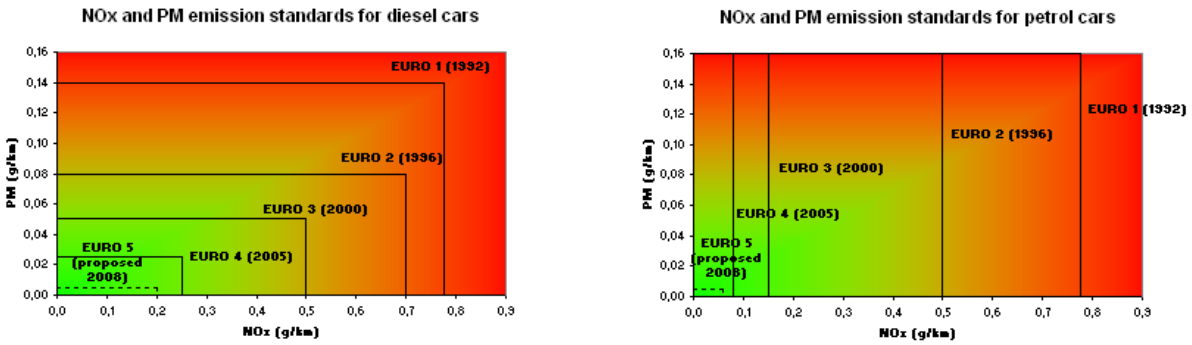

Figure 1: EURO emission reduction development example for NO_x and PM
(source: Academic Dictionaries and Encyclopedias)

The following investigation aims to study the registration curves of passenger cars that meet the vehicle emission standards set by the Euro Standard legislation using the Microsoft Excel Solver tool. The idea is to approach three different mathematical models to the previous Standard curves, collect information and, with this forecast (*Lekić et al., 2021*), the possible future Euro Standard 7 curve. The models selected have been: the Logistics Model, Gompertz Model and Gaussian probability distribution (*Fűr and Csete, 2010*).

2. The mathematical model

The models that will be used are the Logistic model, the Gompertz model and the probabilistic Gaussian distribution. The first two models are used in contexts of exponential population growth. At the same time, the latter model describes natural and human phenomena that tend to group around a central value or are symmetrical. While it is true that the case study is far from being a setting in which these models are typically used, the passenger car sales market can be assumed to be an analogy for population growth.

2.1 The logistic model

The logistic population growth model is a differential equation that describes the growth of a population as a function of time (*Radpour et al., 2021*). It is used when a population grows exponentially initially, but as it approaches the maximum carrying capacity of the environment, growth slows and eventually stabilises. The equation is as shown in (1):

$$N(t) = \frac{N_0 \cdot K}{N_0 + (K - N_0) \cdot e^{-rt}} \tag{1}$$

Where

$N(t)$ is the population at time t ;
N_0 is the population at the initial time ;
K is the maximum capacity of the population or the carrying capacity ;
r is the population growth rate.

2.2 Gompertz model

The Gompertz Model is a function used in various fields, such as biology or automobile market analysis (Rota et al., 2016). It is often used to model the growth of organisms or cells. The function has the formula shown in (2):

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$$f(t) = a \cdot e^{-be^{-ct}} \tag{2}$$

Where

f(t) is the value of the function at time *t*;
a represents the maximum asymptotic value that can be reached ;
b is related to the initial growth rate, a higher *b* indicates faster growth at the beginning;
c is associated with the decrease in growth rate as the population reaches the maximum limit. A higher value indicates a faster decline in growth rate.

2.3 Gaussian model

The Gaussian or normal probability distribution is one of the most widely used statistical distributions and describes a bell-shaped curve (*Wang et al., 2022*). Its equation is shown in (3):

$$f(t) = a \cdot e^{(-b \cdot (x-c)^2} \tag{3}$$

Where

f(t) is the value of the function at the point *t*;
a is the amplitude, a constant which affects the height of the bell;
b is a parameter that determines how fast the bell falls;
c is the central value of the bell, and it is known as the mean-

2.4 Excel Solver Tool

The tool used to achieve the analysis was Excel Solver. This tool has different functions. Among them, it allows the Excel environment to modify the value of a cell dependent on other cells by modifying the value of these other cells. The analysis will provide passenger car (PC) sales year by year for each Euro Standard. Subsequently, it will approximate each mathematical model to the real sales curves. It will minimise the cell containing the sum of the squared errors between reality and the mathematical models by modifying the values of the parameters of the mathematical models. The errors are quadratic to avoid cancellation between positive and negative values. Most smaller players will likely either be bought or merged with other big players. As the industry matures, cost efficiency will play a more significant and prominent role. If a smaller actor wants to stay independent, he must specialise.

3. Results

This section will present the Euro curves of standards 3, 4, 5, and 6. Thanks to the European vehicle market statistics pocketbook, the percentage of registered passenger cars (PC) that meet the different Euro Standards can be obtained, see *(Figure 2)*:

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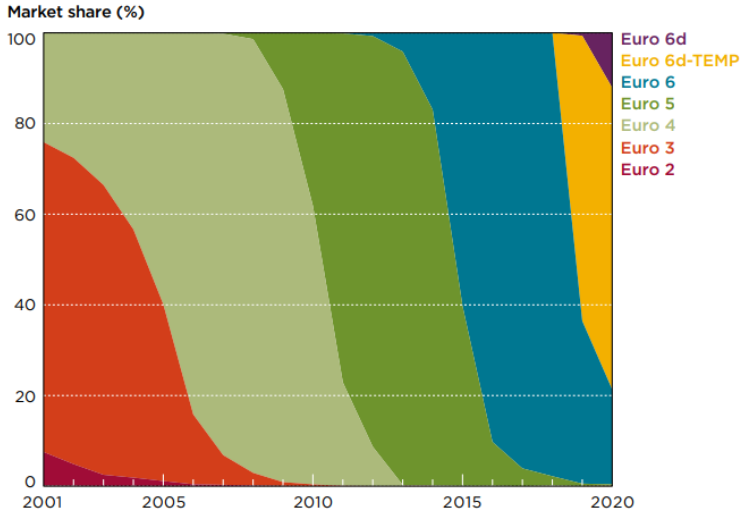

Figure 2. Percentage of new PC registrations
Source: European Vehicle Market Statistics Pocketbook 2021–2022 p50

Besides, the amount of newly registered passenger cars (PC) is known (*Figure 2*). With these sources of information, the number of newly registered PC which accomplishes the different Euro Standards can be obtained and represented *(Figure 3)*:

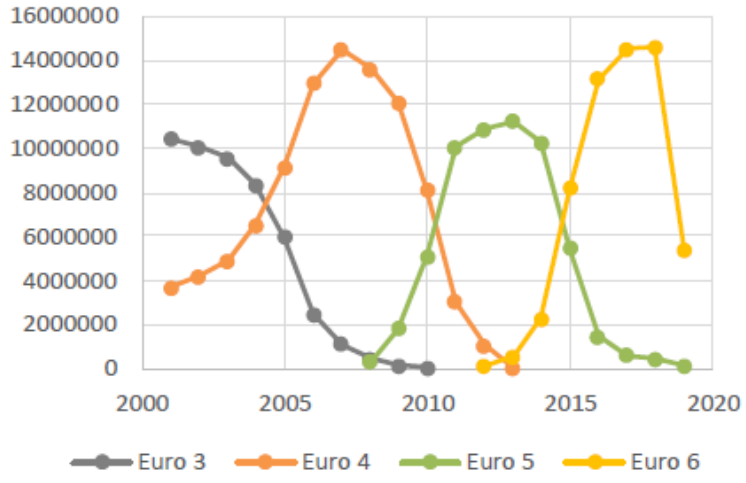

Figure 3. Euro Standard PC registrations curves
Source: own edition based on European vehicle market statistics pocketbook 2021–2022 and ACEA 2022

In this section, the approximation of the mathematical models to the curves will be presented. Only the approximation of the Logistic Model will be presented to avoid repetition. However, the process followed is analogous to the rest of the models. At first, it may not seem possible to approximate the mathematical model to one of these curves. However, with a couple of changes, this is straightforward. First, the curves will be divided into two at the midpoint. The first half of the bell resembles the Logistic Model, so the approximation is straightforward. The procedure for the second bell requires a reverse ordering of the data so that the curve is similar to Logistic Model. Once the approximation is made, the approximated data will be rearranged according to the initial pre-arrangement scheme to maintain the bell shape. Since the central value of the bell is used in both approximations, the mean of the approximations will be taken as the final value. *Figure 4* presents the Logistics Model's final results.

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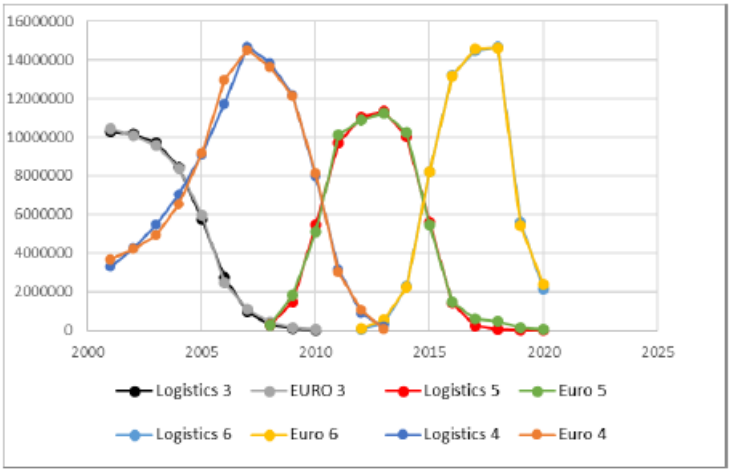

Figure 4. Logistics Model approximations
(Source: Own work)

This procedure is analogous to the Gompertz model (*Figure 5*):

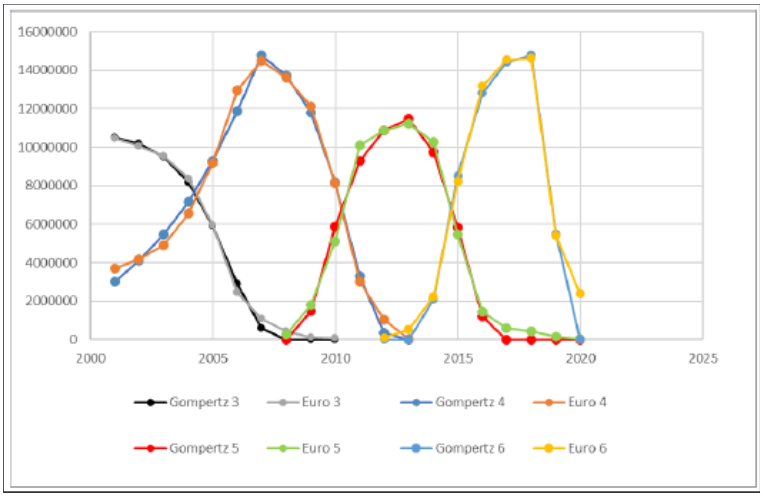

Figure 5. Gompertz Model approximations
(Source: Own work)

As for the Gaussian Model, since the curve is already bell-shaped, it will only require one approximation (*Figure 6*):

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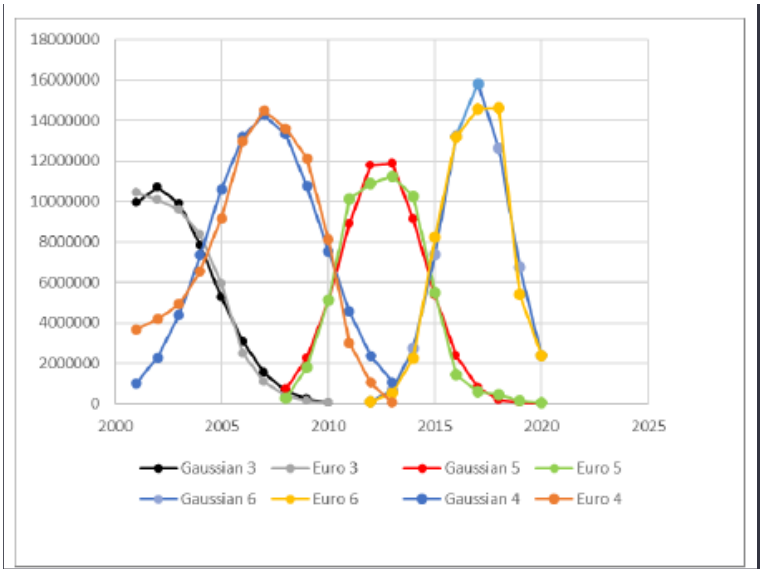

Figure 6. Gaussian Model approximations
(Source: Own work)

4. Analysis and discussion – prediction

There is no predetermined duration for the Euro Standards. Since the only reference information is the previous standards, the average duration of the past curves will be taken as the future duration for Euro Standard 7. This results in a total of 11 years. The average passenger car sales before the Covid-19 crisis were 14 000 000 vehicles. The future standard is expected to enter into force in 2025. By then, it is reasonable to assume that the vehicle market will have recovered from the past crisis, assuming there will be no future recessions. The prediction will assume that the future curve will be symmetric to simplify the analysis. By plotting the percentage of new vehicles registered for the different past Euro Standards (plotted in *Figure 2),* we obtain each standard's percentage curve of new passenger vehicle registrations.

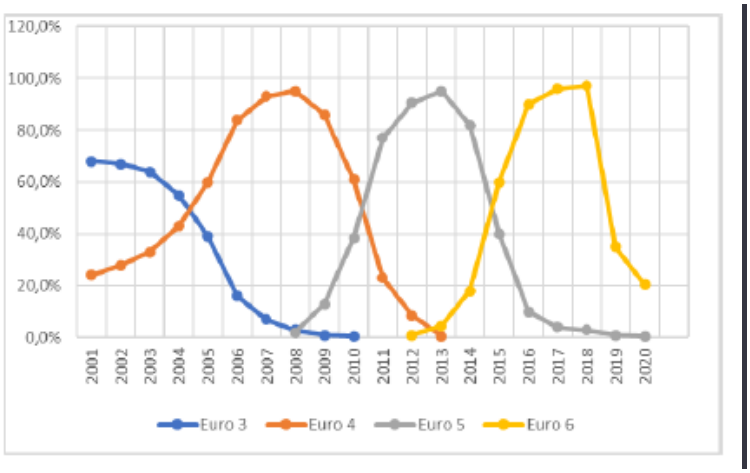

Figure 7. New PC registrations – estimation
(Source: own estimation based on European vehicle market statistics pocketbook 2021–2022)

4.1 The logistics model prediction

As explained above, the logistic model has three parameters. N_0, the number of individuals in the population at time t = 0 (130 000); K, the maximum value attainable by the population (12 000 000) and r, the growth parameter. The latter depends on the situation at the time. Since it is difficult to determine this parameter, the average of the past curves will

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be taken as the future growth coefficient for the Euro 7 curve. Finally, *Figure 8* presents the prediction of the logistic model.

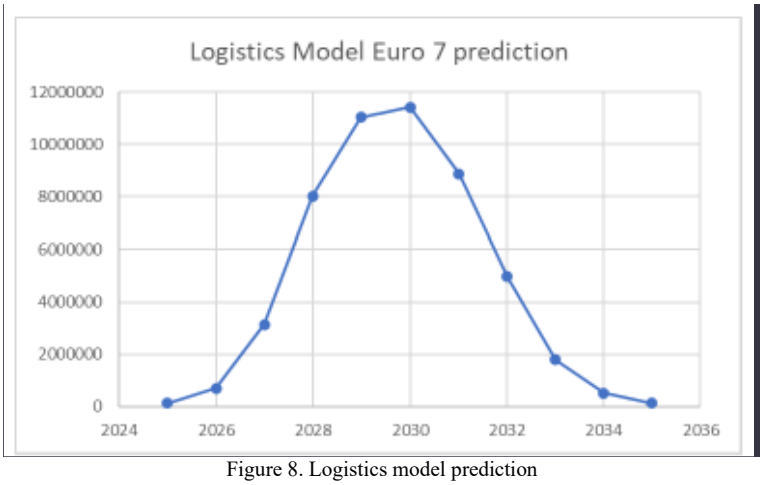

Figure 8. Logistics model prediction
(Source: own estimation)

4.2 The Gompertz model prediction

This model has three parameters. *a* represents the upper limit of the population (12 000 000). *c* represents the growth parameter, proceeding in the same way as with the Logistic Model. The remaining parameter is *b*, a parameter related to the initial growth. An iterative process has been followed to calculate this parameter. Knowing that the estimated initial value is 130 000, Figure 9 presents the prediction from this model.

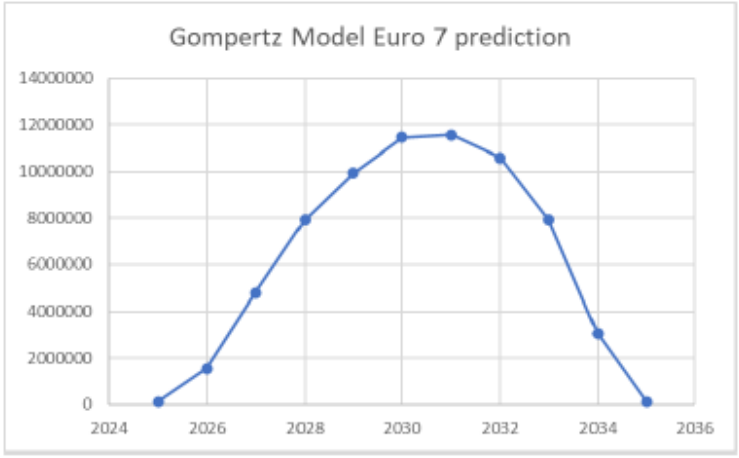

Figure 9. Gompertz model prediction
(Source: own estimation)

4.3 The Gaussian model prediction

In this model, the parameter *a* represents the maximum value of the bell, 12 000 000. The parameter *b* affects how the bell falls. The average of the previous curves has been taken, obtaining $b = 0.12$. The parameter *c* indicates when the maximum value is reached. Symmetry has been assumed in the future curve. According to the prediction, the future curve will last 11 years, with the medium $c = 6$.

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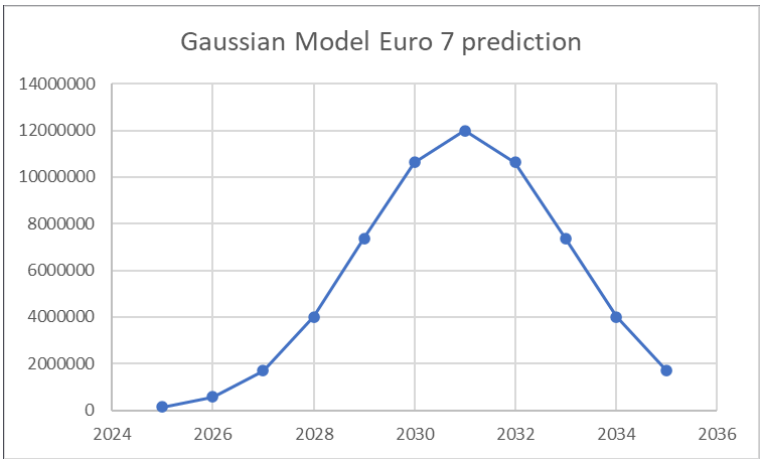

Figure 10. Gompertz model prediction
(Source: own estimation)

5. Conclusion

Table 1 presents the number of new passenger car registrations expected for the new Euro Standard 7 according to the predictions of the three mathematical models.

Table 1. Predictions from Logistics, Gompertz and Gaussian Model for Euro 7

Year	Logistics Model	Gompertz Model	Gaussian Distribution Model
2025	130 000	130 000	150 000
2026	700 000	1 560 000	570 000
2027	3 140 000	4 780 000	1 700 000
2028	8 020 000	7 920 000	4 000 000
2029	11 040 000	9 950 000	7 370 000
2030	11 425 000	11 445 000	10 620 000
2031	8 870 000	11 560 000	12 000 000
2032	4 970 000	10 590 000	10 620 000
2033	1 800 000	7 940 000	7 370 000
2034	510 000	3 060 000	4 000 000
2035	130 000	130 000	1 700 000

Source: Own work

As mentioned above, Euro Standard 7 is expected to come into force in 2025, and the estimated duration is eleven years, until 2035. The analysis has assumed symmetry in the curve, so the registered passenger vehicles should be similar, if not the same, in the start year and the end year. Furthermore, the maximum number of registrations should be reached in 2030. The Gaussian model reaches the maximum number of records in 2031. Moreover, the records in 2035 are much higher than expected. Its prediction has therefore been discarded.

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As for the Gompertz model, although its initial and final records coincide with those stipulated, the maximum is reached in 2031, and the progress of registrations is abrupt. Very different from previous curves, so it has also been discarded. Finally, the prediction taken as possible is presented by the Logistic Model. It meets all the above requirements and has a similar growth to the past curves. It should be marked that this analysis has involved several simplifications to facilitate the study. On the other hand, the future is uncertain, unexpected events may occur, and other factors may make this analysis far from reality.

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