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TWO TYPES OF RANDOM POLYHEDRAL SETS*

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Let $A = \{a_{ij}\}_{i=1,j=1}^{m\ \ n}$, $\alpha = (\alpha_1, \ , \alpha_m)$, be a matrix and a vector (with real coefficients). By a (closed) polyhedral set (PS) we mean the solution set of the system of linear inequalities

(1) $\qquad Ax \geqslant \alpha$

i.e.

(2) $\qquad PS = \{x \in R^n : Ax \geqslant \alpha\}$

If (1) has no solution then we get the "empty polyhedral set", i.e. the empty set is included into the family of PS. Denote by $\mathcal{P}_m$ this family (when m and n are fixed and a_{ij} and α_i run through the real numbers). So the elements of $\mathcal{P}_m$ are all PS of the for m (2) plus the empty set.

One speaks of random polyhedral set (RPS), when a_{ij} and α_i are not real numbers but "real-valued" random variables. We shall assume that they are defined on a common probability-measure space (Ω, Σ, P), i.e. a_{ij} and α_i are real-valued measurable functions defined on Ω.

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The first problem concerning RPS is how to define them?

The study of "random" geometrical objects (points, lines, hyperplanes, circular segments, e.t.c.) has a long history (see M. Kendall-Moran [1], Moran [2],[3]).

On the other hand, in the last few decades the most general random sets have been investigated (Matheron [4], D. Kendall [5], Santaló [6]).

As to RPS, sometimes they are defined as PS where the constants a_{ij} and α_i are randomly generated real numbers (Schmidt-Mattheiss [7]). Another approach is to consider RPS as an intersection of a finite number of random (closed) halfspaces (Rényi-Sulanke [8], Schmidt [9]). A "dual" approach to this is when (a bounded) RPS ("random polytope") is defined as the convex hull of a finite number of points given randomly in R^n.

A special case of Rényi-Sulanke approach is when the (normal vectors of) halfspaces are deterministic and only their translations are random, i.e. a_{ij} are real numbers and α_i are random variables.

In all above concepts of RPS both m an n are arbitrary (but fixed) integers. If we relax this condition, if say, n is fixed but m is also a random integer (i.e. discrete random variable), then we come to a much more complicated concept of RPS. Examples of these are so called Poisson polyhedra (see, e.g., [4] pp. 155-185).

Below we hall restrict ourselves to the study of two possible types of RPS.

The first type is an analogy of the random number: while the latter is a real-valued measurable function on a probability space, the RPS is a "PS-valued" measurable function.

In the second approach a RPS is the collection of those $x \in R^n$ that solve the system (1) with a probability at least p (where $0 \leq p \leq 1$ is a fixed number). The first type RPS stem for the general concept of a random set.

The second one was initiated by the people working in stochastic programming (more exactly "chance constrained" stochastic programming, see Kall [10] for a detailed survey).

There is no similarity between these two types. To study them, completely different techniques are needed and even the questions arising in conection with them are completely different.
We have concentrated to basic questions concerning these two types: in the first type the question of measurability of a "PS-valued" function and in the second one the question of convexity of RPS.

1. RPS of the first type

Let $a_{ij}, \alpha_i, i=1,\dots,m, j=1,\dots,n$, be Borel-measurable real functions on (Ω, Σ, P) and denote $A(\omega) = \{a_{ij}(\omega)\}_{i=1, j=1}^{m \ n}$, $\alpha(\omega) = (\alpha_1(\omega), \dots, \alpha_m(\omega))$, $\omega \in \Omega$. Further denote

$$(3) \qquad X(\omega) := \{x \in R^n : A(\omega)\, x \geq \alpha(\omega)\}, \ \omega \in \Omega .$$

After defining in $\mathcal{P}_m$ a special topology we endowe $\mathcal{P}_m$

with a Borel σ-algebra and prove that $\mathcal{X}$ is a measurable function w.r.t. this σ-algebra. This will mean that $\mathcal{X}$ is a RPS.

Denote by $\mathcal{K}$ the family of all polytopes from R^n (the empty set is included in $\mathcal{K}$). We recall that a polytope is a bounded PS, so $\mathcal{K}$ is a collection of all bounded PS, where m is arbitrary. Denote by $\mathcal{G}$ the family of all open PS (where m is arbitrary, empty set is included in $\mathcal{G}$). (An open PS is the set of the form $\{x \in R^n: \ Ax > \alpha\}$, m is again arbitrary.)

Denote

(4) $$\mathcal{P}_m^K := \{P \in \mathcal{P}_m : P \cap K = \phi\} \ , \ K \in \mathcal{K} ,$$

(5) $$\mathcal{P}_{mG} := \{P \in \mathcal{P}_m : P \cap G \neq \phi\} \ , \ G \in \mathcal{G} .$$

Clearly $\mathcal{P}_m^\phi = \mathcal{P}_m$ and $\mathcal{P}_{m\phi} = \phi$.

Take the family

(6) $$\mathcal{S} := \{\mathcal{P}_m^K : K \in \mathcal{K}\} \cup \{\mathcal{P}_{mG} : G \in \mathcal{G}\}$$

and consider it as a subbase of a topology in $\mathcal{P}_m$. (For some basic facts on topology we refer to Kelley [11].)

Denote

(7) $$\mathcal{M} := \{\bigcap_{i=1}^{r} S_i : S_i \in \mathcal{S} \ , \ i = 1, 2, \dots, r \ , \ r < +\infty\}$$

(this is the basis) and

(8) $$\mathcal{F} := \{\bigcup_{\gamma \in \Gamma} M_\gamma : M_\gamma \in \mathcal{M} \ , \ \gamma \in \Gamma, \ \Gamma \text{ arbitrary}\} .$$

The elements of $\mathcal{F}$ are called open sets in $\mathcal{P}_m$.
We have a topology in $\mathcal{P}_m$ (This is the topology generated by $\mathcal{S}$, or the "roughest" topology containing $\mathcal{S}$)
Now we have

Theorem 1.1. The function $X : \Omega \to \mathcal{P}_m$ defined by (3) is measurable w.r.t. the Borel σ-algebra defined in $\mathcal{P}_m$ by the topology $\mathcal{F}$. □
The truth of the theorem depends on the following lemma

Lemma 1.2. Let $N=(n+1).m$ and let $\varphi : R^N \to \mathcal{P}_m$ be the following map:

$$\varphi(y) := \{ x \in R^n : \sum_{j=1}^{n} y_{(i-1)(n+1)+j} \, x_j \geq y_{i(n+1)}, \; i=1,2,\dots,m \}, \; y \in R^N. \tag{9}$$

Then the φ is continuous in the topology $\mathcal{F}$. □

Proof. We have to prove that $\varphi^{-1}(\mathcal{P})$ is open in R^N for any $\mathcal{P} \in \mathcal{F}$, where $\varphi^{-1}(\mathcal{P}) \subset R^N$ means the set such that $\varphi(\varphi^{-1}(\mathcal{P})) = \mathcal{P}$, (the "inverse domain" of φ). The definition of $\mathcal{F}$ shows that it is enough to prove that $\varphi^{-1}(\mathcal{P}_m^K)$ and $\varphi^{-1}(\mathcal{P}_{mG})$ are open for any $K \in \mathcal{K}$ and any $G \in \mathcal{G}$.
Let $y \in \varphi^{-1}(\mathcal{P}_m^K)$ i.e.

$$\varphi(y) \in \mathcal{P}_m \quad \text{and} \quad \overline{\varphi(y)} \cap K = \phi . \tag{10}$$

$\overline{\varphi(y)}$ is a (closed) PS and K is a compact PS. Hence there is a hyperplane $L \subset R^N$ such that $\varphi(y)$ and K are in two different open halfspaces determined by the L. As K is compact, we can choose L such that it is not parallel to any face of $\varphi(y)$. Let H be the open halfspace containing $\varphi(y)$. Clearly there is a small neighbourhood $\tau(y)$ of y such that $\varphi(z)\in\mathcal{P}_m$ and $\varphi(z)\in H$ for all $z\in\tau(y)$(here we use the fact that L is not parallel to any face of $\varphi(y)$). But this means that $\varphi(z)\cap K=\phi\ \forall z\in\tau(y)$, that proves the openess of $\varphi^{-1}(\mathcal{P}_m^K)$.

Let $y\in\varphi^{-1}(\mathcal{P}_{mG})$, i.e.

(11) $\qquad \varphi(y)\in\mathcal{P}_m$ and $\varphi(y)\cap G\neq\phi$.

Denote by L_y the affine hull of $\varphi(y)$ (i.e. the translated linear subspace of smallest dimension containing $\varphi(y)$). Denote by $\varphi^o(y)$ the relative interior of $\varphi(y)$ (w.r.t. L_y).

The (11) implies

(12) $\qquad \varphi^o(y)\cap G\neq\phi$,

consequently there is an open ball $B\subset R^n$ such that

(13) $\qquad B\subset G,\quad B\cap L_y\subset\varphi^o(y)$

This shows that there is a neighbourhood $\tau(y)$ of y such that $\varphi(z)\in\mathcal{P}_m$ and

(14) $\qquad \varphi(z)\cap B\neq\phi\ \ \forall z\in\tau(y)$.

This proves the openess of $\varphi^{-1}(\mathcal{P}_{mG})$ and by this the lemma is proved. ■

Proof of Theorem 1.1:

Denote

(15) $a=(a_{11},a_{12},\ldots,a_{1n},\alpha_1,a_{21},a_{22},\ldots,a_{2n},\alpha_2,\ldots,a_{m1},\ldots a_{mn},\alpha_m)$

It is easy to see that the map

(16) $a:\Omega\to R^N \quad (N=(n+1)\cdot m)$

is Borel-measurable if all components of a are Borel-measurable. But

(17) $$X(\omega)=\varphi(a(\omega))$$

where φ is the map in the Lemma 2.2.
φ is measurable (being continuous) and the composition of two measurable maps is measurable. ■

2. RPS of the second type

The system of inequalities (1) where a_{ij} and α_i are random variables defined on (Ω,Σ,P) is called system of random inequalities (SRI).
Let $A(\omega)$, $\alpha(\omega)$ be as in the previous section and denote

(18) $$\Omega(x):=\{\omega\in\Omega:\ A(\omega)x\geqslant\alpha(\omega)\}\ .$$

We say that x solve SRI with the probability at least p $(0\leq p\leq 1)$ if

(19) $$P(\Omega(x))\geqslant p.$$

The set

(20) $$V(p) := \{x \in R^n;\ P(\Omega(x)) \geq p\}$$

might be called random polyhedral set of the second type (RPS of the second type).

If all random variables a_{ij}, α_i are discrete, then V(p) may be a PS (i.e. intersection of finite number of closed halfspaces), see, [10] p. 83.

In general we cannot expect that V(p) is a PS. A weaker condition is the convexity of V(p). In fact V(p) is called RPS (of second type) if it is a convex set. Below we shall prove some interesting sufficient conditions for this in the particular case when all a_{ij} are deterministic and only α_i are random variables.

So, let ξ be an m-dimensional random variable defined on the probability space (Ω, Σ, P) and we are asking the convexity of the set

(21) $$V(p) := \{x \in R^n:\ P\ (Ax \geq \xi) \geq p\}.$$

Let F_ξ be the distribution function of ξ, hence

(22) $$V(p) = \{x \in R^n : F_\xi\ (Ax) \geq p\}.$$

By definition a function $\varphi(x)$, $x \in R^k$, is called quasi-concave if all its upper level sets

(23) $$\{x \in R^k:\ \varphi(x) \geq u\}$$

are convex.

It is clear that if the rang of A is n (let us assume this) then the function $F_\xi(Ax)$ (as a function of $x \in R^n$) is quasi-concave if and only if the function $F_\xi(y)$ is quasi concave on the n-dimensional subspace $L := \{y \in R^m : y = Ax, x \in R^n\}$ of R^m. So we can formulate the following simple

Assertion 2.1. If the distribution function $F_\xi(\underline{y})$, $\underline{y} \in R^m$, of the m-dimensional random variable ξ is quasi-concave on the subspace $L := \{y \in R^m : y = Ax, x \in R^n\}$, i.e. if the sets

(24) $\quad \{ y \in L : F_\xi(y) \geqslant p \}$

are convex for all $0 \leq p \leq 1$, then the sets $V(p)$ are convex for all $0 \leq p \leq 1$ i.e. the SRI $Ax \geqslant \xi$ give a RPS of the second type. □

This assertion can be formulated using the probability measure

(25) $\quad \nu_\xi(E) := P(\xi \in E)$, $E \subset R^m$ Borel-measurable, so that

(26) $\quad F_\xi(y) = \nu_\xi(\xi \leq y)$.

In what follows we shall assume that $\nu_\xi(E)$ absolute continuous w.r.t. the Lebesque-measure in R^m, i.e. that $\nu_\xi(E)$ is generated by a density function:

(27) $\quad \nu_\xi(E) = \int_E f_\xi(t)\, dt$, $E \subset R^m$ Lebesque measurable

($\int \cdot\, dt$ means L-integral).

Now

$$(28) \qquad F_\xi(y) = \nu_\xi(\xi \leq y) = \int_{t \leq y} f_\xi(t)\,dt, \qquad y \in R^m.$$

In the rest of the paper we shall deal with the question: <u>What density functions $f_\xi(t)$ generate quasi-concave distributions $F_\xi(y)$?</u>

Our results are a little more restrictive than needed, because in fact we are looking for $f_\xi(t)$ such that $F_\xi(y)$ is quasi-concave on the subspace L only. The whole method below can be adapted to this case, yielding more general results.

It is convenient to express the quasi-concavity in another way.

One can see easily that the function $\varphi: R^k \to R^1_+$ is quasi-concave if and only if

$$(29) \qquad \varphi(\lambda x + (1-\lambda) y) \geq \min\{\varphi(x), \varphi(y)\} \quad \text{for all } x, y \in R^k \text{ and } 0 \leq \lambda \leq 1.$$

For $m=1$, any distribution function $F_\xi(y)$ (define now on R^1) is quasi-concave because it is monotone. For higher dimensions the situation is much more complicated.

A first idea in investigating the quasi concavity of $F_\xi(y)$ (for $m \geq 2$) can be formulated as follows: Is there a quasi-concave density function $f: R^m \to R^1_+$ such that its distribution function F is not quasi-concave? We think the answer is yes, however to find such density functions (even in the case $m=2$) seems to be not an easy task. Hence, the research went into the following direction: Find some well defined subfamilies of quasi-concave density functions f that generate quasi-concave distributions. This direction proved to be fruitful already.

To show the first steps, write analytically the basic question.

Does the condition

(30) $f(\lambda x+(1-\lambda)y) \geq \min\{f(x), f(y)\}$, $x,y \in R^m$, $0 \leq \lambda \leq 1$

implies

(31) $\int\limits_{t \leq \lambda u+(1-\lambda)v} f(t)dt \geq \min\{\int\limits_{x \leq u} f(x)dx, \int\limits_{x \leq v} f(x)dx\}$, $u,v \in R^m$, $0 \leq \lambda \leq 1$?

Let us try to derive (in spite of our doubts) (31) from (30) using a following "trick":

The condition (30) implies

(32) $f(t) \geq \sup\limits_{\lambda x+(1-\lambda)y=t} \min\{f(x), f(y)\}$, $t \in R^m$,

hence

(33) $\int\limits_{t \leq \lambda u+(1-\lambda)v} f(t)dt \geq \int\limits^{*}_{t \leq \lambda u+(1-\lambda)v} \sup\limits_{\lambda x+(1-\lambda)y=t} \min\{f(x), f(y)\}\, dt$,

where $\int^{*}$ is the upper Lebesque-integral, i.e.

$$\int\limits^{*}_{R^m} \psi(t)dt := \inf\{\int\limits_{R^m} \omega(t)dt : \omega(t) \geq \psi(t), t \in R^m, \omega \text{ L-integrable}\}.$$

The upper integral is needed here, because the right hand side of (32) is in general not L-measurable.

If the right hand side of (33) were not less than that of (31) then we were ready.
It is hopeless to prove this (in fact it is in general not true), we try to take on f a more restrictive condition than (30). For this define the following concept: Let a,b≥0, $-\infty < \alpha < +\infty, \alpha \neq 0, 0 \leq \lambda \leq 1$ and denote

(34) $$M_\alpha^{(\lambda)}(a,b) = \begin{cases} 0 & \text{if } a \cdot b = 0 \\ (\lambda a^\alpha + (1-\lambda) b^\alpha)^{1/\alpha} & \text{if } a \cdot b > 0 \end{cases}$$

("the extended α-means").
For a,b fixed, $M_\alpha^{(\lambda)}(a,b)$ is a non-decreasing function of α (see, e.g. Hardy-Littlewood-Pólya, [12]). For $\alpha = -\infty, \alpha = +\infty$ or $\alpha = 0$ we define the means by taking limits to get

(35) $$\begin{aligned} M_{-\infty}^{(\lambda)}(a,b) &= \min\{a,b\} \\ M_0^{(\lambda)}(a,b) &= a^\lambda b^{(1-\lambda)} \\ M_{+\infty}^{(\lambda)}(a,b) &= \begin{cases} 0 & \text{if } a \cdot b = 0 \\ \max\{a,b\} & \text{if } a \cdot b > 0. \end{cases} \end{aligned}$$

We call the function $\varphi : R^k \to R_+^1$ α-concave, $-\infty \leq \alpha \leq +\infty$, if

(36) $$\varphi(\lambda x + (1-\lambda) y) \geq M_\alpha^{(\lambda)}(\varphi(x), \varphi(y)) \quad \forall x,y \in R^k, \ 0 \leq \lambda \leq 1.$$

The $-\infty$-concave functions are quasi-concave, so the monotony of $M_\alpha^{(\lambda)}$ shows that any α-concave function

is quasi-concave. (The 0-concave function are sometimes called logaithmically concave, log-concave).
Now we have

Theorem 2.1. Let $\alpha \geqslant -1/m$.
The distribution function of an α-concave density function $f: R^m \to R^1_+$ is $\alpha\cdot(1+m\alpha)^{-1}$-concave, consequently quasi-concave. $\square$

This theorem is a simple consequence of the following integral inequality

Theorem 2.2. Let $f,g : R^m \to R^1_+$ be Lebesque-measurable functions and denote

(37) $$h_\alpha^{\lambda}(t) := \operatorname*{ess\,sup}_{x\in R^m} M_\alpha^{\lambda}(f(x/\lambda), g((t-x)/(1-\lambda))), \quad t\in R^m.$$

If $\alpha \geqslant -1/m$ then

(38) $$\int_{R^m} h_\alpha^{(\lambda)}(t)\,dt \geqslant M_\beta^{(\lambda)}\left(\int_{R^m} f(x)\,dx, \int_{R^m} g(x)\,dx\right),$$

where $\beta = \frac{\alpha}{1+m\alpha}$. $\square$

Proof of Theorem 2.1. Denote

(40) $$\varphi(x) = \chi_A(x)\cdot f(x), \quad \psi(x) = \chi_B(x)\cdot f(x),$$

where $A = \{t \in R^m : t \leq u\}$, $B = \{t \in R^m : t \leq v\}$.
and χ_A, χ_B are the characteristic functions.
We can write

$$(41) \qquad \int\limits_{t \le \lambda u + (1-\lambda)v} f(t)\, dt = \int\limits_{\lambda A + (1-\lambda)B} f(t)dt$$

(where $\lambda A + (1-\lambda)B$ is the algebraic sum of the sets).
The α-concavity of f implies

$$(42) \qquad f(t) \geq \sup_{\lambda x + (1-\lambda)y = t} M_{\alpha}^{(\lambda)}(\varphi(x), \psi(\ )) \qquad t \in R^m.$$

It is clear that the right hand side of (42) is zero if $t \notin \lambda A + (1-\lambda)B$, hence (after writing the right hand side of (42) in the form (37))

$$(43) \quad \int\limits_{\lambda A + (1-\lambda)B} f(t)dt \geq \int\limits_{R^m} \operatorname*{ess\,sup}_{x \in R^m} M_{\alpha}^{(\lambda)}(\varphi(x/\lambda), \psi(t-x)/(1-\lambda))dt\,.$$

Applying the integral inequality (38) we get the result. ■

For the proof of Theorem 2.2, its sharpenings extensions applications and many other similar results together with a whole history of the integral inequalities of this type, we refer to the papers [15] ÷ [22]
We only note here that for $\alpha > 0$ the inequality (38)$(m=1)$ "almost" coincides with a classical result of Henstock-Macbeath [13] that was extended to higher dimensions by Dinghas [14]. Their paper was almost forgotten and some spacial cases of their inequalities were newly rediscovered nearly 20 years later.

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Summary

An $m \times n$-matrix $A = \{a_{ij}\}$ and a vector $\alpha = (\alpha_i) \in R^m$ define a polyhedral set $PS := \{x \in R^n : Ax \geq \alpha\}$. One speaks of random polyhedral set (RPS) when a_{ij} and α_i are not real number but random variables. In the literature at least three different types of RPS are defined. The paper presents two of them. The first is when RPS is simply a "PS valued" random variable. It is proved in the paper that a topology in the space of PS-s can be defined so that the "PS-valued" function is continuous, consequently measurable. The second type of RPS discussed in the paper comes when only α is a random (m-dimensional) variable. Here the problem is the convexity of the set $V(p) := \{x \in R^n : P(Ax \geq \alpha) \geq p\}$ for all $0 \leq p \leq 1$, when P is the measure related to the α. It is showed that $V(p)$ is convex for many measures P generated by density functions having some well defined concavity-like properties.

VÉLETLEN POLIHEDRIKUS HALMAZOK KÉT TIPUSÁRÓL

Uhrin Béla

Összefoglaló

Egy A $m\times n$-es mátrix és egy $\alpha\in R^m$ m-dimenziós vektor egy $PS := \{x\in R^n : Ax\geq\alpha\}$ polihedrikus halmazt definiál. Véletlen polihedrikus halmazról /RPS/ akkor beszélünk, amikor az A és α elemei valószinüségi változók. Az irodalomban legalább három különböző típusu RPS van. A cikk ezek közül kettőt tárgyal. Az első típus egyszerüen egy "PS-értékü" valószinüségi változó. A cikkben be van bizonyitva, hogy a PS-ek "terében" vett alkalmas topológiában egy "PS-értékü" leképezés folytonos, tehát mérhető, azaz egy valószinüségi változó. A másik tipus, amalyről a cikkben szó van, akkor fordul elő, amikor csak az α véletlen, de az A nem. Itt a fő probléma a $V(p) := \{x\in R^n : P(Ax \geq \alpha) \geq p\}$ halmaz konvexitása, $0 \leq p \leq 1$-re, ahol P az α-hoz tartozó mérték.
A szerző megmutatja, hogy a $V(p)$ konvex, ha a P-t egy bizonyos konkávitás-szerü tulajdonsággal rendelkező sürüségfüggvény generálja.