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SHORT-TERM PRODUCTION AND DISTRIBUTION PLANNING OF STOCKPILING-DISTRIBUTION SUBSYSTEMS OF CRUDE OIL

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ABSTRACT

Stockpiling-distribution subsystems of crude oil products consist of a tankpark, connected together with a crude oil refinery. The system is in bilateral connection with other refineries, tank parks, consumer's terminals. The expected deliveries are forecast on the basis of the medium-term plan of the regional multirefinery system. An optimum short-term planning model of the technological operation to be performed in the subsystem is elaborated, considering the real technological restrictions. The simplified model of the subsystem is a large-size linear programming model.

INTRODUCTION

One of the prominent subsystems of a regional crude oil processing and oil-product stockpiling and distribution system is the high capacity tank park for the storage of crude oils, various intermediate feeds and products connected most frequently with oil refinery. Pipeline network and pumps within the tank park ensure the conveyance of the various materials to the technological units and between the tanks. Hereinafter the stockpiling-distribution system refers to the totality of the tank park and technological units of the refinery.

The system is in bilateral connection with the "outside world", i.e. with other refineries, tank parks, consumers. Crude oil, intermediate feeds (components), products may be transported from the outside into the system, or materials within the listed groups are transported out of the system. Transporation may take place by pipeline, railway, truck, barge.

The technological purpose of the system is outlined as follows:

- to ensure storage of the incoming crude oil, feeds, components and products
- to ensure shipment of the required feeds, components and products;
- to ensure storage of the feeds, components and products (buffer storage in conformity with the seasonal fluctuation of consumption);
- to ensure production of the specified part of the required products by blending from the components;
- to ensure production of the specified part of the components necessary for blending of the products; i.e. it is necessary to specify the quantity of the crude oil to be processed and the operating conditions of processing.

If the stockpiling-distribution system is regarded as the subsystem of a larger, regional system including several oil refineries and tank parks, then it is a metter of course that the proper knowledge of the productive capacity of each subsystem is highly significant under the given circumstances in respect of the optimal production planning of the total system. The production scheduling model to be presented was made with this purpose and as it will be shown at a later stage, the initial data were supplied by the medium term production planning of the regional system.

ASPECTS OF MODELLING

Given is the annual, or quarterly production plan of the regional, high level system. Naturally this includes the task of the individual refineries, tank parks (stockpiling-distribution systems) for the plan period in question. Task of the stockpiling-distribution system's production scheduling model is the following:

- with regard to the restrictions built into the model, to prepare the production program for the shorter periods in a way that the solution should meet the specifications of the production plan (search for the possible solution);
- to ensure the optimal functioning of the system according to given technical-economic objective function (search for the optimal solution);
- to take into consideration the stochastic character of the in- and out-bound deliveries at the specified materials of large volume and at the means of transporation. In present description only a very simple mode of depicting the random effects is being dealt with.

Essential requirement in connection with the model is independence from the concrete technological structure. The same model should be suitable for analysis of the various concrete stockpiling-distribution systems, furthermore for the analysis of the effect of the technological structure variation (e.g. analysing the effect of new investments or operating troubles).

In the following one of the possible, relatively simple varieties of the modelling and production scheduling of the stockpiling-distribution system including the technological units and tank park will be described. In the interest of reducing the dimensions of the task, the production scheduling is restricted to products of large volume (gasolines, motor fuels, fuel oils) and to their components, as well as to the crude oil utilization. Let us assume furthermore that the medium term production plan related to the system is known, i.e. the total quantity of the materials delivered into and transported out of the system is given for a fixed time interval.

DETERMINISTIC MODEL OF THE STOCKPILING-DISTRIBUTION SYSTEM

Let us regard the general stockpiling-distribution subsystem as a graph, the vertices of which are the tanks, while the edges are the pipelines connecting the tanks. For the purpose of general applicability, let us assume that every tank is connectible with every other one in two directions (technologically irrealistic connections are banned), material may arrive into every tak from outside of the system, and material maybe carried from every tank to the outside of the system. Content of the tanks is characterized with the most important quality parameters and every stocking is understood in term of blending, as a result of which "new" material will be stored is the tank.

The technological units are built in between specific tanks, tank groups, their function is regarded as separation or as specified alteration of the quality parameters. The material balance should be fulfilled for the separation-type processes, the lower-upper restrictions for the quantity of components derived in the process of separation, as well as the pertinent quality values are calculated in advance with the aid of the mathematical model of the technological unit. The given quality restrictions should be fulfilled during the process of blending.

Let us introduce the following notations:

$v_{ik}(t)$ - flow velocity of the material from tank-i to tank-k; $i,\ k=1,2,\dots,N;\ i\neq k;\ t\in[0,T];$

$v_i(t)$ - velocity of material flowing into tank-i from outside; $i=1,2,\dots,N;\ \ t\in[0,T];$

$w_i(t)$ - velocity of material carried from tank-i to outside; $i=1,2,\dots,N;\ \ t\in[0,T];$

V_i - capacity of tank-i; $i=1,2,\dots,N;$

$x_i(t)$ - quantity of material in tank-i; $i=1,2,\dots,N;$ $t\in[0,T];$

$\varphi_{ik}(t)$, $\psi_{ik}(t)$ - lower and upper restriction of $v_{ik}(t)$; $i, k=1,2,\ldots,N$; $i\neq k$; $t\in[0,T]$

ω_{ir}, η_{ir} - lower and upper restriction of the separation from tank-i; $i=1,\ldots,N$; $r=1,2,\ldots,R$;

$m_{ij}(t)$ - j-th quality index of the material in the tank-i $i=1,2,\ldots,N$; $j=1,2,\ldots,M$; $t\in[0,T]$;

$M_{ij}(t)$ - j-th quality index of the material delivered into tank-i from the outside; $i=1,2,\ldots,N$; $j=1,2,\ldots,M$; $t\in[0,T]$;

$\underline{m}_{ij}$, $\overline{m}_{ij}$ - lower and upper restriction of the j-th quality index of the material in tank-i; $i=1,2,\ldots,N$; $j=1,2,\ldots,M$;

$[0,T]$ - examined time interval

N - number of tanks

M - number of quality parameters

R - index occurring at separation (for interpretation, see later)

I, K_r - index sets occurring at separation (for interpretation, see later); $r=1,2,\ldots,R$.

In the course of setting up the model the trivial conditions are not detailed. Conditions of the model may be divided into two groups. The first one is a system of ordinary differential equations:

$$\frac{dx_i}{dt}(t) + \sum_{\substack{p=1\\ p\neq i}}^{N} [v_{ip}(t) - v_i(t) + w_i(t)] = 0, \qquad (1a)$$

$$i=1,2,\ldots,N,$$

$$\frac{d(m_{ij}(t)\ x_i(t))}{dt} + \sum_{\substack{p=1\\p\neq i}}^{N} [m_{ij}(t)\ v_{ip}(t) - m_{pj}(t) v_{pi}(t)] +$$

$$+ m_{ij}(t) w_i(t) - M_{ij}(t) v_i(t) = 0, \qquad (1b)$$

$$i=1,2,\dots,N; \quad j=1,2,\dots,M.$$

The second group, includes the restrictions of the functions occurring in the differential equation system:

$$0 \leq x_i(t) \leq V_i, \qquad (2a)$$

$$i=1,2,\dots,N;$$

$$\varphi_{ik}(t) \leq v_{ik}(t) \leq \psi_{ik}(t), \qquad (2b)$$

$$k=1,2,\dots,N, \quad i\neq k,$$

$$\underline{m}_{ij} \leq m_{ij}(t) \leq \bar{m}_{ij}, \qquad (2c)$$

$$i=1,2,\dots,N; \quad j=1,2,\dots,M;$$

$$\omega_{ir} \leq \frac{\sum_{k\in K_r} v_{ik}(t)}{\sum_{\substack{p=1\\p\neq i}}^{N} v_{ip}(t)} \leq \eta_{ir} \qquad (2d)$$

$$i\in I\subset\{1,2,\dots,N\}, \quad r=1,2,\dots R.$$

Brief interpretation of the conditions is given, as follows:

(1a) - condition of conservation of matter (differential material balance)

(1b) - in case of assuming the blending according to linear relationship

(2a) - condition related to capacity of the tank

(2b) - restrictions of the flow velocities, these are generally the functions of the tank pipeline system and pumps

(2c) - following the blending process in the tanks, the lower, upper restrictions of the quality parameters of the product in the tank

(2d) - condition related to the separation type processes, where I represents those tanks, from which the separation takes place, $K_r, r=1,2,\ldots,R$ index set refers to the fact that after the separation type process the same product gets into the tanks pertaining to K_r. In case of given concrete system I and K_r are allocated in advance.

The outlined model is supplemented with certain special conditions necessary for the description of the separation type processes. Noteworthy is the following condition:

$$m_{pj}(t) = m_j(\omega_{pr}) + \frac{m_j(\eta_{pr}) - m_j(\omega_{pr})}{\eta_{pr} - \omega_{pr}} \cdot \left[\frac{\sum\limits_{k \in K_r} v_{pk}(t)}{\sum\limits_{\substack{k=1 \\ k \neq p}}^{N} v_{pk}(t)} - \omega_{pr} \right], \quad \begin{array}{l} p \in I, \\ j=1,2,\ldots,M, \\ r=1,2,\ldots,R \end{array} \qquad (3)$$

Here $m_j(\omega_{pr})$ and $m_j(\eta_{pr})$ are given constants. On the basis of the condition it is apparent that the separation type processes in the model are not characterized with discrete operation conditions, but the technological parameters influencing the operation are continuously variable between the physically determined lower and upper restrictions. Condition (3) expresses that the quality properties of the fractions derived in the process of separation are determined with linear interpolation based on the actual value of the quotient in relationship (2). The quality properties corresponding to the extreme values of the quotient are calculated in advance with the aid of the mathematical model of the technological unit.

In connection with condition (3) let us mention again that combination of the quality properties are expressed with linear approximating relationships. Naturally at certain concrete properties (e.g. flash point, viscosity) the special linearized relationships known from the literature are built into the model.

The *first* problem concerning the model, search of the *possible solution* can be outlined in the following way:

Given $x_i(t)$, $v_i(t)$, $w_i(t)$ and $m_{ij}(t)$ and the knowledge of the other parameters and functions figuring in the conditions, find non-negative functions $v_{ik}(t)$ fulfilling the conditions (1), (2), (3) in the interval $(t_1, t_2) \in [0, T]$.

The *second* problem is the following: such solution of the previous problem is to be found in the interval (t_1, t_2), which is optimal with respect to a linear objective function.

In the concluding section of the paper we shall return to the possible solution methods of the outlined two problems, i.e. the approximative solutions of the problems used by us will be briefly outlined. In the following part those more important random effects will be reviewed the consideration of which is advisable in the modelling process of the stockpilling-distribution system and the methodics applicable under our concrete circumstances will be described.

RANDOM EFFECTS IN THE STOCKPILING-DISTRIBUTION SYSTEM

The external random effects mentioned in the Introduction appear when the precise values of $v_i(t)$ and/or $w_i(t)$ are not known (at least for certain i-s), but they are random. By this the following is understood.

The in- or out-bound transportation of the materials (hereinafter movement) takes place in well separable charges, intermittently, while both the quantity of the material in motion and the length of time elapsed between completion of the previous movement and commencement of the next movement are random (random variables).

In first approximation let us assume that the random quantity of the material is normally distributed, while the random length of time is of exponential distribution.

With regard to those mentioned above, every single random $v_i(t)$ and/or $w_i(t)$ are described with the following type of "process":

Take $\xi_1, \xi_2, \ldots, \xi_m$ and $\eta_1, \eta_2, \ldots, \eta_m$ as the two series of random variables, where ξ_k is the random variable of the length of time in which movement k begins (calculated from completion of movement $k-1$), while η_k is the random variable of the total quantity of the material moving in movement k (charge). Thus, if t_{k-1} represents the moment of the completion of movement $k-1$, then $P(\tau_1 \leqq \xi_k < \tau_2)$ is the probability that movement k has not started until the moment $(t_{k-1} + \tau_1)$, but it starts off before moment $(t_{k-1} + \tau_2)$. Similarly $P(q_1 \leqq \eta_k < q_2)$ represents the probability that the total quantity of the material moving in movement k (charge) is between q_1 and q_2.

As noted above, in first approximation let us assume that ξ_k is of exponential distribution with expection and variance $1/\lambda_k$ $(\lambda_k > 0)$, i.e. its density function:

$$f_k(x) = \begin{cases} \lambda_k \cdot e^{-\lambda_k x} & x \geq 0, \\ 0 & x < 0, \end{cases} \qquad (4)$$

$$k=1,2,\ldots,m,$$

Furthermore let us assume in first approximation that η_k is of normal distribution with expectation p_k and variance δ_k, the density function of which is:

$$g_k(x) = \frac{1}{\sqrt{2\pi}\ \delta_k} e^{-\frac{(x-p_k)^2}{2\delta_k^2}}, \qquad (5)$$

$$x \in R^1, \quad k=1,2,\ldots,m$$

On the basis of (4) and (5), $P(\tau_1 \leqq \xi_k < \tau_2)$, and $P(q_1 \leqq \eta_k < q_2)$ can be easily determined. ξ_k and η_k are generally not independent, their relationship should be examined in every practical case.

In the examined concrete system the probability that the movement begins depends not only on the time elapsed since the previous movement (this resulted in the exponential distribution), but also on the quantity of material still to be moved from the total quantity fixed by the production plan (of the material to be brought into motion in the whole [0,T] interval).

This effect was considered in first approximation by assuming that λ_k will depend also on the quantity of material still to be brought into motion. More precisely this means, that

$$\lambda_k \cong \Phi_k\left(X-D \cdot \sum_{i=1}^{k-1} p_k\right), \quad k=1,2,\ldots,m \qquad (6)$$

(where D and X are constants).

Selection of the functions Φ_k depends on the practical case. Generally it can be stated that the less material quantity was moving during the previous $k-1$ movements, the time period, the movement k will start within, will be probably shorter, i.e. the higher is the argument of Φ_k, the lower is the expectation $1/\lambda_k$. Thus it is advisable to select a simple monotonously increasing function for the function Φ_k.

The fact that exactly quantity X will move for sure during the $[0,T]$ time, is expressed as

$$\sum_{k=1}^{m} p_k = X \tag{7}$$

This refers also to the relationships among the λ_k-s.

Those described so far, give only the first approximation of the random effects occurring in our case.

This can be refined by use of other distributions expressing the reality better than (4) and (5) (e.g. "truncated" exponential, or β-distribution), using in place of (6) and (7) more precise mathematical description of the relationships among ξ_k-s and η_k-s, etc.

However, the most accurate description could be given by the thorough statistical analysis of the random effects, which has to be performed in every concrete case. The "right hand sides" of (1a), (1b) are obtained as a result of such analyses, and solution of the model is tackled only afterwards. This analysis is practically not possible within the technological system, but our model is functionally connectible with a simulation model, which can be used for the simulation of the random transportation into- and out of the system. This question here has not been dealt with.

The stochastic character of $v_i(t)$ and $w_i(t)$ entails not only the mentioned difficulties (namely the precise description of such random effects automatically causes great problems, as it was demonstrated previously), but it extremely aggravates the mathematical discussion and concrete solution of the model. This problem will be dealt with in the next section.

MATHEMATICAL AND COMPUTATIONAL REMARKS

Replacing the differential quotients in (1a) and (1b) by difference ones, we can express $w_i(t+\Delta t)$ and $m_{ij}(t+\Delta t)$ in terms of $v_{ip}(t)\cdot\Delta t$ ... etc. and $m_{ij}(t)\cdot v_{ip}(t)\cdot\Delta t$ ... etc. Taking into account that (2c) holds also for $m_{ij}(t+\Delta t)$, (1b) and (2c) can be approximately replaced by the following two inequalities:

$$
\begin{aligned}
&[m_{ij}(t)-\underline{m}_{ij}]+[M_{ij}(t)-\underline{m}_{ij}]\cdot v_i(t)\cdot\Delta t-\\
&\quad -[m_{ij}(t)-\underline{m}_{ij}]\cdot w_i(t)\cdot\Delta t\ +\\
&\quad + \sum_{\substack{p=1\\ p\neq i}}^{N}[m_{pj}(t)-\underline{m}_{ij}]\cdot v_{pi}(t)\cdot\Delta t - \sum_{\substack{p=1\\ p\neq i}}^{N}[m_{ij}(t)-\underline{m}_{ij}]v_{ip}(t)\cdot\Delta t \geq 0,
\end{aligned}
\tag{8}
$$

and an analogous inequality with $\bar{m}_{ij}$.

Similarly, $x_i(t+\Delta t)$ has to fulfil (2a), hence (1a) and (2a) can also be replaced by two inequalities:

$$
\begin{aligned}
&0 \leqq x_i(t) + v_i(t)\cdot\Delta t - w_i(t)\Delta t\ +\\
&\quad + \sum_{\substack{p=1\\ p\neq i}}^{N}[v_{pi}(t)-v_{ip}(t)]\cdot\Delta t \leqq V_i.
\end{aligned}
\tag{9}
$$

Another possibility to eliminate the differential equations (1a), (1b) from the model (naturally only approximately), is to use (8), thus eliminating (1b), and after that replace the function $x_i(t)$ in (8) by an integral computed from (1a). (Naturally $x_i(t)$ too in (2a) has to be replaced by the integral.) In this case we obtain a system of linear inequalities where unknowns are the functions $v_{ik}(t)$ and their integrals

$$\int_{t_1}^{t} v_{ik}(\tau)\, d\tau .$$

This method is useful especially in the case when the interval (t_1, t_2) over which the model has to be solved, is small.

For solution of the system of linear inequalities many effective computing procedures and computer programmes are available. The programs usually give the so-called basic solution, where many $v_{ik}(t)$ are at zero level, which is quite reasonable from practical point of view.

The mentioned transformation of the model to a (dynamic) system of linear inequalities is also suitable, because the optimization turns now to a series of usual linear programming model(s). Here also many powerful computing packages exist. Use of the LP-technique has also other advantages, e.g. investigations of the sensitivity of the model, interpretation of the duality of LP, i.e. shadow prices, etc.

Also the random effects are more easily handled when we write the model in "linear inequality system" -form. In this case we are dealing with a system of "random linear inequalities". These systems are investigated in detail in stochastic programming (especially in the so-called "chance constrained programming"). The situation is now complicated by the fact that the problem is not a statistical but a dynamic one (i.e. it depends on time t). Hence the random effects are expressed as a t-parameter family of random variables and can be regarded as a general stochastic process.

It is necessary to note that treating of a system of random linear inequalities is after all an easier matter than to investigate a system of random differential equations.

ÖSSZEFOGLALÁS

KŐOLAJTERMÉKEK TERMELÉSI-FORGALMAZÁSI ALRENDSZERÉNEK RÖVID TÁVU TERMELÉS- ÉS ELOSZTÁS-TERVEZÉSÉNEK MODELLJE

Inzelt Péter - Uhrin Béla

Termelési-forgalmazási alrendszeren egy tank park valamint kőolaj-finomitók együttesét értjük. Az alrendszer kapcsolódik egyéb külső finomitókhoz, tank parkokhoz és fogyasztókhoz. Az alrendszerből történő ki- és be-szállitások várható összértékét ismertnek tekintjük. A modell a tank parkon belüli anyag-áramlást irja le valós technológiai korlátok figyelembevételével.

A modell célja az anyagáramlás optimális időbeli megadása, hogy a mindenkori ki- ill. be-szállitások teljesithetők ill. fogadhatók legyenek. A cikk a modell egyszerüsitését valamint bizonyos /külső/ véletlen effektusokat is tárgyalja.

МОДЕЛИРОВАНИЕ КРАТКОСРОЧНОГО ПЛАНИРОВАНИЯ ПРОДУКЦИИ И РАСПРЕДЕЛЕНИЯ В НЕФТЯНЫХ ПРОДУКЦИОННЫХ И СОХРАНЯЮЩИХ ПОДСИСТЕМАХ

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Цель модели, рассматриваемой в статье, заключается в планировании оптимального потока материалов между цистернами подсистемы, для выполнения данного годового плана. В статье разработаны методы упрощения модели /чтобы их можно было решить на ЭВМ/, а также влияние случайных эффектов.